

Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21102367**

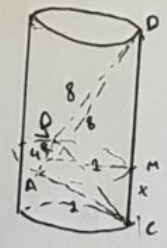
ID профиля: **378514**

Вариант 24

$40^2 - 64 < -4$
 $40^2 - 64 < -4$

Исходник

Логовина И.И.



Заметим, что любая образующая параллельна оси цилиндра. Если $C'D \perp C'D$ лежит на доп. поверхности, т.е. она лежит на образующей. При данном CP , т.е. высоте $|BA|; |PC| \Rightarrow R_{min}$ при найденном $CP \Rightarrow C'D \perp C'D$ лежит на основании цилиндра

$\perp PA \perp DC \Rightarrow$ они принадлежат плоскости сечения // основанию

$AP = \sqrt{64 - 2^2} = \sqrt{60}$

$PC = \sqrt{49 - 4} = \sqrt{45} \quad AM = \sqrt{49 - 2^2}$

~~$\perp C'D \perp BC = \sqrt{60^2 - 2^2}$~~

$\perp MC = x \Rightarrow PM = \sqrt{45 - 2^2}$

~~$PA^2 = \{60 - PM^2 = 45 - 2^2 \Rightarrow PA = \sqrt{15 + x^2}$~~

~~$\cos(\angle PRC) = \frac{60 + 45 - (\sqrt{15 + x^2} + x)^2}{2 \cdot 60 \cdot 45}$~~

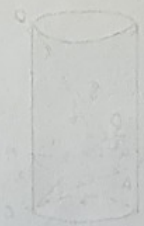
$PM^2 = PA^2 + 64 \Rightarrow 15 + x^2 = 49 - 2^2 + 64 \Rightarrow 2x^2 = 49 \Rightarrow x^2 = 49 \Rightarrow x = 7\sqrt{1}$

$PC = 2\sqrt{11} + \sqrt{15 + 49} = 2\sqrt{11} + \sqrt{64}$

Ответ: $PC = 2\sqrt{11} + 8$

$$\begin{aligned}
 &< 64 \\
 &+ 40d^2 + 64 < 0 \\
 &40d^2 - 64 \\
 &40d^2 - 64 < -4
 \end{aligned}$$

$$\begin{aligned}
 &t \in [-4; 40d^2 - 64] \\
 &t \in [-4; 60 - 40d^2] \\
 &C = 1 = 0
 \end{aligned}$$



The text on this page is extremely faint and mostly illegible. It appears to be a series of lines of text, possibly describing a mathematical derivation or a problem statement. Some words like "Determine" and "find" are faintly visible.

$$a_1^2 + 21da_1 + 108d^2 < 60 + 9a_1 + 36d$$

$$\frac{68}{28} \frac{1}{12} \quad \frac{441}{432} \quad \frac{117}{19}$$

$S =$ *problem.*

$$= \frac{2a_1 + 8d}{2} \cdot 9 = 9(a_1 + 4d) = 9a_1 + 36d$$

$$\frac{17}{8}$$

$$\begin{cases} a_5 - a_1 > S - 4 \\ a_{10} - a_1 < S + 60 \end{cases} \Leftrightarrow \begin{cases} (a_1 + 4d)(a_1 + 12d) > 9a_1 + 36d - 4 \\ (a_1 + 9d)(a_1 + 12d) < 9a_1 + 36d + 60 \end{cases}$$

$$\Leftrightarrow \begin{cases} a_1^2 + 21ad + 68d^2 - 9a_1 - 36d + 4 > 0 \\ a_1^2 + 21ad + 108d^2 - 9a_1 - 36d - 60 < 0 \end{cases} \Leftrightarrow \begin{cases} a_1^2 + (21d-9)a_1 + 68d^2 - 36d + 4 > 0 \\ a_1^2 + (21d-9)a_1 + 108d^2 - 36d - 60 < 0 \end{cases}$$

$P_1 =$

$$68d^2 - 36d + 4 = 4(17d^2 - 9d + 1)$$

$$9 \pm \frac{\sqrt{81 - 68}}{2} = \frac{9 \pm \sqrt{13}}{2}$$

$$\frac{a_1^2 + (21d-9)a_1 + 108d^2 - 36d - 60}{a_1^2 + 4ad} \Big|_{\frac{a_1+4d}{a_1+12d-9}}$$

$$(17d-9)a_1 + \dots$$

$$\begin{aligned} &< 64 \\ &+ 40d^2 + 60 < 0 \\ &40d^2 - 64 \\ &40d^2 - 64 < -4 \\ &60 - 40d^2 \end{aligned}$$

$$\begin{aligned} t &\in [-4; 40d^2 - 64] \\ t &\in [-4; 60 - 40d^2] \end{aligned}$$

$$d=1 \Rightarrow$$

Gegeben:

v2.

$d > 0 \Rightarrow S = \frac{2a_1 + d(n-1)}{2} \cdot n \Rightarrow a_1, n \in \mathbb{Z}$

$S_9 = \frac{2a_1 + 8d}{2} \cdot 9 = (a_1 + 4d) \cdot 9$

$a_5 - a_1 \Rightarrow S - 4$
 $a_{10} - a_1 < S + 60$

$(a_1 + 4d)(a_1 + 17d) > 9(a_1 + 4d) - 4$
 $(a_1 + 8d)(a_1 + 12d) < 60 + 9(a_1 + 4d)$

$a_1^2 + 21ad + 68d^2 > 9a_1 + 36d - 4$

$a_1^2 + (21d - 9)a_1 + 68d^2 - 36d + 4 > 0$

$D = (21d - 9)^2 - 4(68d^2 - 36d + 4) = 441d^2 - 378d + 81 - 272d^2 + 144d - 16 = 169d^2 - 234d + 65$

$\frac{81-16}{65} \quad 234 \sqrt{26} \quad \frac{234 \sqrt{26}}{2} \quad \frac{13}{2} \quad d > \frac{9}{13}$

$\frac{9 - 21d \pm (17d - 9)}{2} = \frac{(-21+13)d}{2}; \quad \frac{13 - 34d}{2} = (-4d)(9 - 17d)$

$(a_1 + 4d)(a_1 - 9 + 17d) > -4$

$a_1^2 + 21da_1 + 108d^2 < 60 + 9a_1 + 36d$

$a_1^2 + (21d - 9)a_1 + 108d^2 - 36d - 60 < 0$

$D = 441d^2 - 378d + 81 - 432d^2 + 144d - 240 < 0 \Rightarrow 9d^2 - 234d + 39^2$

$68d^2 - 36d + 4 \approx 35d^2$

$17d^2 - 9d + 1$
 $-6 \pm \sqrt{36 - 12} = -6 \pm \sqrt{24}$

$\frac{6 \pm \sqrt{36 - 12}}{21} = \frac{6 \pm \sqrt{24}}{21} = \frac{6 \pm 2\sqrt{6}}{21}$

$a = \frac{-12 \pm \sqrt{444 - 43}}{2} =$

$a = 24 + 12 \quad \frac{36 \sqrt{24}}{10} \quad (a+2)(a+10)$

$a_1^2 + 12a_1 + 3254 > 0$
 $a_1^2 + 12a_1 + 39 - 20 < 0$

$a_1^2 + 12a_1 + 36 > 0$
 $a_1^2 + 12a_1 + 12 < 0$
 $(a_1 + 6)^2 > 0$
 $a = 24 - 6 \pm 36 - 48$
 $a_1 = \frac{-6 \pm \sqrt{36 - 12}}{2} = -6 \pm \sqrt{12} = -6 \pm 2\sqrt{3} = -2 \pm 10$

$$-6a - 2b < 10$$

reproduces

$$a^2 + b^2 \leq -6a - 2b$$

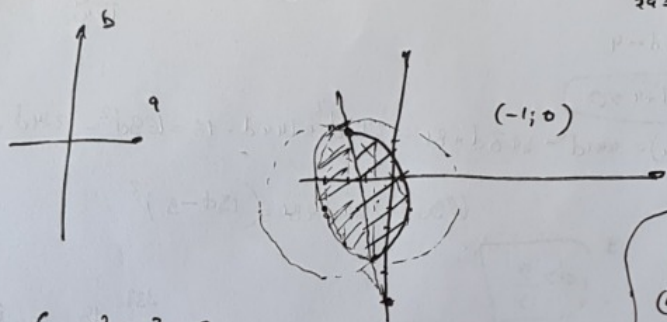
$$(a+3)^2 - 9 + (b+1)^2 - 1 \leq 0$$

$$\begin{cases} (a+3)^2 + (b+1)^2 \leq 10 \\ -6a - 2b > -5 \end{cases} \Rightarrow 3a + b > -5 \Rightarrow b > -5 - 3a$$

$$\begin{cases} a^2 + b^2 < 10 \\ 3a + b < -5 \end{cases}$$

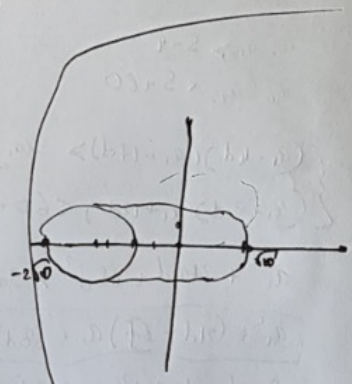
$$x^2 - 2a + a^2 + y^2 - 2b + b^2 - 10 < x^2 - 2a + y^2 - 2b$$

$$x^2 - 2a + y^2 - 2b$$



$$(a+3)^2 + (b+1)^2 = 10$$

$$3a = -5$$



$$\begin{aligned} b=0 \\ (a+3)^2 = 9 \\ a+3 = \pm 3 \\ a = -6 \end{aligned}$$

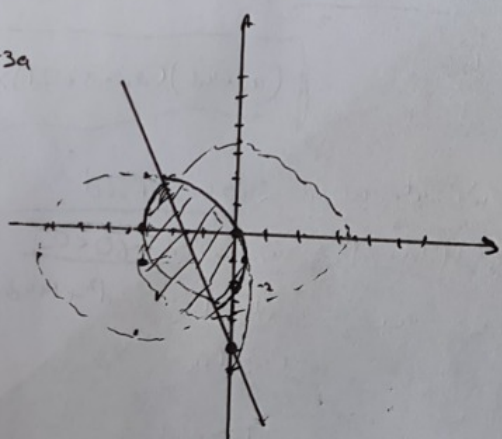
$$\begin{cases} a^2 + b^2 \leq -6a - 2b \\ -6a - 2b < 10 \end{cases} \Leftrightarrow \begin{cases} (a+3)^2 + (b+1)^2 \leq 10 \\ 3a + b > -5 \end{cases}$$

$$\begin{cases} a^2 + b^2 \leq 10 \\ -6a - 2b > 10 \end{cases} \Rightarrow \begin{cases} a^2 + b^2 \leq 10 \\ 3a + b < -5 \end{cases}$$

$$\begin{aligned} b &= -5 - 3a \\ a^2 + 25 + 30a + 9a^2 &= 10 \\ 10a^2 & \end{aligned}$$

$$a \in [-\sqrt{10}; -3 + \sqrt{10}]$$

$$a \in [-\sqrt{10}; 0] \Rightarrow$$



○

Умножение

1.

т.к. $a_n \in \mathbb{Z} \Rightarrow \begin{cases} d \in \mathbb{Z} \Rightarrow d \in \mathbb{N} \\ d \geq 0 \end{cases}$

$S = \frac{2a_1 + 8d}{2} \cdot 9 = 9(a_1 + 4d)$

$\begin{cases} a_5 \cdot a_{13} > S - 4 \\ a_{10} \cdot a_{18} < S + 60 \end{cases} \Leftrightarrow \begin{cases} (a_1 + 4d)(a_1 + 8d) > 9(a_1 + 4d) - 4 \\ (a_1 + 9d)(a_1 + 12d) < 9(a_1 + 4d) + 60 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} (a_1 + 4d)(a_1 - 9 + 12d) > -4 \\ (a_1 + 4d)(a_1 - 9 + 12d) < 60 - 40d^2 \end{cases}$

$\exists (a_1 + 4d)(a_1 - 9 + 12d) = t \Rightarrow \begin{cases} t > -4 \\ t < 60 - 40d^2 \end{cases}$

т.к. $d > 1$ найдем: $\begin{cases} t > -4 \\ t < -100 \end{cases} \Rightarrow t \in \emptyset \Rightarrow d = 1$

$\begin{cases} (a_1 + 4)(a_1 + 8) > -4 \\ (a_1 + 4)(a_1 + 8) < 20 \end{cases} \Leftrightarrow \begin{cases} a_1^2 + 12a_1 + 36 > 0 \\ a_1^2 + 12a_1 + 32 < 0 \end{cases} \Leftrightarrow a \in \begin{cases} (a+6)^2 > 0 \\ a \in (-6 - \sqrt{24}; -6 + \sqrt{24}) \end{cases} \Leftrightarrow \begin{matrix} (a \in \mathbb{Z}) \\ \Leftrightarrow \end{matrix}$

$\Leftrightarrow \begin{cases} a \neq 6 \\ a \in [-10; -2] \\ a \in [-10; -2] \end{cases} \Rightarrow \begin{cases} a \in [-10; -6) \cup (-6; -2] \\ a \in \mathbb{Z} \end{cases}$

Ответ: $a \in \{-10; -9; -8; -7; -5; -4; -3; -2\}$.

11

$x^2 + 15 = 49 - x^2$

$$A = \sqrt{4b^2 - c^2}$$

problem

Let a, b, c be positive real numbers such that $a + b + c = 1$.

Prove that $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}$.

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = \frac{a^2}{a(b+c)} + \frac{b^2}{b(c+a)} + \frac{c^2}{c(a+b)} + \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$$

$$= \frac{a^2}{a(1-a)} + \frac{b^2}{b(1-b)} + \frac{c^2}{c(1-c)} + \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$$

$$= \frac{a^2}{a-b} + \frac{b^2}{b-a} + \frac{c^2}{c-a} + \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$$

$$= \frac{a^2}{a-b} - \frac{b^2}{a-b} + \frac{c^2}{c-a} + \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$$

$$= \frac{a^2 - b^2}{a-b} + \frac{c^2}{c-a} + \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$$

$$= \frac{a+b}{1} + \frac{c^2}{c-a} + \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$$

$$= \frac{a+b}{1} + \frac{c^2}{c-a} + \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$$

$$= \frac{a+b}{1} + \frac{c^2}{c-a} + \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$$

Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21102367**

ID профиля: **378514**

Вариант 24

Условие

✓4

$$\begin{aligned} a &= 3^{d_1} \cdot \beta_1 \\ b &= 3^{d_2} \cdot \beta_2 \\ c &= 3^{d_3} \cdot \beta_3 \end{aligned} \Rightarrow$$

$$\text{НОК}(a; b; c) = 3^{19} \cdot 15 \Rightarrow \begin{cases} \max(d_i) = 19 \\ \max(\beta_i) = 15 \end{cases}$$

$$\text{НОД}(a; b; c) = 33 \Rightarrow \begin{cases} a: 33 \\ b: 33 \Leftrightarrow \begin{cases} d_i \geq 1 \\ \beta_i \geq 1 \end{cases} \\ c: 33 \end{cases}$$

$$\begin{cases} \min(d_i) = 1 \\ \min(\beta_i) = 1 \end{cases}$$

(1) Исходное условие среди d_i и β_i такое, что $\max = 1$, $\min = 19$,

$$d_3 \neq 1; 19 \Rightarrow \text{беру } d_3 \text{ минимально } \frac{19-2}{2} = 12 \text{ пар.} \Rightarrow (1) = 3 \cdot 2 \cdot 1 \cdot 17 = 6 \cdot 17 = 102$$

$$(2) \begin{cases} d_3 = 1 \\ d_3 = 19 \end{cases} \Rightarrow 2C_3^2 = 6 \Rightarrow (1) + (2) = 102 + 6 = 108$$

(3) Исходное условие среди β_i $\begin{cases} \beta_i = 1 \\ \beta_j = 15 \\ \beta_k \neq 1; 15 \end{cases} \Rightarrow (3) = 12 \cdot 3 \cdot 2 = 6 \cdot 12 = 72$

(4) $\begin{cases} \beta_i = 1 \\ \beta_j = 15 \\ \beta_k = 1; \\ \beta_k = 15 \end{cases} \Rightarrow (4) = 2 \cdot C_3^2 = 6 \Rightarrow (3) + (4) = 6 + 72 = 78$

$$((1)-(2)) \cdot ((3)+(4)) = 102 \cdot 78 = 8568$$

Ответ: 8568

5.

ОДЗ: $\begin{cases} 29-x > 0 \\ 29-x \neq 1 \\ x+49 > 0 \\ x+1 \neq 0 \\ \frac{x}{7} + 7 \neq 1 \\ -x-1 > 0 \end{cases} \Leftrightarrow \begin{cases} x < 29 \\ x \neq 28 \\ x > -49 \\ x \neq 0 \\ x \neq -42 \\ x < -1 \end{cases} \Leftrightarrow x \in (-49; -1) \setminus \{0, 28\}$

На ОДЗ $\sqrt{(x+1)^2} = |x+1|$ раскрывается в знаменателе -

$\begin{cases} a = \ln(29-x) \\ b = \ln(\frac{x}{7} + 7) \\ c = \ln(-x-1) \end{cases}, a+b+c \neq 0 \begin{cases} a \neq 0 \\ b \neq 0 \\ c \neq 0 \end{cases}$

Ищем: $\frac{2a}{b}; \frac{b}{2c}; \frac{2c}{a}$

~~1) $\frac{2a}{b} = \frac{2c}{a} \Leftrightarrow 2a^2 = 2bc \Leftrightarrow a^2 = bc$~~

~~$\frac{b}{2c} - 1 = \frac{2a}{b} \Leftrightarrow \frac{b^2 - 2bc - 2ac}{2cb} = 0$~~

~~2) $\begin{cases} \frac{2a}{b} = \frac{b}{2c} \Rightarrow \begin{cases} kac = b^2 \\ \frac{2c}{a} - 1 = \frac{2a}{b} \end{cases} \begin{cases} kac - b^2 = 0 \\ 2cb - ab - 2a^2 = 0 \end{cases}$~~

1) $\begin{cases} \frac{2a}{b} = \frac{b}{2c} \\ \frac{2c}{a} - 1 = \frac{b}{2c} \end{cases} \Leftrightarrow \begin{cases} kac = b^2 \\ 4c^2 - 2ac = ab \end{cases} \Rightarrow \begin{cases} (2c-b)(2c+b) - 2ac - ab + kac = 0 \Rightarrow \\ (2c-b)(2c+b) + a(2c-b) = 0 \Rightarrow \\ (2c-b)(2c+b+a) = 0 \Rightarrow \end{cases}$

$\Rightarrow \begin{cases} 2c = b \\ 2c+b+a = 0 \end{cases} \Leftrightarrow \begin{cases} \ln \frac{(x+1)^2}{29-x} = 0 \\ \ln \left(\frac{(x+1)^2 \cdot (29-x)(\frac{x+49}{7})}{7} \right) = 0 \end{cases} \Leftrightarrow \begin{cases} (x+1)^2 = 29-x \\ (x+1)^2 \cdot (29-x) \cdot \frac{(x+49)}{7} = 1 \end{cases}$

на ОДЗ $\begin{cases} 29-x > 0 \\ \frac{x+1}{7} > 0 \\ \frac{x+49}{7} < 7 \end{cases} \Rightarrow 2c+b+a \neq 0 \Rightarrow x^2 + 2x + 1 + x - 29 < 0 \Rightarrow x^2 + 3x - 28 < 0 \Rightarrow x \in (-7; 4)$
 $\Rightarrow \begin{cases} x = 4 \\ x = -7 \end{cases} \Rightarrow \boxed{x = -7}$
 ОДЗ

2

(2)

Умножим

$$\begin{cases} \frac{2a}{b} = \frac{b}{2c} \\ \frac{2a}{b} - 1 = \frac{b}{2c} \\ \frac{2a}{b} - 1 = \frac{2c}{a} \end{cases} \Leftrightarrow \begin{cases} 4c^2 = ab \\ 4ac - 2bc - b^2 = 0 \\ 2a^2 - ab - 2cb = 0 \end{cases} \rightarrow \begin{cases} (2c-b)(2c+b) + 4ac - 2bc - ab \\ 2c^2 - 4c^2 - 2cb = 0 \Leftrightarrow a^2 - 2c^2 - cb = 0 \\ \Leftrightarrow a^2 - 2c^2 - \frac{4c^3}{a} = 0 \Leftrightarrow a^3 - 2ac^2 - 4c^3 = 0 \\ \sqrt{a=2c} \end{cases}$$

(3)

$$\begin{cases} \frac{a}{b} = \frac{c}{a} \\ \frac{b}{2c} - 1 = \frac{2a}{b} \\ \frac{b}{2c} - 1 = \frac{2c}{a} \end{cases} \Leftrightarrow \begin{cases} a^2 = bc \\ b^2 - 2bc = 2ac \\ ab - 2ac = ac^2 \end{cases} \Leftrightarrow \begin{cases} b=2c \\ b=2a \end{cases}$$

(2)

$$\frac{x}{7} + 7 = (x+1)^2 \Leftrightarrow x^2 + 2x + 1 = \frac{x}{7} + 7 \Leftrightarrow 7x^2 + 14x + 7 = x + 49 \Leftrightarrow 7x^2 + 13x - 42 = 0$$

$$\Leftrightarrow x = \frac{-13 \pm \sqrt{169 + 1176}}{2 \cdot 7} = \frac{13 \pm 35}{14} = -\frac{12}{14} = -\frac{6}{7}$$

DPS

(3)

$$29 - x = \frac{x}{7} + 7 \Leftrightarrow 203 - 7x = x + 49 \Leftrightarrow 8x = 154 \Rightarrow x = 19.25$$

DPS

Ответ: $\begin{cases} x = -7 \\ x = -\frac{6}{7} \end{cases}$

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371

репроду

$$x = -3 \pm \sqrt{9+12} = -3 \pm \sqrt{21}$$

28
112

репроду:

$$\text{KOR}(a; b; c) = 33 = 3 \cdot 11$$

$$\text{KOK}(a; b; c) = 3^{13} \cdot 11^{15} \quad \log_2(614) =$$

$$a = 3^2 \cdot 11^8$$

$$b = 3^4 \cdot 11^9$$

$$c = 3^6 \cdot 11^7$$

канн. сдв. парнас

66.2

d_1, d_2, d_3

1 13 X

13 1 X

1 X 19

19 X 1

19 1 X 19

1 19

1 15 3

$$\log_{\sqrt{29-x}} \left(\frac{x}{7} + 7 \right) \quad \log_{(x+1)^2} (9-x) \quad \log_{\sqrt{\frac{x}{7}+2}} (-x-1)$$

$$\text{OP3: } 29-x > 0 \quad x < 29$$

$$29-x \neq 1 \quad x \neq 28$$

$$\frac{x+49}{7} > 0 \quad x > -49$$

$$x \neq 0 \quad x \neq 0$$

$$9-x > 0 \quad x < 9$$

$$\frac{x+49}{7} \neq 1 \quad x \neq 9-49 = -40$$

$$-x-1 > 0 \quad x < -1 \quad x > -49 \quad x \neq 0 \quad x \neq -42$$

OP3
 $x \in (-49; -1) \cup (-42; 0)$

4
19
6
102

$$\log_{\sqrt{29-x}} \left(\frac{x}{7} + 7 \right) = \log_{(x+1)^2} (9-x)$$

$$2 \log_{29-x} \left(\frac{x+49}{7} \right) = 2 \log_{29-x} (x+1)$$

$$2 \log_{(29-x)^2} \left(\frac{x}{7} + 7 \right); \frac{1}{2} \log_{(x+1)^2} (9-x); 2 \log_{\sqrt{\frac{x}{7}+2}} (-x-1)$$

$$a = \frac{x}{7} + 7$$

$$b = 29-x$$

$$c = -x-1$$

1191
1919
1311
19191
1919
191919

$$\frac{\ln a}{\ln b} = \frac{\ln b}{\ln c} \Leftrightarrow \frac{\ln a \ln c - \ln^2 b}{\ln b \ln c} = 0$$

$$\ln \left(\frac{x}{7} + 7 \right) \cdot \ln (-x-1)$$

$$2 \log_b a \quad \frac{1}{2} \log_c b \quad 2 \log_a c \quad \ln a = \ln b \cdot \ln c$$

$$\log_b a = \log_a c \Leftrightarrow \frac{\ln a \cdot \ln a - \ln b \cdot \ln c}{\ln b \cdot \ln a} = 0$$

$$2 \log_a c - 1 = 2 \log_b a \Leftrightarrow 2 \left(\frac{\ln c \cdot \ln b - \ln^2 a}{\ln a \cdot \ln b} \right) - 1 = 0$$

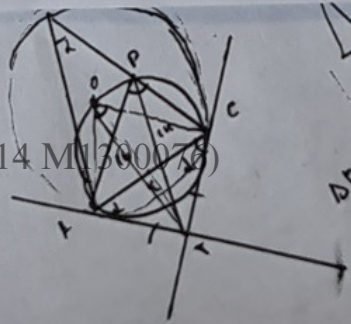
$$2 \log_a c = 1 = \frac{1}{2} \log_c b \Rightarrow \frac{2 \ln c}{\ln a} - 1 = \frac{1}{2} \frac{\ln b}{\ln c} \Leftrightarrow \ln^2 c - \ln a \ln c - \ln b \ln a = 0$$

1
13
6

98
6
84

108
84
408
816
8568

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$AK = KC$
 $\angle AQP = 90^\circ$

$$\frac{PX \cdot AC}{2} = 30 \quad \frac{PX \cdot KC}{2} = 14$$

a: 33
b: 33
c: 33

reproben

$\frac{28}{112}$

$$x = \frac{-3 \pm \sqrt{9 + 112}}{2} = \frac{-3 \pm \sqrt{121}}{2} = \frac{-3 \pm 11}{2} = 4; -2$$

$1 \leq d_1 \leq 19 \rightarrow -19$
 $1 \leq d_2 \leq 15 \rightarrow -15$
 $b = 3^{d_2} \cdot 11 \cdot p_2 \quad 1 \leq d_2 \leq 15 \rightarrow -15$
 $c = 3^{d_3} \cdot 4 \cdot p_3 \quad 1 \leq p_i \leq 15 \rightarrow -15$

$d_1 = 19 \Rightarrow \exists 3 \text{ numbers } d_i = 19$
 $d_2 = 15 \Rightarrow \exists 3 \text{ numbers } p_i = 15$

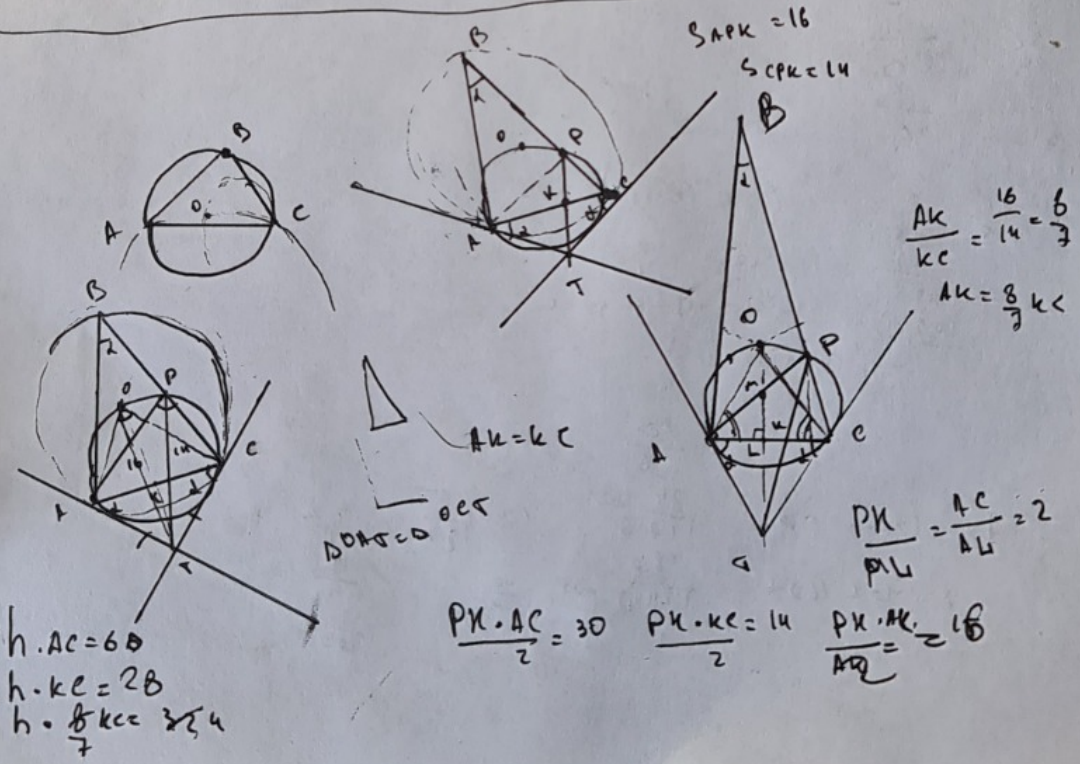
19 · 15 · 10 · 5 · 9

$m_{\frac{a}{b}} = a$ $m_{\frac{b}{c}} = b$ $m_{\frac{c}{a}} = c$

$\frac{2a}{b} = \frac{2c}{a}$ $\frac{2b}{c} = \frac{2a}{b}$ $\frac{2c}{a} = \frac{2b}{c}$ (1) & (2)

$\frac{2a}{b} = \frac{2c}{a} \Leftrightarrow 4ac = b^2$

$\frac{2c}{a} - 1 = \frac{2c}{a} \Leftrightarrow 2cb - ab - 2a^2 = 0$
 $\frac{2c}{a} - 1 = \frac{b}{2c} \Leftrightarrow 4c^2 - 2ac - ab = 0$
 $2cb - ab - 2a^2 = 0$
 $4c^2 - 2ac - ab = 0$
 $2cb - ab - 2a^2 = 2c^2 - 2ac$
 $c^2 + a^2 = ac + b \Rightarrow c(c+b) = a^2 + c^2$



reprobleme

$$2 \log_b a \quad \frac{1}{2} \log_c b \quad 2 \log_a c$$

$$1) 2 \log_b a = 2 \log_a c \Leftrightarrow \frac{\ln^2 a - \ln b \cdot \ln c}{\ln a \cdot \ln b} = 0 \Rightarrow \ln^2 a - \ln b \cdot \ln c = 0$$

$$2 \log_b a + 1 = 2 \log_a c \Leftrightarrow 2 \left(\frac{\ln^2 a - \ln b \cdot \ln c}{\ln a} \right) + 1 = 0$$

$$2) 2 \log_b a = \frac{1}{2} \log_c b \Leftrightarrow \frac{4 \ln a \ln c - \ln^2 b \cdot \ln c}{2 \ln b \cdot \ln c} = 0 \Rightarrow \ln^2 b = 4 \ln a \ln c$$

$$2 \log_a c - 1 = 2 \log_b a = 2 \left(\frac{\ln c \ln b - \ln^2 a}{\ln a \cdot \ln b} \right) = 0$$

$$2 \ln c \ln b - \ln^2 a - \ln a \ln b = 0$$

$$2 \ln c \ln b - \ln a \ln b - \ln^2 a = 0$$

$$\ln b (2 \ln c - \ln a) - \ln^2 a = 0$$

$$(2 \log_a c) - 1 = \frac{1}{2} \log_c b \Leftrightarrow 4 \log_a c - 2 - \log_c b = 0 \Leftrightarrow \frac{4 \ln^2 c - \ln b \cdot \ln a - 2 \ln a \cdot \ln c}{\ln} = 0$$

$$6 \quad 4 \ln^2 c - \ln b \cdot \ln a - \frac{\ln^2 b}{2} = 0$$

$$\frac{29}{2}$$

$$6 \ln^2 c - 2 \ln b \cdot \ln a - \ln^2 b = 0$$

$$\begin{array}{r} 12 \\ 35 \\ \hline 1 \\ 42 \\ 28 \\ \hline 336 \\ 64 \\ \hline 1176 \\ 169 \\ \hline 1245 = \frac{35}{35} \\ 125 \\ \hline 105 \end{array}$$

$$\begin{aligned} x + 9 &= (x+4)^2 && 169 + \\ x^2 + 2x + 41 &= \frac{x}{2} + 9 && 42 \\ 2x^2 + 14x + 17 &- x - 49 = 0 \end{aligned}$$

$$2x^2 + 13x - 42 = 0$$

$$2x^2 + 13x - 42 = 0$$

$$x = -13 \pm 169 +$$

$$\begin{array}{r} 42 \\ 28 \\ \hline 336 \\ 84 \\ \hline 1176 \\ 169 \\ \hline 1245 \end{array} \quad \begin{array}{r} 1245 \overline{) 5} \\ 10 \\ \hline 24 \\ 20 \\ \hline 45 \end{array} \quad \begin{array}{r} 42 \\ 17 \\ \hline 119 \\ 12 \\ \hline 259 \end{array}$$