Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21100637**

ID профиля: **285707**

Вариант 24

черновик

$a_5 a_{18} = (a_1+4d)(a_1+17d)$

$S_9 = \frac{2a_1+8d}{2} \cdot 9 = (a_1+4d)\cdot 9$

$a_{10} a_{13} = (a_1+9d)(a_1+12d)$

$d>0$; $a_i \in Z$

$(a_1+4d)(a_1+17d) > 9(a_1+4d)-4$

$(a_1+4d)(a_1+17d-9) > -4$

$(a_1+4d)(9-a_1-17d) < 4$

$(a_1+9d)(a_1+12d) < 9(a_1+4d)+60$

$\cancel{(a_1+9d)(a_1+12d-9) < 60}$

$a_1^2+21a_1d+68d^2 > 9a_1+36d-4$

$a_1^2+21a_1d-9a_1 > 36d-68d^2-4 \quad \cdot(-1)$

$d \in Z$

$+\ a_1^2+12a_1d-9a_1+9a_1d+108d^2-81d < 60$

$\ \ -a_1^2-21a_1d+9a_1 < 68d^2+4-36d$

$108d^2-81d < 64+68d^2-36d$

$40d^2-45d-64<0$

$D = 2025+64\cdot160$

$2^6\cdot2^4\cdot10 = 10240$

$+2025$

12265

$\sqrt{12265} = 115,$

$9a_1-a_1^2-17a_1d+36d-4a_1d-68d^2<4$

$+\ -a_1^2-21a_1d+9a_1-68d^2+36d<4$

$a_1^2+21a_1d+108d^2 < 9a_1+36d+60$

$108d^2-68d^2+\cancel{36d}-\cancel{36d} < 64$

$40d^2<64$

$10d^2<16$

$d^2 < \frac{16}{10} = \frac{8}{5}$ $\quad d\in Z,\quad d=1$

$a_1^2+21a_1-9a_1+108-36<60$

$a_1^2+12a_1+12<0.$

24 $\sqrt{24}\approx5$

$a_1 = -6\pm2\sqrt6$

$-6+2\sqrt6$

$-6-2\sqrt6 \approx$ $\quad -6-\sqrt{24}\approx-11$

$\cancel{-16<a_1<}$ $\quad -6+\sqrt{24}$

$-11<a_1<-1$; $a_1\in Z$

$b=-\frac{a}{6}a+c$

$(a;b)$

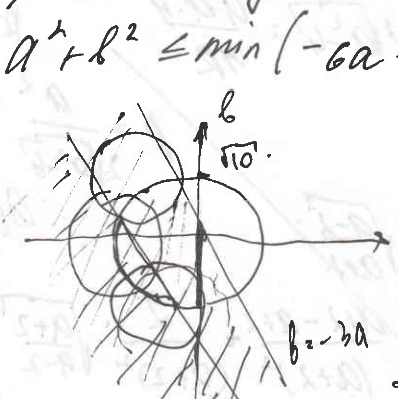
$b=d\cdot a$

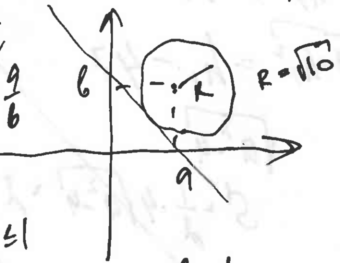
$d=\frac{b}{a}$

$k_2=-\frac{9}{6}$

$(x-a)^2+(y-b)^2\le10$

$a^2+b^2\le\min(-6a-2b;10)$





$10a^2\le10$

$a^2\le1$

$-6a-2b>0$

$-2b>6a$

$2b<-6a$

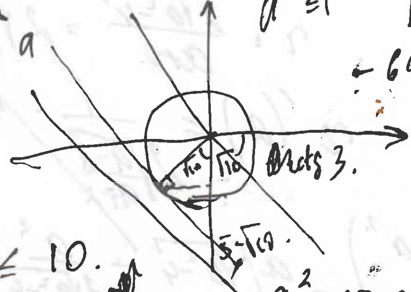
$b<-3a$

$-6a-2b=1$

$2b+6a+1=0.$

$b=-\frac12-3a$

$-6a-2b>10$



$-5-3a\le b\le-3a$

$a^2+b^2\le-6a-2b$

$10a^2\le0 \Rightarrow a=0.$

$\begin{cases} a^2+b^2\le10. \\ b<-5-3a \end{cases}$

$-2b>10+6a$

$b<-5-3a$

$a^2+25+9a^2+30a\le10.$

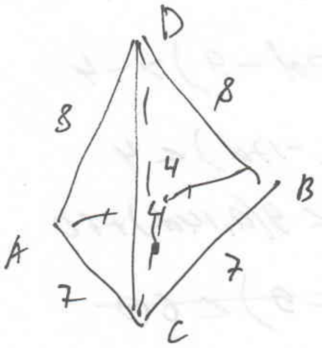
$10a^2+30a+15\le0.$

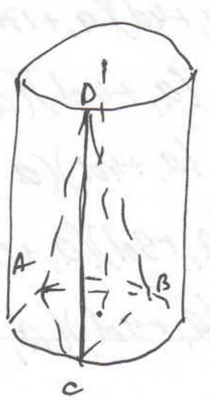
$2a^2+6a+3\le0.$

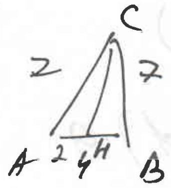
$36-24=12.$

$\frac{-3\pm\sqrt3}{2}$

черновик







$CH = \sqrt{45} = 3\sqrt{5}$

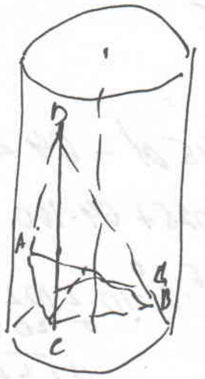
$S_{ABC} = \frac{1}{2}\cdot 4\cdot 3\sqrt{5} = 6\sqrt{5}$

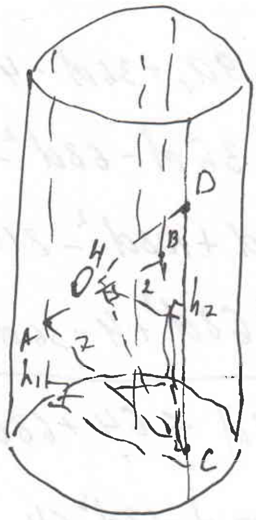
$6\sqrt{5} = 2\cdot 9 \cdot r$ $\quad r = \frac{2\sqrt{5}}{3}$

$h_2 = \sqrt{60-4} + \sqrt{45-4} = \sqrt{56} + \sqrt{41}$

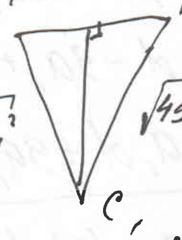
$\cancel{h_2 = \sqrt{a^2-4} = 2}$ $\quad 8\cdot 7 = 4\cdot 14$

$2\sqrt{14} + \sqrt{41}$





A_1 $\sqrt{16-(h_2-h_1)^2}$ B_1

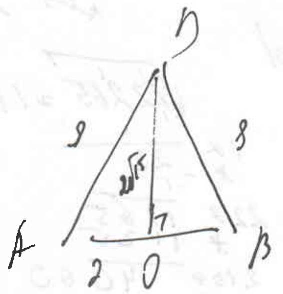


R_{min}: $2\sqrt{2}\,a = 2\sqrt{2}$.

$a = \sqrt{49-h^2} = \sqrt{8}$

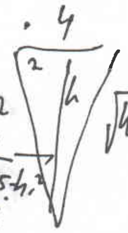
$49-h^2 = 8$

$h^2 = 41$



$AB \perp (COD) \Rightarrow AB$ $\quad h_1 = h_2.$ $\quad h = \sqrt{41}$

$h = \sqrt{49-h_1^2-4} = \sqrt{45-h_1^2}$



$r'(h_1) = 2\cdot\left(\frac{-h_1}{\sqrt{45-h_1^2}}\left(2+\sqrt{49-h_1^2}\right) + \frac{h_1\sqrt{45-h_1^2}}{\sqrt{49-h_1^2}}\right.$

$S = \frac{1}{2}\cdot 4\cdot\sqrt{45-h_1^2} = 2\sqrt{45-h_1^2}$

$p = \left(\sqrt{49-h_1^2}+2\right) r = 2\sqrt{45-h_1^2}$

$h_1 = 0.$

$\cancel{r = \frac{2\sqrt{45-h_1^2}}{2+\sqrt{49-h_1^2}}}$

$\left(2+\sqrt{49-h_1^2}\right)\sqrt{49-h_1^2} + 45 - h_1^2$

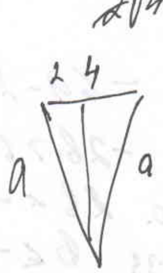
$2\sqrt{49-h_1^2} + 49 - h_1^2 + 45 - h_1^2 = 0. \Rightarrow h_1 = 0.$

$-\sqrt{2}$ $\quad 0$ $\quad \sqrt{2}$ $\quad - \quad +$

$2\sqrt{49-h_1^2} = h_1 - 47$

$2\sqrt{a^2-4} = \frac{a\,a\cdot 4}{4R}$

$R = 2\sqrt{2}.$



$\sqrt{a^2-4}$

$S = \frac{1}{2}\cdot 4\sqrt{a^2-4} = 2\sqrt{a^2-4} = (a+2)r$

$r = \frac{2\sqrt{a^2-4}}{a+2} = 2\sqrt{\frac{a-2}{a+2}}$

$R = \frac{a^2}{2\sqrt{a^2-4}}$

$a(a^2-8)$

$r'(a) = \frac{1}{2\sqrt{\frac{a-2}{a+2}}}\cdot\frac{a+2-a+2}{(a+2)^2} = \frac{2}{(a+2)^2}\cdot\sqrt{\frac{a+2}{a-2}}$ $\quad a =$

$R = \frac{a^2}{2\sqrt{a^2-4}}$

$R'(a) = \frac{1}{2}\left(\frac{2a\cdot\sqrt{a^2-4} - \frac{2a}{2\sqrt{a^2-4}}\cdot a^2}{a^2-4}\right) = \frac{a\left(\sqrt{a^2-4} - \frac{a^2}{2\sqrt{a^2-4}}\right)}{a^2-4} = \frac{a}{a^2-4}\cdot\frac{2a^2-8-a^2}{2\sqrt{a^2-4}}$

чистовик

№1.

$a_i \in Z$ $\qquad a_5 \cdot a_{18} > S_9 - 4$

$d>0;\ d \in Z$ $\qquad (a_1+4d)(a_1+17d) > \frac{2a_1+8d}{2}\cdot 9 - 4$

$$a_1^2+21a_1d+68d^2 > 9a_1+36d-4 \quad |\cdot(-1)$$

$$-a_1^2-21a_1d-68d^2+9a_1+36d<4 \quad (1)$$

$a_{10}\cdot a_{13} < S_9+60$

$(a_1+9d)(a_1+12d) < 9a_1+36d+60$

$a_1^2+21a_1d+108d^2-9a_1-36d<60 \quad (2)$

$(1)+(2) \Rightarrow 40d^2<64 \Leftrightarrow d^2<\frac{8}{5}$ т.к. $d>0$ и $d\in Z$ $\Rightarrow d=1$

подставим $d=1$ в (2)

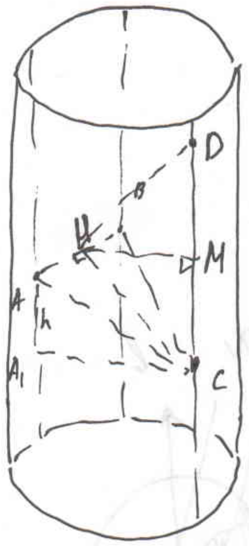
$a_1^2+12a_1+12<0 \Leftrightarrow -6-\sqrt{24}<a_1<-6+\sqrt{24}$, т.к. $a_1\in Z$,

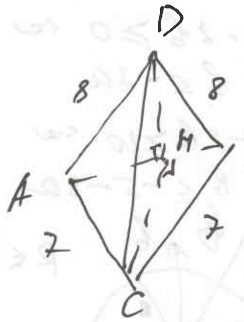
то $-11<a_1<-1$, $a_1\in Z$

$a_1\in\{-10;-9;-8;-7;-6;-5;-4;-3;-2\}$

Ответ: $\{-10;-9;-8;-7;-6;-5;-4;-3;-2\}$

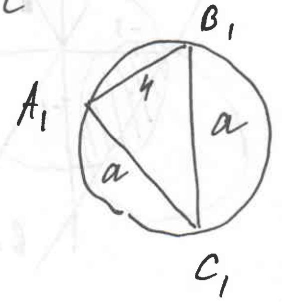
№2.





$\left.\begin{array}{l} AB\perp DH \\ AB\perp CH \end{array}\right| \Rightarrow AB\perp(CDH)\Rightarrow$

$\Rightarrow AB\perp CD$



спроектируем ΔABC на одно из оснований

$A_1B_1=4$; $A_1C_1=C_1B_1=a$

т.к. $AB\perp CD$, то расстояние от т. A и т. B до плоскости основания равны $\Rightarrow$

$\Rightarrow A_1C_1=B_1C_1=a$

$S_{A_1B_1C_1}=\frac{1}{2}\cdot 4\cdot\sqrt{a^2-4}=2\sqrt{a^2-4}$

$S_{A_1B_1C_1}=\frac{a\cdot a\cdot 4}{4R}=\frac{a^2}{R}$

Чистовик

продолжение №2

$$2\sqrt{a^2-4} = \frac{a^2}{R} \; ; \quad R = \frac{a^2}{2\sqrt{a^2-4}}$$

$$R'(a) = \frac{1}{2}\cdot\left(\frac{2a\sqrt{a^2-4} - \frac{2a}{2\sqrt{a^2-4}}\cdot a^2}{a^2-4}\right) = \frac{a}{a^2-4}\cdot\frac{a^2-8}{2\sqrt{a^2-4}}$$

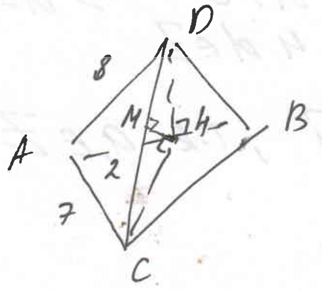
$$R'(a) = 0 \iff \left[\begin{array}{l} a = 0 \\ a = \pm 2\sqrt{2} \end{array}\right.$$

$-\;+\;-\;+$; $-2\sqrt{2}$, 0, $2\sqrt{2}$; a ; $a > 0$

$a_{min} = 2\sqrt{2}$

$\rho(AB; CD) = \sqrt{a_{min}^2 - 4} = 2$. , т.е. $HM = 2$

$DC = DM + MC$



$DH = \sqrt{64-4} = \sqrt{60}$

$CH = \sqrt{49-4} = \sqrt{45}$

$DM = \sqrt{60-4} = \sqrt{56} = 2\sqrt{14}$

$MC = \sqrt{45-4} = \sqrt{41}$

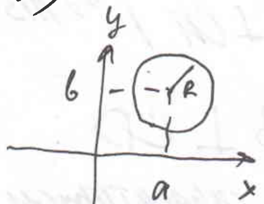
$DC = 2\sqrt{14} + \sqrt{41}$

Ответ: $2\sqrt{14} + \sqrt{41}$

№3.

$(x-a)^2 + (y-b)^2 \le 10$

$a^2 + b^2 \le \min(-6a-2b; 10)$



$-6a - 2b \ge 0 \iff$

$\iff b \le -3a$

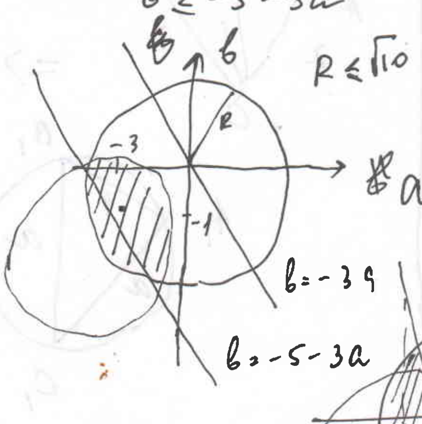
$-6a - 2b \ge 10 \iff$

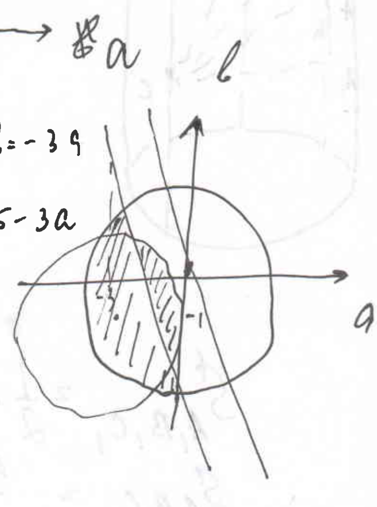
$\iff b \le -5 - 3a$

при $\begin{cases} b \le -5-3a \\ a^2+b^2 \le 10 \end{cases}$

при $\begin{cases} -5-3a \le b \le -3a \\ a^2+b^2 \le 10 - 6a - 2b \end{cases} \iff$

$\iff \begin{cases} -5-3a \le b \le -3a \\ (a+3)^2 + (b+1)^2 \le 10 \end{cases}$





множество т. $(a; b)$ содерж. в закрашенной области

Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21100637**

ID профиля: **285707**

Вариант 24

черновик

$\log_{\sqrt{29-x}}\left(\frac{x+49}{7}\right)$; $\log_{(x+1)^2}(29-x)$; $\log_{\sqrt{\frac{x}{7}+7}}(-x-1)$

$\log_{\sqrt{29-x}}\left(\frac{x}{7}+7\right) = \log_{\sqrt{\frac{x}{7}+7}}(-x-1) = 2\log_{\frac{x}{7}+7}(-x-1) = \log_{\frac{x}{7}+7}(x+1)^2$

$\frac{1}{\log_{\frac{x}{7}+7}\sqrt{29-x}} = \log_{\frac{x}{7}+7}(x+1)^2$; $1 = \log_{\frac{x}{7}+7}\sqrt{29-x}\cdot\log_{\frac{x}{7}+7}(x+1)^2$

$\log_{\sqrt{29-x}}\left(\frac{x}{7}+7\right) = \log_{(x+1)^2}(29-x)$; $\log_{\sqrt{29-x}}\left(\frac{x}{7}+7\right) = \log_{\sqrt{\frac{x}{7}+7}}(-x-1)$

$29-x = \frac{x}{7}+7$ $x = \frac{27}{4}$

$\frac{8}{7}x = 22$

$x = -7$

1) $x = -7$; 1 ; 1 ; 2. $x > -49$; $x \neq -42$

$x < 29$; $x \neq 28$ 13^2 45 $\sqrt{5}$ 13 5

$(29-x)^{\alpha} = \frac{x}{7}+7$ $\log_{(x+1)^2}(29-x) = \log_{\sqrt{\frac{x}{7}+7}}(-x-1) = \frac{1}{\log_{(x+1)^2}\left(\frac{x}{7}+7\right)}$

$\left(\frac{x}{7}+7\right)^{\alpha} = (-x-1)$

$\left(\frac{7(29-x)}{x+49}\right)^{\alpha} = \frac{x+49}{7(x+1)}$ НОД$(a;b;c) = 33 = 3\cdot 11$ $3\cdot 11$ $3^m\cdot 11^n$ $3^k\cdot 11^z$ (a, b, c)

НОК$(a;b;c) = 3^{19}\cdot 11^{15}$. m или k $= 19$

z или n $= 11$

$m \geq 1$, n пусть $k = 19$, $z = 15$ C^m 1……19 = 19 вар. $m, n, k, z \geq 1$

1…15 $19\cdot 15$

$\begin{cases} k = 19 \\ z = 15 \end{cases}$ $19\cdot 15$ вар.

$\begin{cases} k = 19 \\ n = 15 \end{cases}$ m 1… $19\cdot 15$ вар. $12\cdot 19\cdot 15$ если одно равно 33

если не равно. $3\cdot 11$; $3^m\cdot 11$; $3^b\cdot 11^z$ (a)

$\begin{cases} m = 19 \\ n = 15 \end{cases}$ $4\cdot 19\cdot 15$ вар. $\cdot 3$ $\begin{cases} b = 19 \\ z = 15 \end{cases}$ $19\cdot 15$ $\begin{cases} m = 19 \\ z = 15 \end{cases}$

$\begin{cases} m = 15 \\ z = 15 \end{cases}$ x ; x ; $(x+1) = 2$. $\begin{cases} a = 15 \\ m = 19 \end{cases}$ $\begin{cases} a = 15 \\ b = 15 \end{cases}$

$4\cdot 3\cdot 19\cdot 15$

$24\cdot 19\cdot 30$ $x^3 + x^2 = 2$. $2\cdot 12\cdot 19\cdot 15 = 24\cdot 19\cdot 15$ $-\frac{8}{27}+\frac{4}{9} = \frac{4}{27}$

$\log_{x+1}\sqrt{29-x} = \log_{\frac{x}{7}+7}(x+1)^2 = 2\log_{\frac{x}{7}+7}(x+1)$ $x = 0$.

$\begin{array}{r} 24 \\ \times\ 19 \\ \hline 216 \\ +\ 24 \\ \hline 456 \\ \times\ 3 \\ \hline 1368.0 \end{array}$

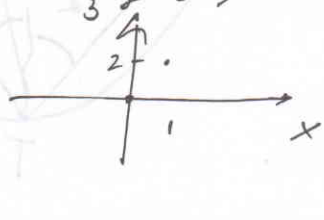
$f(x) = x^3 + x^2$ $x(3x+2)$

$f'(x) = 3x^2 + 2x$ $\log_{\frac{x}{7}+7}(x+1)\sqrt{29-x} = \frac{2}{\log_{x+1}\left(\frac{x}{7}+7\right)}$

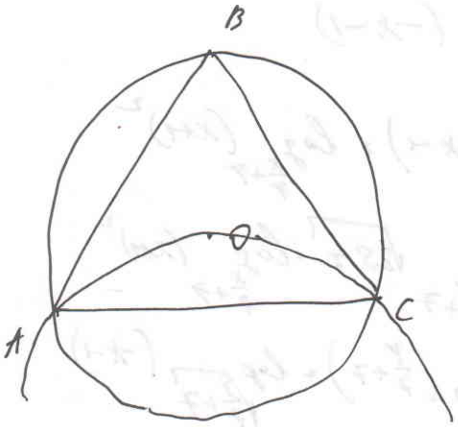
+ − +

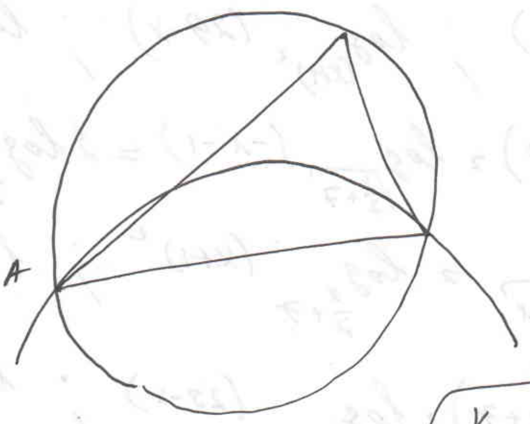
$-\frac{2}{3}$ 0

$2\log_{29-x}\left(\frac{x}{7}+7\right)\cdot\log_{(x+1)^2}(29-x)\cdot\log_{(x+1)}\ \log_{\left(\frac{x}{7}+7\right)}(x+1)^2 = 2$.



черновик





$+28$
42

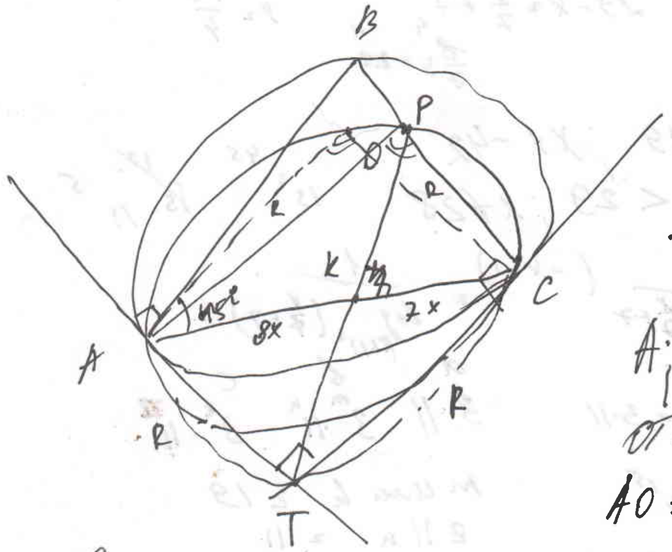
$\sqrt{\frac{x}{7}+7} = (-x-1)$

$\frac{x}{7}+7 = x^2+2x+1$

$x+49 = 7x^2+14x+7$ 169

$7x^2+13x-42=$

$\frac{-13\pm\sqrt{\ }}{14}$



$S_{APK} = 16$

$S_{CPK} = 14$

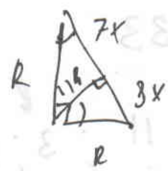
$T \in$ окр. пр. через 3 т. A, P, C

A, P, C, T леж. на одн. окр.

~~OT~~ ~~PT~~ - диаметр.

$AO = OC$; $\Rightarrow AOCT$ - прямоуг. (квадрат)

$\frac{S_{ABK}}{S_{CPK}} = \frac{AK}{\cancel{AK}KC} = \frac{8}{7}$



$\frac{h}{8x} = \frac{7x}{h}$

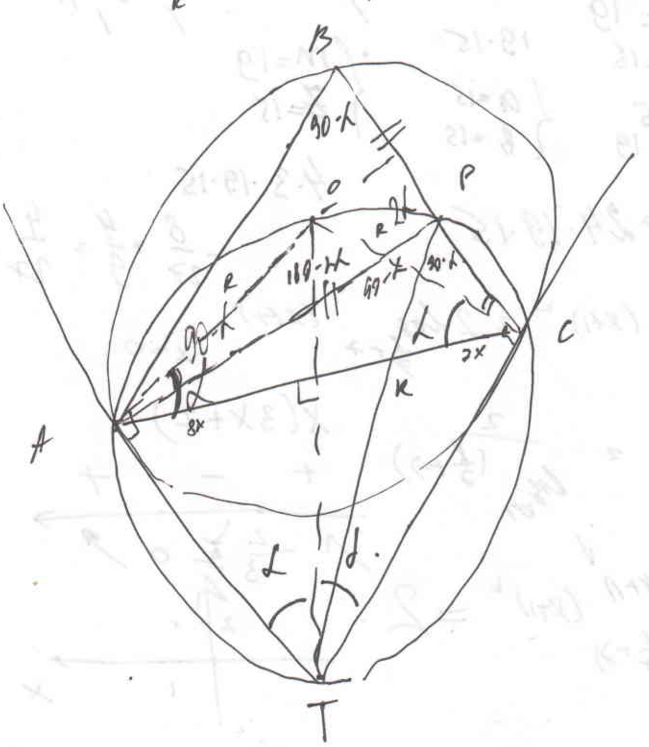
$\cancel{h = \sqrt{56}\cdot x = 2\sqrt{14}\cdot x}$

$56x^2 = TK\cdot KP$

TK - высота из прям. угла.

$\overset{28}{\cancel{56}}x^2 = \cancel{2}\sqrt{14}\cdot \cancel{x}\cdot KP$

$\cancel{KP = \frac{28}{\sqrt{14}} = 2\sqrt{14}}$



$\cos\alpha = \frac{\cancel{3x}}{\cancel{R}}\ \frac{15x}{2R}$

$1+tg^2\alpha = \frac{4R^2}{225x^2}$

$tg^2\alpha = \frac{4R^2-225x^2}{225x^2}$

$tg\alpha = \frac{\sqrt{4R^2-225x^2}}{15x}$

$1\, a\cdot\frac{\sqrt{4R^2-225x^2}}{15x} = \frac{15x}{2}$

$a = \frac{225x^2}{2\sqrt{4R^2-225x^2}}$

Чистовик

№4.

$НОД(a; b; c) = 33$

$НОК(a; b; c) = 3^{19} \cdot 11^{15}$

1) одно число 33

$3 \cdot 11$; $3^{\alpha} \cdot 11^{\beta}$; $3^{\gamma} \cdot 11^{\varphi}$

максим. степень у 3 — 19

у 11 — 15

варианты

$\begin{cases} \alpha = 19 \\ \beta = 15 \end{cases}$ $\begin{cases} \gamma = 19 \\ \varphi = 15 \end{cases}$ $\alpha, \beta, \gamma, \varphi \geq 1$

$\begin{cases} \alpha = 19 \\ \varphi = 15 \end{cases}$ $\begin{cases} \gamma = 15 \\ \beta = 15 \end{cases}$

В каждой из 4 систем кол-во вариантов $19 \cdot 15$

Всего 4 системы и, если $(a; b; c)$ - подходит, то и $(a; c; b)$ и т.д.

$P_3 = 3! = 6$ ш. - каждая тройка даёт 6 вариантов

для 1 сл. $4 \cdot 15 \cdot 19 \cdot 6$ вариантов

2)

$3 \cdot 11^{\alpha}$; $3^{\beta} \cdot 11$; $3^{\gamma} \cdot 11^{\varphi}$ $\alpha, \beta, \gamma, \varphi \geq 1$

$\begin{cases} \alpha = 15 \\ \beta = 19 \end{cases}$ $\begin{cases} \varphi = 15 \\ \gamma = 19 \end{cases}$ аналогично $4 \cdot 15 \cdot 19 \cdot 6$ вариантов.

$\begin{cases} \alpha = 15 \\ \gamma = 19 \end{cases}$ $\begin{cases} \varphi = 15 \\ \beta = 19 \end{cases}$

Всего $4 \cdot 15 \cdot 19 \cdot 6 \cdot 2 = 12680$ вариантов.

Ответ: 12680

№5.

$\log_{\sqrt{29-x}}\left(\frac{x}{7}+7\right)$; $\log_{(x+1)^2}(29-x)$; $\log_{\sqrt{\frac{x}{7}+7}}(-x-1)$

$\begin{cases} x < -1 \\ x > -49 \end{cases}$; $x \neq -2$; $x \neq -42$

$\log_{\sqrt{29-x}}\left(\frac{x}{7}+7\right) = \log_{(x+1)^2}(29-x)$

при $x = -7$

при $x = -7$

значения 1; 1; 2 удовлетворяет условию

$\log_{\sqrt{29-x}}\left(\frac{x}{7}+7\right) \cdot \log_{(x+1)^2}(29-x) \cdot \log_{\sqrt{\frac{x}{7}+7}}(-x-1) = 2.$

чистовик

продолжение № 5.

$\alpha \quad \alpha \quad \alpha+1 \quad \alpha\cdot\alpha(\alpha+1)=2$

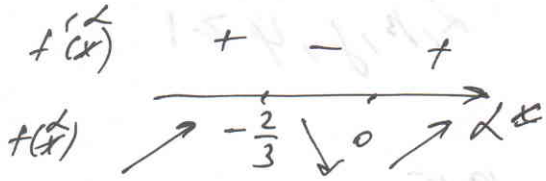
по условию должно быть ~~x; x; $x+1$; $x\cdot x(x+1)=2$~~

перемножим их $\cancel{x^3+x^2=2}$ $\alpha^3+\alpha^2=2$

рассмотрим ф-ю $\cancel{f(x)=x^3+x^2}$ $f(\alpha)=\alpha^3+\alpha^2$

$\cancel{f'(x)=3x^2+2x}$ $\cancel{f'(x)=0}$ $\Leftrightarrow \left[\begin{array}{l}\alpha=0\\ \alpha=-\frac{2}{3}\end{array}\right.$

$f'(\alpha)=3\alpha^2+2\alpha$ $\quad f'(\alpha)=0$

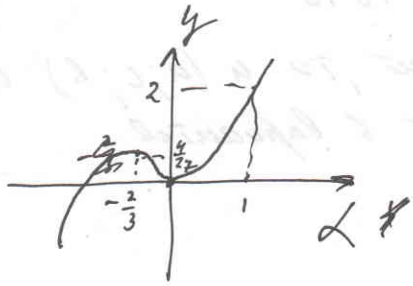


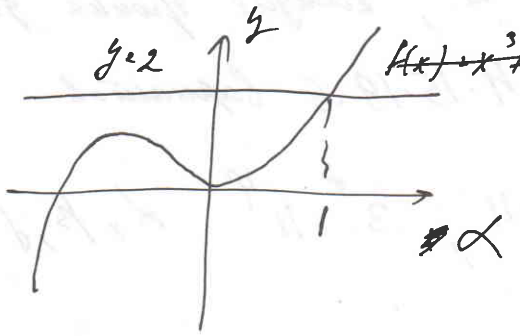
$\alpha=0$ – т. миним.

$\alpha=-\frac{2}{3}$ т. максим.

$f(0)=0$

$f\left(-\frac{2}{3}\right)=-\frac{8}{27}+\frac{4}{9}=\frac{4}{27}$





$\cancel{f(x)=x^3+x^2}$ $f(\alpha)=\alpha^3+\alpha^2$

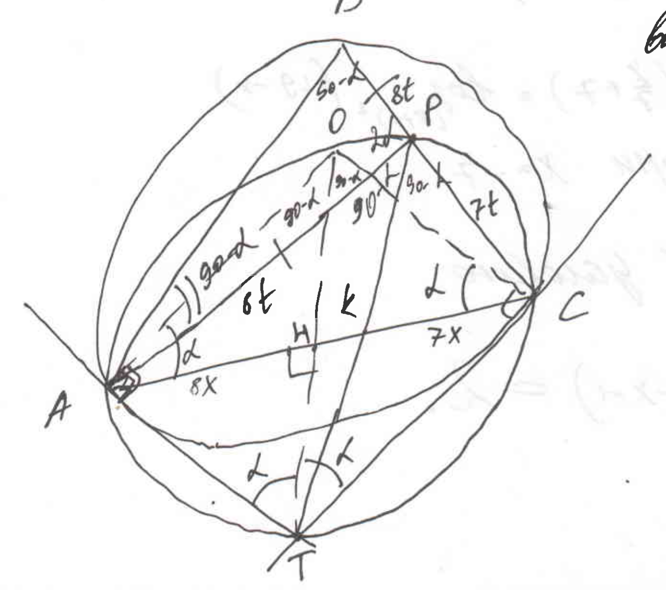
~~$f(x)=2$~~
~~имеет ед. решение~~
~~$x^3+x^2=2 \Leftrightarrow x=1$~~

$f(\alpha)=2$
имеет ед. решение
$\alpha^3+\alpha^2=2 \Leftrightarrow \alpha=1 \Rightarrow$

$\Rightarrow x=-7$ – единств. значение

Ответ: -7

№ 6.



вокруг четырехугольника AOCT можно опис. окр. т.к. $\angle OAT+\angle OCT=180°$ $(OA\perp AT)$ $(OC\perp CT)$

$\Rightarrow$ т. A; O; C; T лежат на одной окр-ти

$\frac{S_{APK}}{S_{CPK}}=\frac{AK}{KC}=\frac{16}{14}=\frac{8}{7}$

Чистовик

продолжение № 6

$\angle APT = 90 - \alpha$
$\angle TPC = 90 - \alpha$ $\Rightarrow$ PT - биссектриса

$\angle BPA = 2\alpha$

$\angle ABC = 90 - \alpha$

$\angle BAP = 180 - 90 + \alpha - 2\alpha = 90 - \alpha$

$\Rightarrow$ ΔABP равнобедр. $BP = AP$

по св-ву биссектрисы $\frac{AK}{KC} = \frac{AP}{PC} = \frac{8}{7}$

$S_{APC} = 16 + 14 = 30$

$\frac{S_{APC}}{S_{ABC}} = \frac{7}{15}$; $S_{ABC} = \frac{15}{7} \cdot S_{APC} = \frac{15}{7} \cdot 30 = \frac{450}{7}$

$tg(90 - \alpha) = ctg\,\alpha = \frac{3}{5}$; $tg\,\alpha = \frac{5}{3}$

$S_{APC} = \frac{1}{2} \cdot h \cdot 15x = 30$; $hx = 4$

$TH \cdot \frac{5}{3} = 7x$; $TH = \frac{21x}{5}$

$OH = 7x \cdot \frac{5}{3} = \frac{35x}{3}$ ~~35~~

Ответ: а) $S_{ABC} = \frac{450}{7}$