Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21100275**

ID профиля: **256842**

Вариант 24

N1

$$S = \frac{9(2a_1+8d)}{2} = 9(a_1+4d)$$

① $a_5 \cdot a_{18} > S-4 \Rightarrow (a_1+4d)(a_1+17d) > 9(a_1+4d)-4$

② $a_{10} \cdot a_3 < S+60 \Rightarrow 9(a_1+4d)+60 > (a_1+9d)(a_1+12d)$ +

$$a_1^2 + 21a_1d + 17\cdot 4d + 9a_1 + 36d + 60 >$$

$$> 9a_1 + 36d - 4 + a_1^2 + 21a_1d + 9\cdot 12d \Rightarrow$$

$$\Rightarrow 17\cdot 4d + 60 > 9\cdot 12d - 4 \quad \Big| :4$$

$$17d + 15 > 27d - 1$$

$$\begin{cases} 16 > 10d \\ d > 0 \\ d \in \mathbb{Z} \end{cases} \Rightarrow d = 1$$

① $a_1^2 + 21a_1 + 17\cdot 4 > 9a_1 + 36 - 4$

$a_1^2 + 12a_1 + 36 > 0$

$a_1 \neq -6$

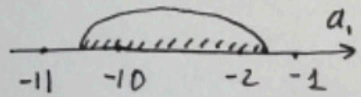
② $a_1^2 + 21a_1 + 12\cdot 9 < 9a_1 + 36 + 60$

$a_1^2 + 12a_1 + 12 < 0$

$D = 4^2 \cdot 6$

$a_1 = \frac{-12 \pm 4\sqrt{6}}{2} = \pm 2\sqrt{6} - 6$

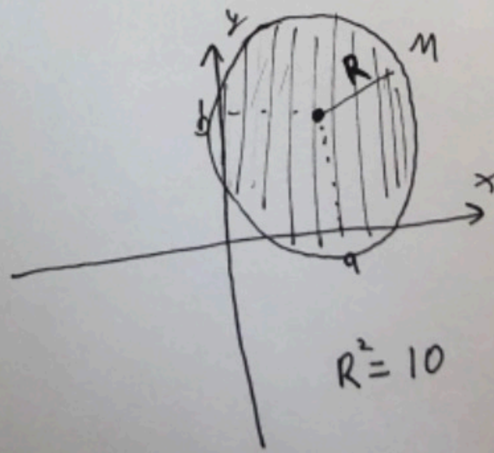
$4 < 2\sqrt{6} < 5$



Ответ: −10, −9, −8, −7, −5, −4, −3, −2

N3

$$\begin{cases} (x-a)^2 + (y-b)^2 \le 10 \\ a^2 + b^2 \le \min(-6a-2b,\ 10) \end{cases}$$



пусть $a = b = -1$

$1+1 \le \min(6+2, 10)$ – верно ⇒

⇒ фигура M существует

$(x-a)^2 + (y-b) \le 10$ = круг радиусом 10

$R^2 = 10$

$S = \pi R^2 = 10\pi$

Ответ: 10π

$a_1^2 + 21a_1 + 12\cdot 9a_1 < 9a_1 + 36 + 60$ черновик

$a_1^2 + 12a_1 + 12 < 0$

$D = 12^2 - 4\cdot 12 = 12(12-4) = 2^2\cdot 3\cdot 2^2(3-1) =$

$= 2^2\cdot 3\cdot 2^2\cdot 2$

$a_1 = \frac{-12 \pm 4\sqrt{6}}{2} = \pm 2\sqrt{6} - 6$

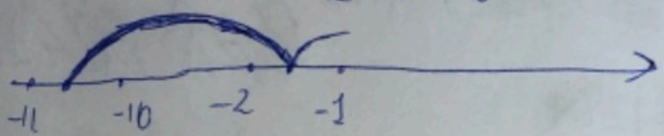
$2\sqrt{6} \vee 5$

$4\cdot 6 \vee 25$

$24 < 25$

$2\sqrt{6} - 6$

$4 < 2\sqrt{6} < 5$



$4,5 - 6 = -1,5$

$-4,5 - 6 = -10,5$

$$\begin{cases} (x-a)^2 + (y-b)^2 \le 10 \\ a^2 + b^2 \le \min(-6a - b, 10) \end{cases}$$

$a^2 + b^2 \le -6a - 2b$

$a^2 + b^2 \le 10$

$a = -1 \quad b = -1$

$1 + 1 \le \min(6 + 1; 10) \to 2 < 7$

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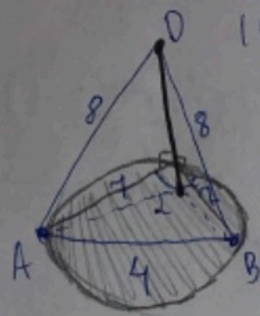
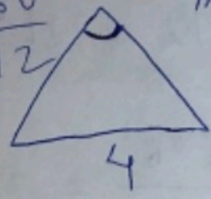
$$\begin{array}{r} 12 \\ \times\ 9 \\ \hline 108 \end{array} \qquad \begin{array}{r} 108 \\ -36 \\ \hline 72 \\ -60 \\ \hline 12 \end{array}$$





$\min \underbrace{2R} = \frac{4}{\sin\alpha}$

$\underbrace{\sin\alpha = \max}$

$\sqrt{1-\cos^2\alpha} = \max$

$D = 12^2 - 4\cdot 12 = 3\cdot 4\cdot 4(3-1) = 2(4\sqrt{6})^2$

$$\frac{16x^{\cancel{2}} - 32x^{\cancel{8}} + 128\cancel{x}}{x^{\cancel{x}3}} =$$

$$= \frac{128 - 16x}{x^3} = 0$$

$x = 8$

$h = 2$

$4^2 = h^2 + h^2 - 2h^2\cdot\cos\alpha$

$4^2 = 2h^2(1 - \cos\alpha)$

$\cancel{\cos\alpha}$

$49 - 8 = 41$

$$\frac{4^2}{2h^2} - 1 = -\cos\alpha$$

$5\cdot 9$

$$1 - \frac{8}{h^2} = \cos\alpha$$

$49 - 4 = 45$

$$\sqrt{1-\cos^2\alpha} = \sqrt{1 - \frac{h^4 - 16h^2 + 64}{h^4}} =$$

$$\cos^2\alpha = \left(\frac{h^2-8}{h^2}\right)^2 \qquad = \sqrt{\frac{h^4 - h^4 + 16h^2 - 64}{h^4}} =$$

$$= \frac{\sqrt{16h^2 - 64}}{h^2}$$

$h^2 = x$

$$\begin{array}{r} 17 \\ \times\ 2 \\ \hline 34 \\ \times\ 2 \\ \hline 68 \end{array}$$

$$(1 - \cos^2\alpha)' = \frac{16x - 64}{x^2} =$$

$$\max \frac{16x - 64}{x^2}$$

$$= \frac{(16x-64)'x^2 - (16x-64)2x}{x^4} =$$

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$S_{9} = S_9 = \frac{9\cdot(2a_1+8d)}{2} = 9(a_1+4d)$ черновик

$a_5 \cdot a_{18} > S-4$ $\quad a_1 - ?$

$a_{10}a_{13} < S+60$

$68+17-36+4=$

$a_5 = a_1+4d$

$a_{10} = a_1 +$

$(a_1+4d)(a_1+17d) > 9(a_1+4d)-4$

$(a_1+9d)(a_1+12d) < 9(a_1+4d)+60$

$a_1^2+4a_1d+17a_1d+17\cdot4d > 9a_1+36d-4$

$(a_1+4d)(a_1+17d-9) > -4$

$\cancel{a_1^2}\underbrace{\cancel{+4a_1d+17a_1d}}_{21}+17\cdot4d > \cancel{9a_1}+\cancel{36d}-4$

$\cancel{9a_1}+\cancel{36d}+60 > \cancel{a_1^2}\underbrace{\cancel{+9a_1d+12a_1d}}_{21}+9\cdot12d$

$17\cdot4d+60 > 9\cdot12d-4$

$9\cdot3=27$

$17d+15 > 27d-4$

$19 > 10d \rightarrow \boxed{d=1}$

a_1:

$a_1^2+\cancel{4a_1}+17+\overset{68}{\cancel{17\cdot4}} > \overset{5a_1}{\cancel{9a_1}}+36-4$

$a_1^2+12a_1+19 > 0$

$D \geq 0$

$(a_1+6)^2$

$D = 12^2-4\cdot19 =$

$= 2^2(4\cdot9-19) = 2^2(36-19) =$

$= 17\cdot2^2$

$6+4,5=$

$25-19\cdot4<0$

$\begin{array}{r}17\\ \times\ 2\\ \hline 34\\ \times\ 2\\ \hline 68\end{array}$ $\quad \begin{array}{r}68\\ -17\\ \hline 51\end{array}$ $\quad \begin{array}{r}55\\ -36\\ \hline 19\end{array}$

$\begin{array}{r}12\\ \times\ 9\\ \hline 108\end{array}$ $\quad \begin{array}{r}108\\ -36\\ \hline 72\end{array}$

$72-60$

$\begin{array}{r}68\\ -36\\ \hline 32\\ +\ 4\\ \hline 36\end{array}$

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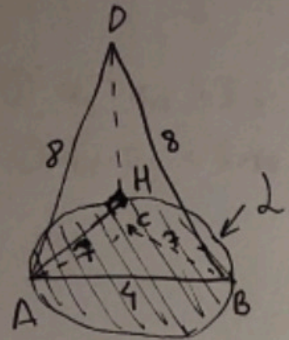
Чистовик лист 2

N2

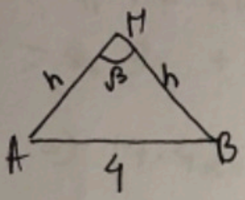


проведём через A пл $\alpha \perp CD$

$\alpha \cup CD = H$

$\Delta ADC = \Delta BDC \to AH = BH$ и $BH \perp CD \Rightarrow$

$\Rightarrow B$ лежит в α



радиус цилиндра

$$2R = \frac{4}{\sin\beta}$$

$R = \min \Rightarrow \sin\beta = \max \Rightarrow$

$\Rightarrow \sqrt{1-\cos^2\beta} = \max \to$

$\to 1 - \cos^2\beta = \max$

$$4^2 = h^2 + h^2 - 2h^2 \cdot \cos\beta \Rightarrow \cos\beta = 1 - \frac{8}{h^2}$$

$$\cos^2\beta = \left(\frac{h^2-8}{h^2}\right)^2$$

$$1 - \cos^2\beta = 1 - \frac{h^4 - 16h^2 + 64}{h^4} = \frac{16h^2 - 64}{h^4}$$

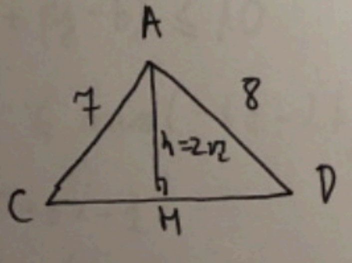
$h^2 = x$

$$f(x) = \frac{16x-64}{x^2}$$

$$0 = f'(x) = \frac{(16x-64)' x^2 - (16x-64)(x^2)'}{x^2 \cdot x^2} =$$

$$= \frac{16x^2 - 32x^2 + 128}{x^3} = 0$$

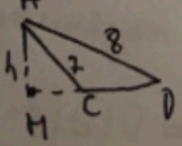
$x = 8 \to h = 2\sqrt{2}$

1) 

$HD = \sqrt{8^2 - 8} = 2\sqrt{16-2} = 2\sqrt{14}$

$CH = \sqrt{7^2 - 8} = \sqrt{41}$

$CD = 2\sqrt{14} + \sqrt{41}$

2) 

$CD = HD - HC = 2\sqrt{14} - \sqrt{41}$

Ответ: $2\sqrt{14} \pm \sqrt{41}$

Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21100275**

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Вариант 24

N 2 чистовик Лист 2

$$\left.\begin{array}{l}\sqrt{29-x}=a \rightarrow x<29 \\ \sqrt{\frac{x}{7}+7}=b \rightarrow x>-49 \\ -x-1=c \rightarrow x<-1\end{array}\right\} \Rightarrow -49<x<-1$$

$$\log_a b=\log_{c^2} a^2=\log_b c-1=k$$

$$\log_{c^2} a^2 = \log_c a$$

$$\left.\begin{array}{l}a^k=b \\ c^k=a\end{array}\right\} \rightarrow (c^k)^k=b$$

$$b^k=\frac{c}{b} \rightarrow c=b^{k-1}$$

$$\Rightarrow (c^{2k})^{k-1}=c$$

$$c^{2k(k-1)}=c$$

$$2k(k-1)=1$$

$$2k^2-2k-1=0$$

$$k=\frac{2\pm2\sqrt{3}}{2\cdot2}=\frac{1\pm\sqrt{3}}{2}$$

$$-x-1=\left(\frac{x}{7}+7\right)^{\frac{1}{2}\left(\frac{1}{2}(1\pm\sqrt{3})-1\right)}$$

$$\left[\begin{array}{l}-x-1=\left(\frac{x}{7}+7\right)^{\frac{\sqrt{3}}{2}-\frac{1}{2}} \\ -x-1=\left(\frac{x}{7}+7\right)^{-\frac{\sqrt{3}}{2}-\frac{1}{2}}\end{array}\right. \Rightarrow \left[\begin{array}{l}(-x-1)\sqrt{\frac{x}{7}+7}=\left(\frac{x}{7}+7\right)^{\frac{\sqrt{3}}{2}} \\ (-x-1)\sqrt{\frac{x}{7}+7}=\left(\frac{x}{7}+7\right)^{-\frac{\sqrt{3}}{2}}\end{array}\right.$$

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Чистовик

№1 лист 1

НОД $(a, b, c) = 33 \to$ одно из чисел $\vdots\, 3$ и $\not\vdots\, 9$

одно из чисел $\vdots\, 11$ и $\not\vdots\, 11^2$

$a, b, c \;\vdots\; 33 = 3 \cdot 11$

НОК $(a, b, c) = 3^{19} \cdot 11^{15} \to$ одно из чисел $\vdots\, 3^{19}$

одно из чисел $\vdots\, 11^{15}$

a, b, c содержат в разложении на простые только 11^* и 3^*

$a = 3^{a_1} \cdot 11^{a_2}$

$b = 3^{b_1} \cdot 11^{b_2}$

$c = 3^{c_1} \cdot 11^{c_2}$

a_1, b_1, c_1 – 1 из них $= 1$, 1 из них $= 19$, 1 из них ≥ 1 и ≤ 19 пусть это n

$n = 1$	$n = 19$	$1 < n < 19$
$(a_1, b_1, c_1) = (1, 1, 19)$	$(a_1, b_1, c_1) = (1, 19, 19)$	$17 \cdot 3 \cdot 2$ вариантов $= 17 \cdot 6$
3 варианта	3 варианта	$\uparrow$ выбрать n

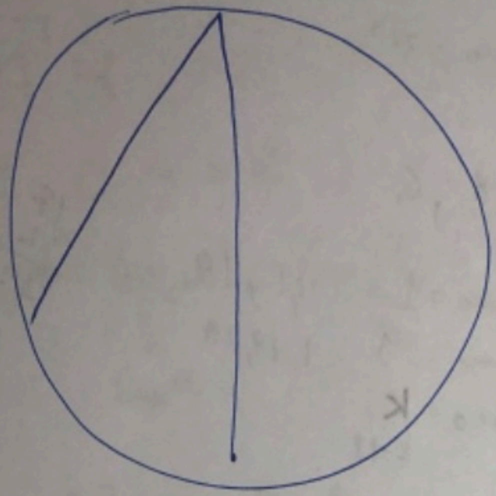
аналогично для $a_2\, b_2\, c_2$

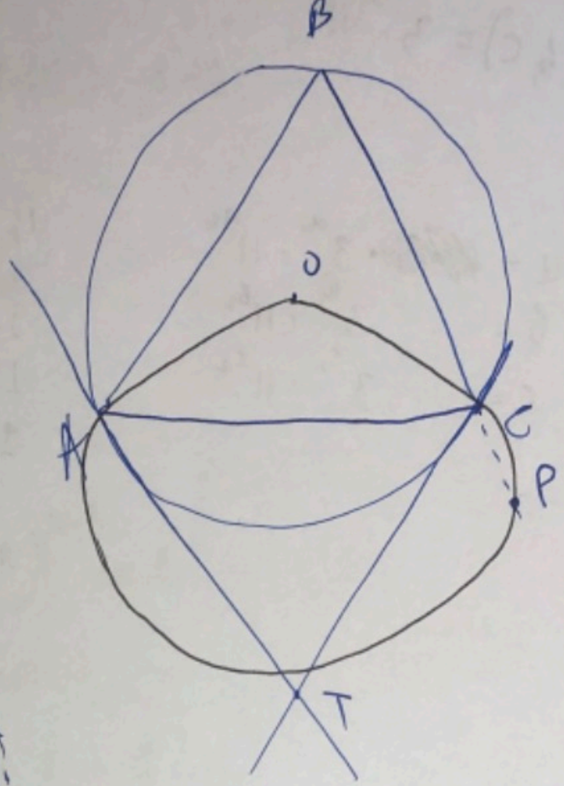
3 вар. $+ 3$ вар $+ 9 \cdot 3 \cdot 2$ вар $= 3 \cdot 20$

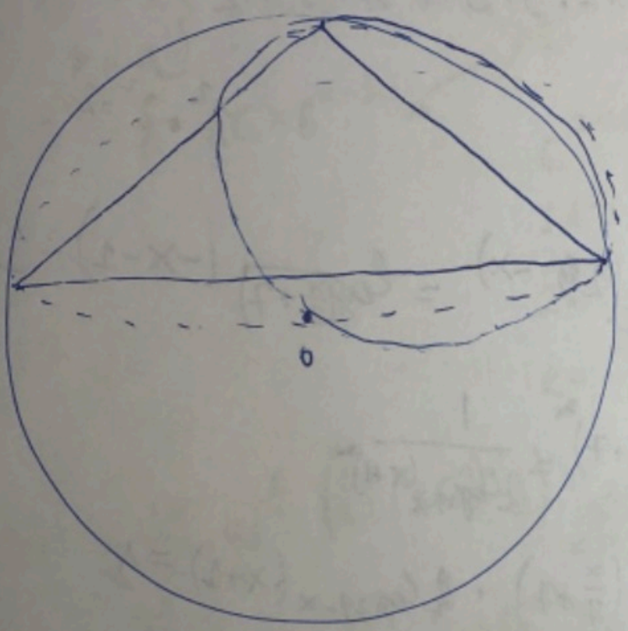
Т.к. a_1, a_2 не зависят, $b_1 b_2$ не зависят и $c_1 c_2$ не зависят друг от друга. $\Rightarrow$

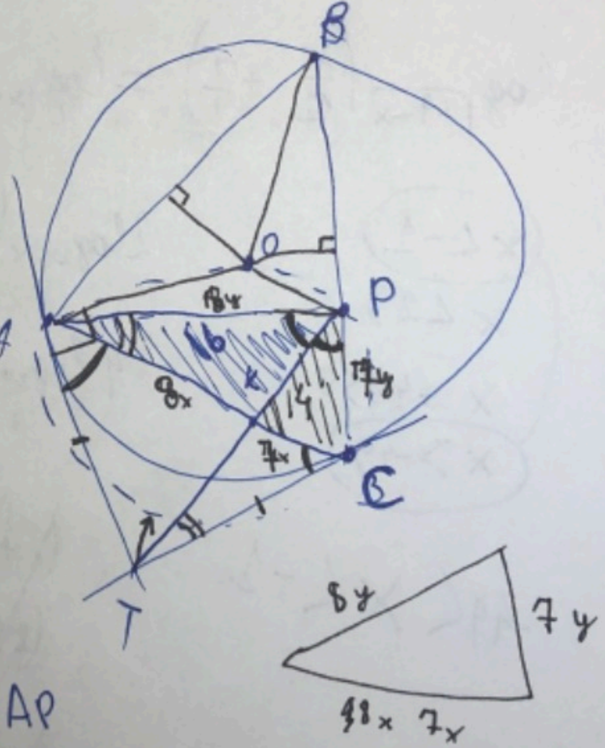
$\Rightarrow \Sigma_{\text{кол. вар.}} = (3 + 3 + 17 \cdot 6) \cdot (3 \cdot 20) = 9 \cdot 20 \cdot 36 = 3^4 \cdot 5 \cdot 2^4$

Ответ: $3^4 \cdot 2^4 \cdot 5$









AP

НОД $(a,b,c)=33=3\cdot 11$ → хотя бы 1 число $\vdots\, 3$ и $\not\vdots\, 9$

НОК $(a,b,c)=3^{19}\cdot 11^{15}$ 1 число $\vdots\, 11$ и $\not\vdots\, 11^2$

1 число $\vdots\, 3^{19}$

$a = 3^{a_1}\cdot 11^{a_2}$

$b = 3^{b_1}\cdot 11^{b_2}$

$c = 3^{c_1}\cdot 11^{c_2}$

a_1, b_1, c_1

1 число $=1$ 1, 1, 19

1 число $=19$ 1, 19, 19

1 число $=k$ 2–18

$17\cdot 3\cdot 2 + 3 + 3$

$\begin{array}{r}17\\ \times\ 2\\ \hline 34\end{array}$

$2^2\cdot 2^2=2^4$

$$\log_{\sqrt{29-x}}\left(\frac{x}{7}+7\right)=\log_{(x+1)^2}(29-x)=\log_{\sqrt{\frac{x}{7}+7}}(-x-1)-1$$

$x<-1$

$x<29$

$x+49>0$

$x>-49$

$-49<x<-1$

$$2\log_{29-x}\left(\frac{x}{7}+7\right)=\frac{1}{2\log_{29-x}(x+1)}$$

$$4\log_{29-x}\left(\frac{x}{7}+7\right)\cdot \log_{29-x}(x+1)=1$$

$\log_{29-x}\left(\frac{x}{7}+7\right) = a$, $\log_{29-x}(x+1) = b$

$\cancel{(29-x)^a=\frac{x}{7}+7}$

$\cancel{(29-x)^b=(x-1)}$

$\cancel{(29-x)^{a+b}=\left(\frac{x}{7}+7\right)(x-1)}$

$\cancel{(29-x)^{a-b}=\frac{\frac{x}{7}+7}{x-1}}$

$$\frac{2}{\log_{\frac{x}{7}+7}29-x}=\log_{\sqrt{\frac{x}{7}+7}}(-x-1)-\log_{\sqrt{\frac{x}{7}+7}}\sqrt{\frac{x}{7}+7}$$

$$\log_{\sqrt{\frac{x}{7}+7}}\frac{-(x+1)}{\sqrt{\frac{x}{7}+7}}=2\log_{\frac{x}{7}+7}\frac{(x+1)^2}{\frac{x}{7}+7}$$

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$\sqrt{29-x} = a \qquad a^2 = 29 - x$

$\sqrt{\frac{x}{7}+7} = b \qquad b^2 = \frac{x}{7}+7$

$-x+1 = c$

$\log_a b = \log_{c^2} a^2 = \log_b c - 1$

$\log_c a$

$\log_b c - 1 = \log_b c - \log_b b = \log_b \frac{c}{b}$

$D = 4 + 8 = 12$

$\log_a b = \log_c a = \log_b \frac{c}{b} = k$

$a^k = b$

$c^k = a$

$b^k = \frac{c}{b}$

$\boxed{\begin{cases} a = c^k \\ b = a^k \end{cases}}$

$\sqrt{29-x} = (x+1)^k$

$\sqrt{\frac{x}{7}+7} = ((x+1)^k)^k$

$\sqrt{\frac{x}{7}+7} = \sqrt{29-x}^k$

$\begin{cases} a^2 + 7b^2 = 29 + 49 = 78 \\ a^2 + c = 30 \end{cases}$

$c = 30 - a^2$

$\begin{cases} a = c^k \\ b = c^k \\ c = b^{k-1} \end{cases}$

$c^1 = (c^k)^{k-1} = c^{k(k-1)}$

5

6

$c^{2k} + c = 30$

$c = 30 -$

$k(k-1) = 1$

$k^2 - k - 1 = 0$

$D = 1 + 4 = 5$

$c^{2k} + 7a^{2k}$

$k = \frac{1 \pm \sqrt{5}}{2}$

$\sqrt{29-x} = (x+1)^{\frac{1\pm\sqrt{5}}{2}}$

$\frac{1}{2} + \frac{\sqrt{3}}{2} - 1$

$\downarrow 29 - x = (x+1)^{1\pm\sqrt{5}} \quad \uparrow^{+} \quad \downarrow^{-}$

$\frac{\sqrt{3}}{2} - \frac{1}{2}$

$-\frac{\sqrt{3}}{2} - \frac{1}{2}$