

Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

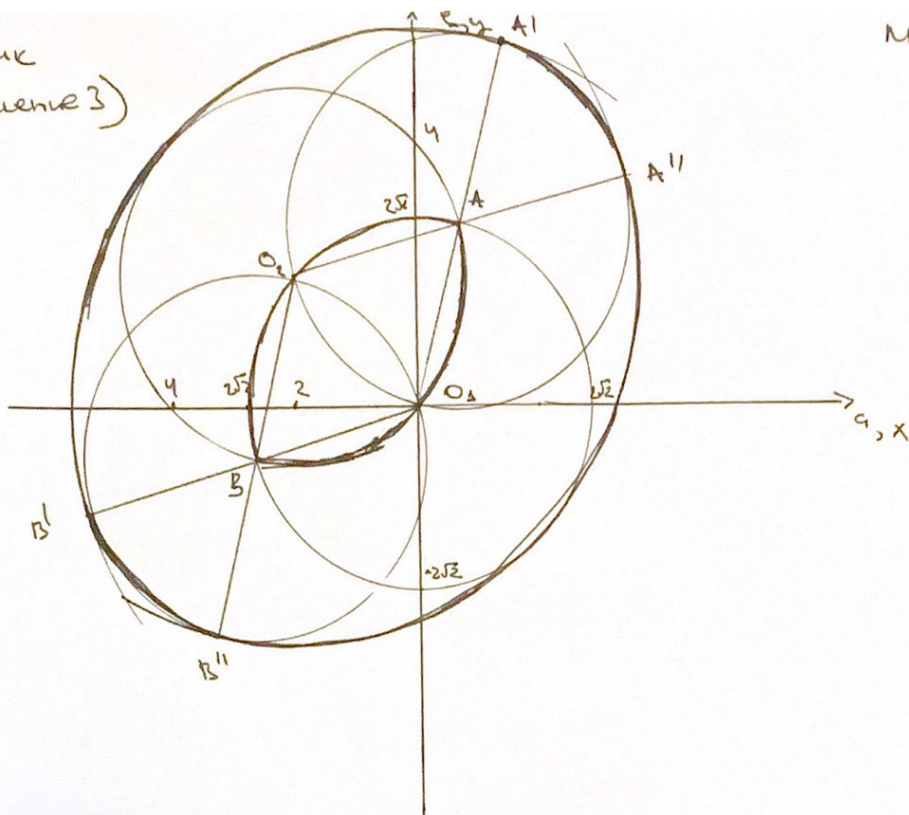
Шифр: **21103439**

ID профиля: **322702**

Вариант 23

Система
Проекции 3)

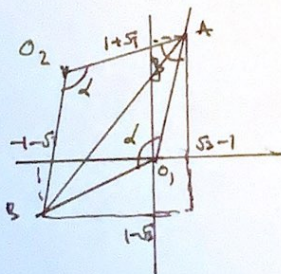
МАТЕМАТИКА
2 часть



Если от любой точки на дуге (верхней) AB (окружности с центром O_1) отложить окружность с радиусом $2\sqrt{2}$, то получится сектор окружности с радиусом $4\sqrt{2}$ и центром O_1 и 2 полуокружности с центрами A, B и радиусами $2\sqrt{2}$.

Если от любой точки на дуге AB (нижней) (окружности с центром O_2) отложить окружность с радиусом $2\sqrt{2}$, то получится сектор окружности с радиусом $4\sqrt{2}$ и центром O_2 и 2 полуокружности с центрами A, B и радиусами $2\sqrt{2}$.

При наложении получается то, что на рисунке ↑.



$$\angle BO_2A = \angle BO_1A = \alpha$$

$$\beta = 2 \cdot \frac{180 - \alpha}{2} = 60' = \frac{\pi}{3}$$

$$AB = \sqrt{(2\sqrt{3})^2 + (2\sqrt{3})^2} = \sqrt{24} = 2\sqrt{6}$$

$$24 = 8 + 8 - 2 \cdot 8 \cdot \cos \alpha$$

$$\cos \alpha = -\frac{1}{2}$$

$$\alpha = 120' = \frac{2\pi}{3}$$

↓

$$S_{\text{сектора } O_1 B A} = \frac{2\pi}{3} \cdot 8$$

$$S_{\text{сектора } O_2 B A} = \frac{2\pi}{3} \cdot 8$$

$$S_{\text{сектора } A A' A''} = \frac{\pi}{3} \cdot 8 \quad 2 \text{ и } 3 \cdot 8$$

$$S_M = 8 \cdot \left(\frac{2\pi}{3} + \frac{2\pi}{3} + \frac{\pi}{3} + \frac{\pi}{3} \right) = \frac{S_{\text{сектора } B B' B''} = \frac{\pi}{3} \cdot 8}{16\pi} \leftarrow \text{ответ}$$

Задание

Д)

$$S = 6a_1 + 15d$$

$$d > 0$$

$$a_1 \in \mathbb{Z}$$

$$d \in \mathbb{Z}$$

Математика

Задача

$$a_{10} = a_1 + 9d$$

$$a_{16} = a_1 + 15d$$

$$(a_1 + 9d)(a_1 + 15d) \geq 6a_1 + 15d + 35$$

$$a_{11} = a_1 + 10d$$

$$a_{15} = a_1 + 14d$$

$$(a_1 + 10d)(a_1 + 14d) < 6a_1 + 15d + 55$$

$$\begin{aligned}
 & \cancel{a_1^2 + 24a_1d + 135d^2} \geq \cancel{6a_1 + 15d + 35} \\
 + & \quad \quad \quad \cancel{6a_1 + 15d + 55} > \cancel{a_1^2 + 24a_1d + 140d^2}
 \end{aligned}$$

$$16 > 5d^2$$

$$\frac{16}{5} > d^2 \quad (d > 0)$$

$$\frac{4}{\sqrt{5}} > d$$

$$\boxed{d \leq 1} \leftarrow \text{невозможно}$$

$$d \leq 2 \leftarrow \text{невозможно}$$

$$\begin{cases}
 a_1^2 + 24a_1 + 135 \geq 6a_1 + 54 \\
 a_1^2 + 24a_1 + 140 < 6a_1 + 70
 \end{cases}$$

$$a_1^2 + 18a_1 + 81 \geq 0$$

$$\begin{aligned}
 D_1 &= 81 - 81 = 0 \leftarrow \text{верно} \\
 &\quad \quad \quad \uparrow \\
 &\quad \quad \quad \text{верно} \geq 0
 \end{aligned}$$

~~$$a_1^2 + 18a_1 + 140 < 0$$~~

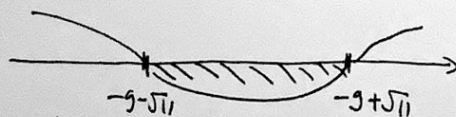
$$a_1^2 + 18a_1 + 70 < 0$$

$$\frac{D_2}{4} = 81 - 70 = 11$$

корни:

$$a_1 = -9 \pm \sqrt{11}$$

$$(a_1 + 9 - \sqrt{11})(a_1 + 9 + \sqrt{11}) < 0$$



$$a_1 \in (-9 - \sqrt{11}; -9 + \sqrt{11})$$

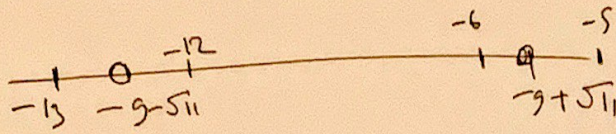
Знач

Задача

Прогноз

Математика
1 часть

$$a_1 \in \mathbb{Z}$$



$$3 < \sqrt{11} < 4$$

$$-6 < -9 + \sqrt{11} < -5$$

$$-3 > -\sqrt{11} > -4$$

$$-12 > -9 - \sqrt{11} > -13$$

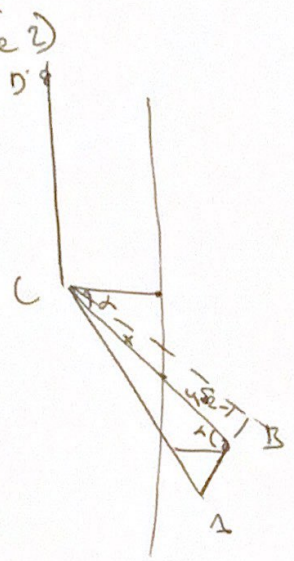
$$a_1 = \{-12; -11; -10; -9; -8; -7; -6\}$$

Ответ ↗

4.26

Задача
Продолжение 2)

Математика
1 часть

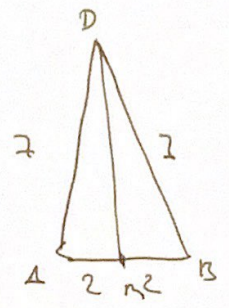


$$x \cdot \cos \alpha = (4\sqrt{2} - x) \cos \alpha$$

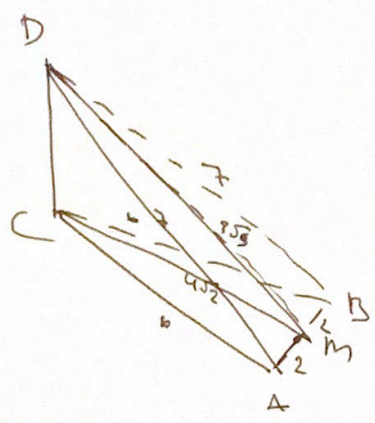
$$2x \cos \alpha = 4\sqrt{2} \cos \alpha$$

$$x = \frac{4\sqrt{2}}{2} = \boxed{2\sqrt{2}}$$

$$V = 2\sqrt{2} \cdot \cos \alpha \quad (\alpha \in [-\frac{\pi}{2}; \frac{\pi}{2}])$$



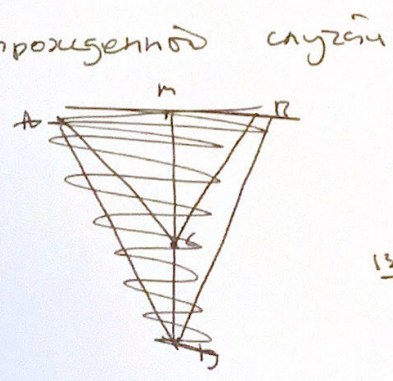
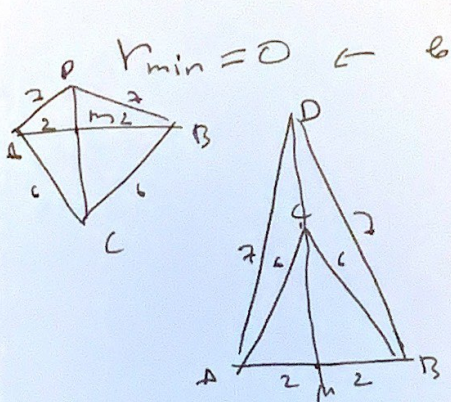
$$DM = 3\sqrt{5}$$



$$45 = CD^2 + 32 - 2 \cdot 4\sqrt{2} \cdot CD \cdot \cos(90^\circ + \alpha)$$

$$CD^2 - 13 = -8\sqrt{2} CD \cdot \sin \alpha$$

$$\sin \alpha = \frac{13 - CD^2}{8\sqrt{2} CD}$$



$V_{\min} = 0$ ← возникает нуль из-за нулевой площади

$$\cos \alpha = 0$$

$$\sin \alpha = \pm 1$$

$$\frac{13 - CD_1^2}{8\sqrt{2} CD_1} = 1$$

$$13 - CD_1^2 - 8\sqrt{2} CD_1 = 0 \quad \text{Quadratic}$$

$$CD_1^2 + 8\sqrt{2} CD_1 - 13 = 0$$

$$CD_1 = -4\sqrt{2} + 3\sqrt{5} > 0 \Rightarrow$$

$$\Rightarrow CD_1 = 3\sqrt{5} - 4\sqrt{2}$$

$$\frac{13 - CD_2^2}{8\sqrt{2} CD_2} = -1$$

$$\frac{13 - CD_2^2 + 8\sqrt{2} CD_2}{8\sqrt{2} CD_2} = 0$$

$$CD_2^2 - 8\sqrt{2} CD_2 - 13 = 0$$

$$CD_2 = 4\sqrt{2} + 3\sqrt{5} > 0 \Rightarrow$$

$$\Rightarrow 4\sqrt{2} + 3\sqrt{5}$$

$$\boxed{CD = 3\sqrt{5} \pm 4\sqrt{2}}$$

$$S = a + a^2d + a^4 + 2d + \dots + a + sd$$

$$63 + S = 916 \Rightarrow S = 853$$

$$55 + S = 915 \Rightarrow S = 860$$

$$55 + 59 + 25$$

$$PS + Pn + P2 + P2 + P$$

$$\frac{135}{54} = \frac{15}{6}$$

OLP
 $\begin{matrix} \nearrow a \\ \nearrow d \end{matrix}$

$$a_1^2 + 9a_1d + 15a_1d + 135d^2 \geq 6a_1 + 15d + 35$$

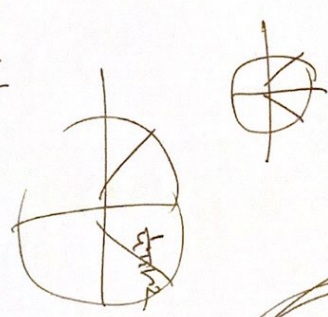
$$d^2 + 24a_1d + 135d^2 \geq 6a_1 + 15d + 35$$

$$6a_1 + 15d + 35$$

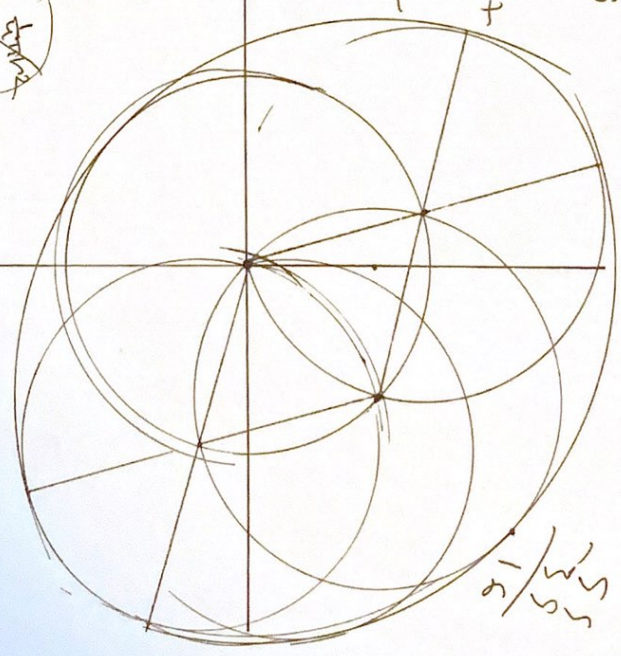
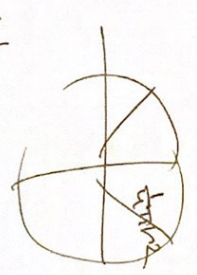
$$d^2 + 16a_1d + 14a_1d + 14d^2$$

$$16 \Rightarrow 5d^2$$

$$d = 2 \Rightarrow \delta = 4 \Rightarrow \sin \alpha = 20$$



$$157 + 524$$



$$\frac{15}{35}$$

$$(-1 + \sqrt{3})^2 = -16 \cos \delta = 8$$

$$\cos \delta = -\frac{1}{2}$$

$$= 1 - \sqrt{3} - \sqrt{3} + 3$$

$$-2 + 2\sqrt{3} - 2$$

$$a^2 + 24 - 2 = 0$$

$$1 + 2 = \sqrt{3}$$

$$-1 \pm \sqrt{3}$$

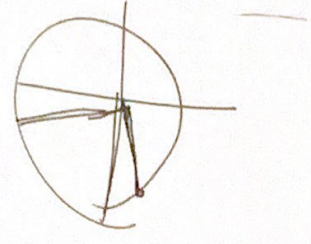
$$u_1 + u_2 = 24$$

$$\sqrt{8.2} = 2\sqrt{2}$$

$$\frac{6T}{2} = 2\pi r = 16\pi$$

$355 - 4\sqrt{2}$

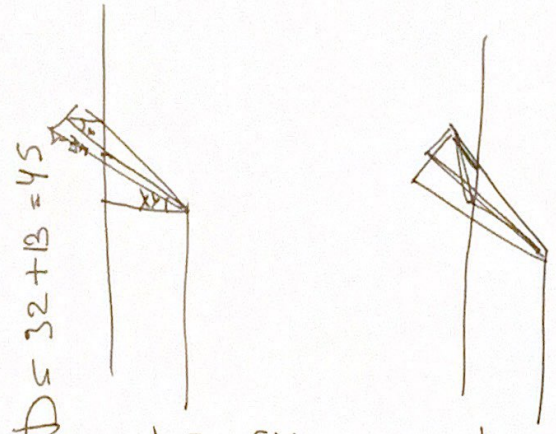
$36 - 4 = \sqrt{32} = \sqrt{4 \cdot 8} = 2 \cdot 2\sqrt{2} = 4\sqrt{2}$
 $\cos(90^\circ + \alpha) = \cos \alpha \cdot 0 - \sin \alpha \cdot 1$



$\frac{13 - 4B}{8\sqrt{2}CP} \leq 1$

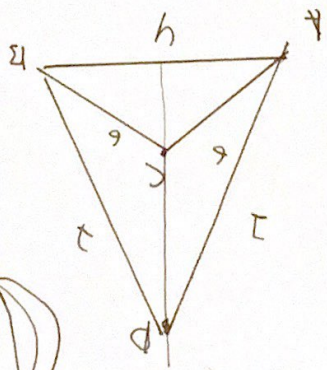
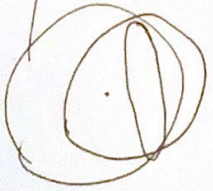
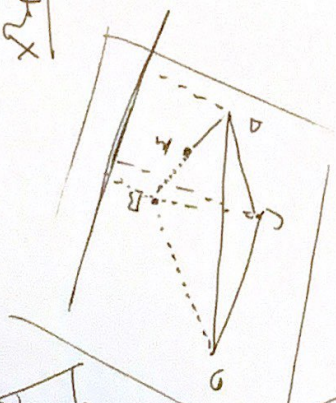
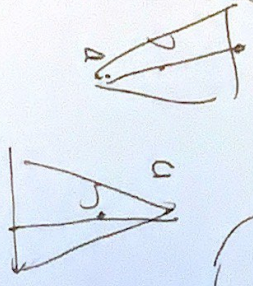
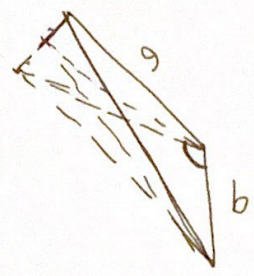
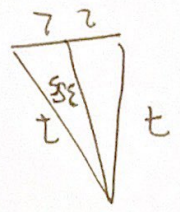
$\frac{13 - X^2 - 8\sqrt{2}X}{8\sqrt{2}X} \leq 0$

$\frac{X^2 + 8\sqrt{2}X - 13}{8\sqrt{2}X} \geq 0$
 $X = -4\sqrt{2} \pm 3\sqrt{5}$
 $\phi \leq 32 + 13 = 45$

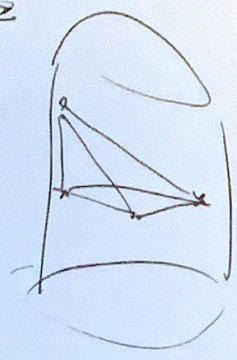


$4S - 4 = \sqrt{32} = 3\sqrt{2}$

$\frac{13}{32} - 4S$



$(4\sqrt{2})$
 $32 + 13 = 45$

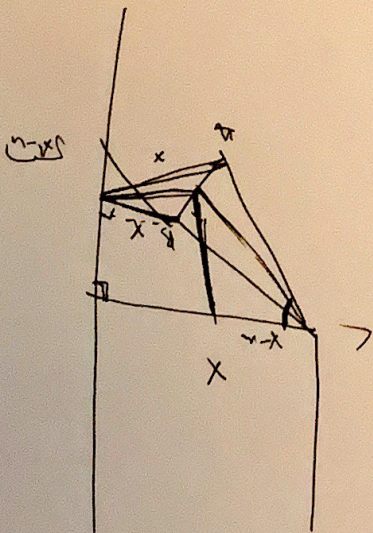


CD // OA

CD < 9
 (min)

$AB = 4$
 $AC = CB = 6$
 $AD = DB = 7$

$$\cos \alpha = \frac{x - \sqrt{x^2 - u}}{4\sqrt{u}}$$



Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21103439**

ID профиля: **322702**

Вариант 23

Задание

Математика Задание

$$\begin{aligned} 5) \\ X &= \log_{\sqrt{x+34}}^{(2x+23)} \\ Y &= \log_{(x+4)^2}^{(x+34)} \\ Z &= \log_{\sqrt{2x+23}}^{(x-4)} \end{aligned}$$

$$\begin{aligned} A &= \sqrt{x+34} \\ B &= \sqrt{2x+23} \\ C &= -x-4 \end{aligned}$$

$$\begin{aligned} 2x+23 > 0 \\ x+34 > 0 \\ -x-4 > 0 \\ \sqrt{x+34} \neq 1 \\ (x+4)^2 \neq 1 \\ \sqrt{2x+23} \neq 1 \\ \sqrt{x+34} > 0 \\ (x+4)^2 > 0 \\ \sqrt{2x+23} > 0 \end{aligned}$$

$$X = \log_A B^2 ; \quad Y = \log_C A^2 ; \quad Z = \log_B C$$

$$\frac{X}{2} \cdot Y \cdot Z = \log_A B \cdot \log_C A \cdot \log_B C = 1$$

$$XYZ = 2$$

Для n_3 $nux = n$ $metho - n+1$

$$n \cdot n \cdot (n+1) = 2$$

$$n^3 + n^2 - 2 = 0$$

$$(n-1)(n^2+2n+2) = 0$$

$\frac{b}{a} = 1-2 < 0$

$$\boxed{n=1}$$

$$\begin{cases} \log_{\sqrt{x+34}}^{2x+23} = 2 \\ \log_{(x+4)^2}^{(x+34)} = 2 \\ \log_{\sqrt{2x+23}}^{(x-4)} = 2 \end{cases}$$

$$\begin{cases} 2x+23 = x+34 \\ x+34 = (x+4)^2 \\ -x-4 = 2x+23 \end{cases}$$

↓ 425

Задача переопределенная 5)

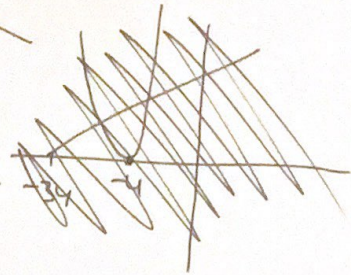
Математика
2 часть

$$x = 11$$

$$x + 34 = x^4 + 16x^3 + 96x^2 + 256x + 256$$

$$x = -\frac{27}{3} \Rightarrow$$

это решать
сложно
попробуй
и правдо
2 пути
1



$$x > -\frac{23}{2}$$

$$x > -34$$

$$x < -4$$

$$x \neq -33$$

$$x \neq -3$$

$$x \neq -1$$



$$x = 11 \quad \text{✗}$$

$$\boxed{x = -9} \quad \text{✓}$$

$$\log_{\sqrt{2x+23}}^{-x-4} = 1$$

$$-x-4 = \sqrt{2x+23}$$

$$x^2 + 8x + 16 = 2x + 23$$

$$x^2 + 6x - 7 = 0$$

$$(x+7)(x-1) = 0 \quad \begin{cases} x=1 \quad \text{✗} \\ x=-7 \end{cases}$$

$$\log_{\sqrt{27}}^{(23+4)} = 1?$$

$$9 = \sqrt{27} \leftarrow x$$

↓
варианта
 $x = 7 = 1$
 $y = 2$

не может быть

$$\log_{\sqrt{-9+34}}^{(-9+23)} = 1?$$

$$5 = \sqrt{25} \quad \text{✓}$$

$$\log_{(-9+4)^2}^{(-9+34)} = 1?$$

$$25 = 25 \quad \text{✓}$$

Решение

$$\text{возможный } x - \boxed{x = -9}$$

2ч25

4)

$$a = 22 \cdot a_1$$

$$b = 22 \cdot b_1$$

$$c = 22 \cdot c_1$$

$$\text{НОД}(a; b; c) = 1 \Rightarrow \text{НОК}(a; b; c) =$$

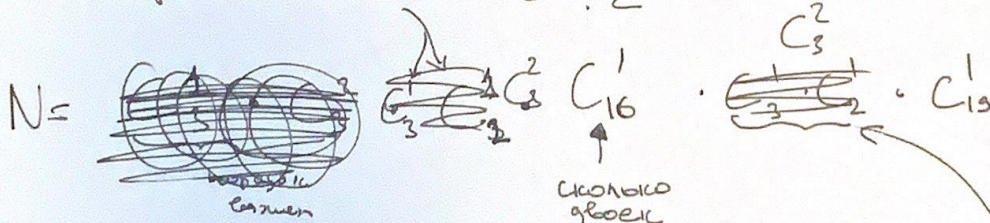
$$= 22 \cdot a_1 \cdot b_1 \cdot c_1 = 2^{16} \cdot 11^{19}$$

$$a_1 \cdot b_1 \cdot c_1 = 2^{15} \cdot 11^{18}$$

Максимум 2 \cup_3 раз числа : 2

Максимум 2 — на 11

Вот перем, куча 2 : 2



циклоко
 группа
 в осях \cup_3
 вограничен (от 0 до 15)
 автоморфизмы образует
 локало группа в
 группах

* куча гла : 11

циклоко 11
 вограничен
 в осях \cup_3
 вограничен (от 0 до 18)
 автоморфизмы, циклоко в
 группах

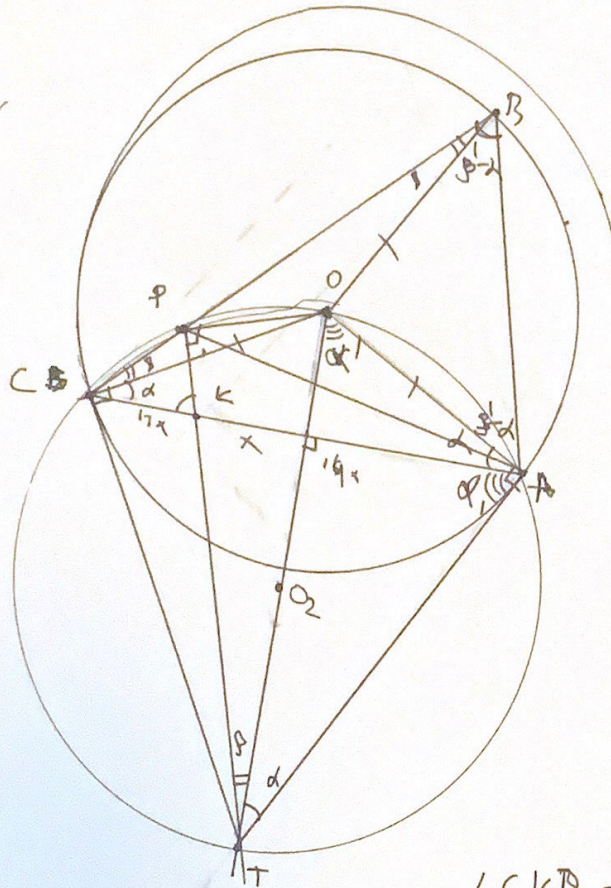
~~$$3 \cdot 16 \cdot 3 \cdot 19 = 2736$$~~

$$N = 16 \cdot 19 \cdot 3 \cdot 3 = 2736$$

3 \cup_3 5

Задача
6)

Математика
2 часть



Касательная перпендикулярна в точке, следовательно $\angle O_2AT = 90^\circ$ (по свойству окружности)
 $\angle LOAT = \angle OCT = 90^\circ$

высота из P на CA

$$S_{CPK} = \frac{1}{2} h \cdot CK = 13$$

$$S_{APK} = \frac{1}{2} h \cdot AK = 15$$

$$\frac{CK}{AK} = \frac{13}{15}$$

$$\angle CKP = 90 - \beta$$

$$\angle COB = 180 - 2\beta \Rightarrow$$

$$\Rightarrow \angle CAB = 90 - \beta \Rightarrow$$

$$\Rightarrow PK \parallel AB \Rightarrow$$

$$\Rightarrow S_{ABC} = \frac{13 \cdot 288}{13^2} = \frac{288}{13}$$

$$\angle CBA = \arcsin \frac{4}{7} \approx 34^\circ$$

$$\frac{AC}{\sin \angle CBA} = 2R$$

$$AC = \sin \arcsin \frac{4}{7} \cdot 2R$$

и из 5

Знаходимо

mathematics
2202

6. ~~напоказуємо~~

$$\alpha' = 90 - \alpha \quad \beta' = 90 - \beta$$

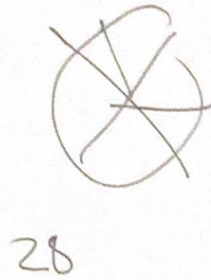
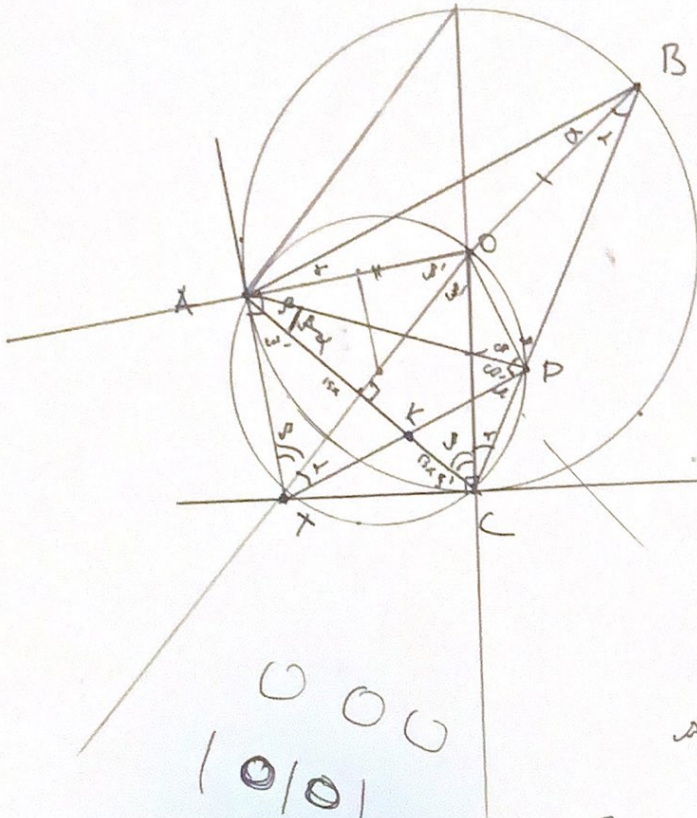
$$S = 4R^2 \cdot \sin \alpha' \cdot \sin \alpha' \cdot \sin(\alpha + \beta) = \frac{784}{13}$$

$$AC = 2R \cdot \sin \alpha'$$

543 S

SAPK = 15
 SCPK = 13

SABS



28

$$\begin{array}{r} \times 28 \\ 28 \\ \hline 224 \\ 56 \\ \hline 284 \end{array}$$

$$13 \times 9 \cdot \sin \alpha = 13$$

$$28 \times 9 \cdot \sin \alpha = 28$$

$\beta = \alpha$

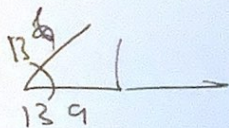
$\cos(\alpha + \beta) = 1$

$\cos 2\alpha = 1$

$2\alpha = 0^\circ$

$\cos(180 - 2\alpha) = -\cos 2\alpha$

$$\begin{array}{c} \circ \circ \circ \\ | \circ / \circ | \end{array}$$



$\frac{1}{2} \cdot 28 \cdot 25$

$$\frac{1}{2} \cdot 12 \cdot 13 \cdot \sin B = \frac{1}{2} \cdot 19 \cdot 16$$

$$25 \cdot \sin B = 152$$

$$\sin B = \frac{152}{25}$$

$50 - \alpha = \beta + \gamma$

$$\begin{array}{r} \times 204 \\ 9 \\ \hline 2736 \end{array}$$

$$\begin{array}{r} 15 \\ 15 \\ \hline 225 \end{array}$$

$\angle LAB = 50 - \beta$

$180 - \beta - \beta - \beta$

$$a = 22a_1 \quad \Delta(a, b, c) = 1$$

$$b = 22b_1$$

$$c = 22c_1$$

$$22a_1 b_1 c_1 = 2^{16} \cdot 11^{19}$$

$$a_1 \cdot b_1 \cdot c_1 = 2^{15} \cdot 11^{18}$$

$$2^k \cdot 11^p \quad 2^m \cdot 11^s \quad 2^r \cdot 11^t$$

$$k + m + r = 15$$

$$2^2 \cdot 3^3$$

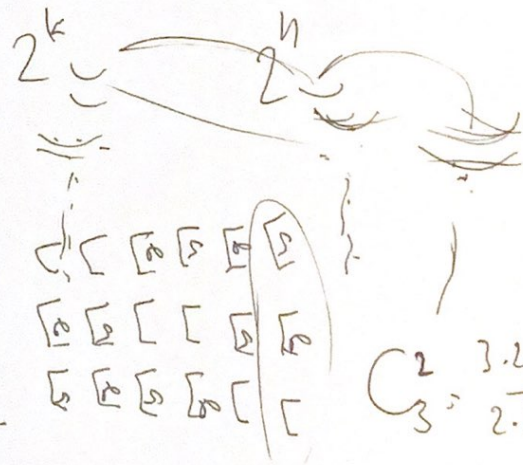
2	2	0	0	0
1	4	0	0	0
5	1	0	0	0

$$6 \cdot 3 = 18$$

$$C_3^2 = \frac{3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 1} = 3$$

$$C_{16}^2$$

$$C_{15}^2$$



$$C_3^2 = \frac{3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 1}$$

$$\begin{array}{r} 36 \\ \times 16 \\ \hline 216 \\ + 36 \\ \hline 576 \end{array}$$

$$C_{16}^2 \cdot C_{15}^2 \cdot C_3^2$$

$$\begin{array}{r} 4 \quad 1 \\ 4 \quad 4 \quad 1 \\ 4 \quad 1 \quad 1 \end{array}$$

$$\begin{array}{r} \times 576 \\ 19 \\ \hline 5184 \\ 576 \\ \hline 10944 \end{array}$$

$$3 \cdot 3 \cdot 3 \cdot 3 = 81$$

$$3 \cdot 2 \cdot 16 \cdot 3 \cdot 2 \cdot 15 = 10944$$

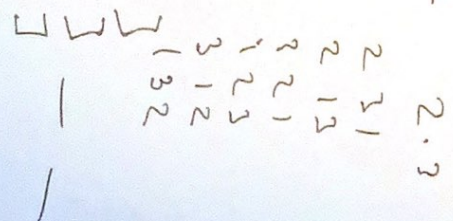
$$= 36 \cdot 16 \cdot 15$$

$$2^2 \cdot 3^3$$

$$(3 \cdot 3 \cdot 3) = 27$$

$$4$$

$$9$$



$$\frac{2 \sin \frac{\alpha}{2} + 2 \sin \frac{\alpha}{2}}{2}$$

$$\frac{AC}{\sin \beta} = \frac{CB}{\sin \alpha} = 2R$$

$$AC \cdot CB \cdot \sin \gamma = S$$

$$2R \cdot \sin \beta \cdot 2R \cdot \sin \alpha \cdot \sin \gamma = S$$

$$\log_{\sqrt{x+34}}(2x+23)$$

$$\log_{(x+4)^2}(x+34)$$

$$\log_{\sqrt{2x+23}}(-x-4)$$

$$A > 0$$

$$B > 0$$

$$C > 0$$

$$\log_{A^2} B^2$$

$$\log_{C^2} A^2$$

$$\log_B C$$

x

y

z

$$x = y = z - 1$$

$$x + y + z = 1$$

$$2x + z = 1$$

$$3x + 1 = 1$$

$$x = 0$$

$$\log_A B^2 + \frac{\log_A A^2}{\log_A C^2} + \frac{\log_A C}{\log_A A}$$

$$\log_A C$$

$$3x + 1 = 1$$

$$x = 0$$

$2 \log_{x+34}$
 $\frac{1}{2} \log_{x+34}$
 $2 \log_{2x+23}$
 $x+34$
 $x+34$
 $x+34$
 $2x+23$
 $2x+23$

$\log_A B \cdot \log_B C$
 $R = 2x + 2z$

$$\log_A B^2 \cdot \log_A C^2 \cdot \log_A B + \log_A A^2 \cdot \log_A B + \log_A C \cdot (\log_A C^2)$$

$$\frac{2(\log_A B)^2 \cdot 2 \log_A C \cdot \log_A B + \sqrt{2} \cdot \log_A A^2 + 2(\log_A C)^2}{2 \log_A C \cdot \log_A B}$$

$$2N^2 \cdot M + N + 2M^2$$

$$\frac{2N^2 \cdot M + N + 2M^2}{MN} = 2N + \frac{1}{M} + \frac{2M}{N}$$

$$\frac{2 \cos \theta + \frac{1}{\cos \theta} + \cos \theta}{\cos \theta + \cos \theta} = \frac{2 \cos \theta + \frac{1}{\cos \theta} + \cos \theta}{2 \cos \theta}$$

$$2 \cos \theta + \frac{1}{\cos \theta} + \cos \theta = 2 \cos \theta$$

1	5	9	1
2	5	1	2
3	9	4	3
4	9	1	4
5	1	9	4
6	1	4	9
7	5	3	5
8	5	3	5
9	3	5	5
10	3	5	5

$5-4=5$
 23
 $9-4=5$
 23
 $0=7-2=5$
 $X+9=0$

11	3	3	5
12	9	4	3
13	2	9	2
14	2	2	5
15	9	2	2
16	9	2	2
17	2	9	2
18	2	2	5
19	2	3	8
20	2	6	3

21	6	2	3
22	6	3	2
23	3	2	6
24	3	6	2
25			
26			
27			

$27-4=23$
 $-27=3X$
 $-9=5X$
 $-9-4=13$
 -23
 -18
 4

23
 53
 4
 16
 111
 81
 34

$X^4 + 6X^3 + 16X^2 + 128X + 256$
 $6X^3 + 60X^2 + 166X + 256$
 $416X^2 + 166X + 256$

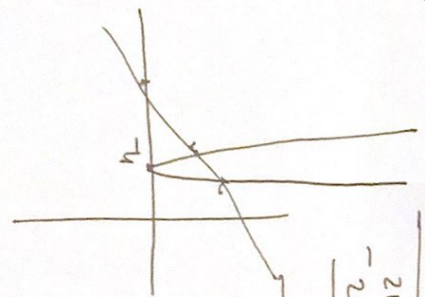
$(X+8)(X+16)$

$(X+U)$

$\frac{X^2+27}{9}$

$1-16+96-255+222$

$X^4 + 16X^3 + 96X^2 + 255X + 222 = 0$



$\frac{16n^2-2}{-16n^2+2} \cdot \frac{16n-2}{16n^2+2n+2}$

$\frac{2n-2}{2n^2-2n}$
 $\frac{2n-2}{2n-2}$

$\frac{16}{96} \cdot \frac{16}{256}$
 $\frac{16}{96}$
 $\frac{16}{256}$

$\sqrt{16+256} > 1$

$2X+23 = X+34$

$9-4=-13$
 $18+23$

$4X-4=2X+23$
 $-27=3X$
 $X=-\frac{27}{3}$
 $X=-9$
 $4X+23=1$
 $X=-2$
 $X < -4$
 $X > -4$
 $-9-4=-13$