Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21100076**

ID профиля: **892436**

Вариант 23

№1 Чистовик

Пусть $x = a_1 \in \mathbb{Z}$; $y = \alpha \in \mathbb{Z} > 0$

Тогда:

$$\begin{cases}(x+9y)(x+15y) > 6x+15y+39 \\ (x+10y)(x+14y) < 6x+15y+55\end{cases}$$

$$\begin{cases}x^2+24xy+135y^2-6x-15y-39>0 \quad (1) \\ x^2+24xy+140y^2-6x-15y-55<0 \quad (2)\end{cases}$$

(2) - (1): $5y^2 < 16 \Rightarrow y = 1$

$$\begin{cases}x^2+18x+81>0 \\ x^2+18x+70<0\end{cases}$$

$x \neq 9$

$x \in (-9-\sqrt{11}; -9+\sqrt{11})$

$-13 < -9-\sqrt{11} < -12$ $\qquad$ $-6 < -9+\sqrt{11} < -5$

Ответ: $-12; -11; -10; -8; -7; -6$

1

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Чистовик

№3

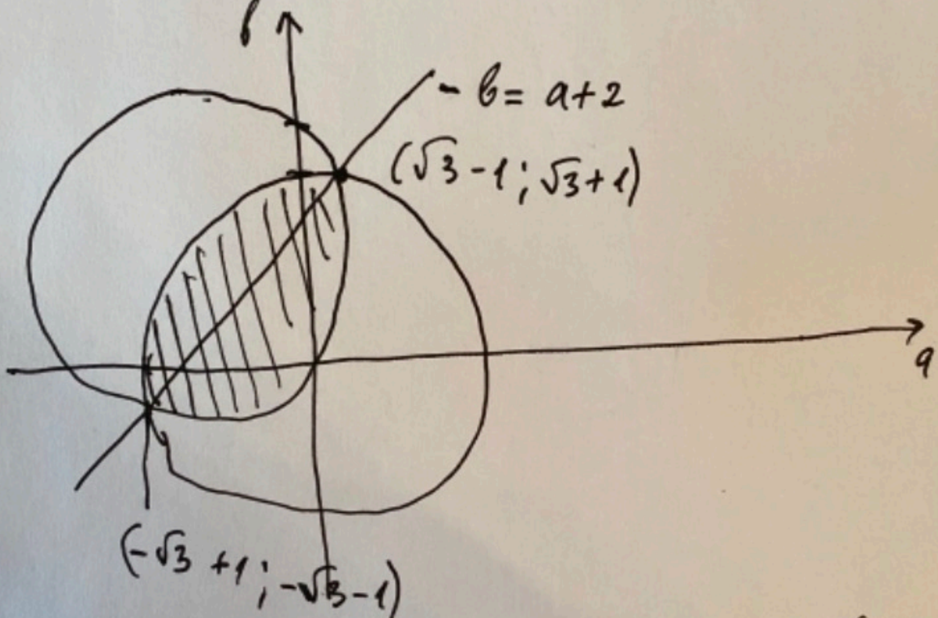
$$a^2+b^2 \leq \min(-4a+4b,\ 8)$$

$$\Updownarrow$$

$a^2+b^2 \leq 8$ при $-4a+4b > 8$

$a^2+b^2 \leq -4a+4b$ при $-4a+4b > 8$

$$\begin{cases} a^2+b^2 \leq 8 & \text{при } b > a+2 \\ (a+2)^2+(b-2)^2 \leq 8 & \text{при } b \leq a+2 \end{cases}$$



~~Для которых (a, b) [illegible]~~

$\cancel{(x-a)^2+(y-b)^2=8}$

Θ $(x-a)^2+(y-b)^2 \leq 8$

2

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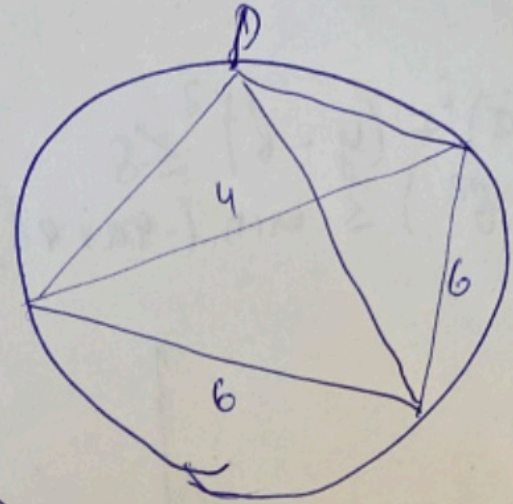

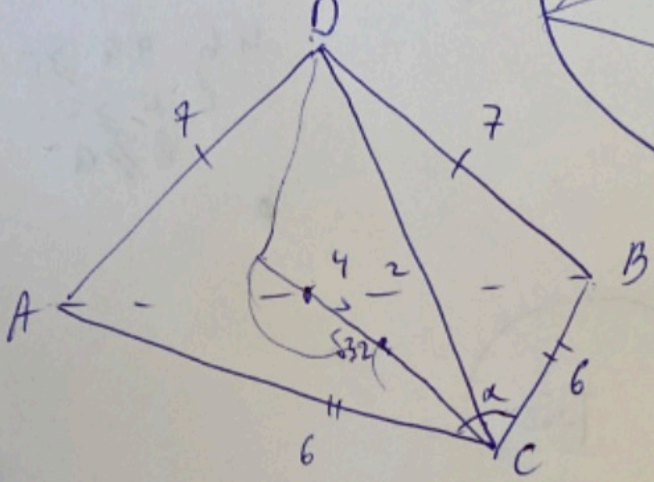

$36-4$

$2R =$

$\sqrt{32} < x < 13$

$16 = 36+36-2\cdot 36\cdot \cos\alpha$ $\quad 4\sqrt{2} < x < 13$

$6 < x < 13$

$72\cos\alpha = 52$

$\cos\alpha = \frac{52}{72} = \frac{26}{36} = \frac{13}{18}$

$\sin\alpha = \sqrt{\frac{5}{18}} =$

$2R = \frac{4}{\sqrt{\frac{5}{18}}} = \frac{\sqrt{18}\cdot 2}{\sqrt{5}} = R$

$\frac{36}{6}$ $\quad \frac{5}{2}$

$\frac{-72}{16}$ $\quad \frac{72}{-16}$ $\quad 56$

$6^2 + x < 169$

$a^2+b^2=c$

$x+6$

8

$72\cos\alpha = 56$

$\cos\alpha = \frac{56}{72} = \frac{23}{36}$

$\frac{56}{72} = \frac{28}{36} = \frac{14}{18} = \frac{7}{9}$

$\sin\alpha = \sqrt{\frac{2}{9}}$

$\sin\alpha = \frac{\sqrt{2}}{9}$

$2R = \frac{4\sqrt{2}}{8}$ $\quad \frac{36}{\sqrt{2}}$

$R = \frac{18}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2}} = 3$

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$S = a_1 + a_2 + \dots + a_6 = 6a + 15d$

$$\begin{cases} a_{10}\, a_{16} > S + 39 \\ a_{11}\, a_{15} < S + 55 \end{cases}$$

$$\begin{cases} (a_1+9d)(a_1+15d) > 6a_1 + 15d + 39 \\ (a_1+10d)(a_1+14d) < 6a_1 + 15d + 55 \end{cases}$$

$$\begin{cases} a_1^2 + 9da_1 + 15da_1 + 135d^2 > 6a_1 + 15d + 39 \\ a_1^2 + 10da_1 + 14da_1 + 140d^2 < 6a_1 + 15d + 55 \end{cases} \quad \Big| \ominus$$

$\cancel{a_1^2 + 24da_1 - 6a}$

$\cancel{5d^2}$

$$\begin{cases} a_1^2 + 24da_1 - 6a_1 + 135d^2 - 15d - 39 > 0 \\ a_1^2 + 24da_1 - 6a_1 + 140d^2 - 15d - 55 < 0 \end{cases}$$

$$\begin{cases} 135d^2 - d(15-24a) + a_1^2 - 6a - 39 > 0 \\ 140d^2 - d(15-24a) + a^2 - 6a - 55 < 0 \end{cases}$$

$\cancel{D = 225 - 720a + 576a^2 - 540a^2 + 3240a + 126360 =}$

$= \cancel{36a^2 + 3168a + 126485}$

$36a^2 + 2520a + \cancel{126485}$

21285

$D = 225 - 720a + 576a^2 - 560a^2 + 3360a + 30800 =$

$= 16a^2 + 2640a + 31025$

$6205 \cdot 5 = 1249 \cdot 5^2$

$(2) - (1) \Rightarrow 5y^2 < 16 \Rightarrow y^2 < \frac{16}{5}$

$y \in \left(-\frac{4}{\sqrt{5}};\ \frac{4}{\sqrt{5}}\right)$

1 2 3 4 5

$3360 - 720 = 2640$

$30800 + 225 = 31025$

$31025 : 5 = 6205$

$560 \times 6 = 3360$

$3360 - 720 = 2640$

$560 \times 55 = 30800$

2640

$540 \times 39 = 21060$

$21060 + 225 = 21285$

$30 \times 24 = 720$

$24 \times 30 = 720$

$24 \times 24 = 576$

$24 \times 24 = 576$

$135 \times 4 = 540$

$540 \times 6 = 3240$

$3240 \times 39 = 126360$

$126360 + 125 = 126485$

$21285 : 5 = 4257$

$21185 : 5 = 4257$

$3240 - 720 = 2520$

$126485 : 5 = 25297$

$625 \times 5 = 3125$

$6205 \times 5 = 31025$

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$$\begin{array}{r|l} 6205 & 5 \\ -5 & \overline{1249} \\ \overline{12} & \\ -10 & \\ \overline{24} & \\ -20 & \\ \overline{45} & \end{array} \qquad \begin{array}{r|l} 1249 & 9 \\ -9 & \overline{13} \\ \overline{34} & \\ -27 & \\ \overline{79} & \end{array} \qquad \begin{array}{r} 25 \\ \times\ 16 \\ \hline 150 \\ 25 \\ \hline 400 \end{array}$$

$$\begin{array}{r|l} 1249 & 3 \\ -12 & \overline{41} \\ \overline{4} & \\ -3 & \\ \overline{1} & \end{array} \qquad \begin{array}{r|l} 1249 & 7 \\ -7 & \overline{17} \\ \overline{54} & \\ -49 & \\ \overline{59} & \end{array}$$

$\cancel{a^2+a(24d}$

$a^2 + 2a(12d-3) + 135d^2 - 15d - 39 > 0$

$\frac{D}{4} = 144d^2 - 72d + 9 - 135d^2 + 15d + 39 > 0$

$9d^2 - 57d + 48$

$D = 3249 - 1728$

$$\begin{array}{r} 3249 \\ -1728 \\ \hline 2521 \end{array} \qquad \begin{array}{r|l} 2521 & 11 \\ -22 & \overline{22} \\ \overline{32} & \\ -22 & \\ \overline{101} & \end{array}$$

$a_{1,2} = 3 - 12d \pm \sqrt{9d^2 - 57d + 48}$

$$\begin{array}{r} 24 \\ \times\ 3 \\ \hline 72 \\ -15 \\ \hline 57 \end{array} \qquad \begin{array}{r} 57 \\ \times\ 57 \\ \hline 399 \\ 285 \\ \hline 3249 \end{array} \qquad \begin{array}{r} 36 \\ \times\ 48 \\ \hline 288 \\ 144 \\ \hline 1728 \end{array}$$

$a^2 + 2(12d-3) + 140d^2 - 15d - 55 < 0$

$\frac{D}{4} = 144d^2 - 72d + 9 - 140d^2 + 15d + 55 < 0$

$4d^2 - 57d + 64$

$(2d-8)^2 - 25d$

$(2d - 8 - 5\sqrt{d})(2d - 8 + 5\sqrt{d})$

$$\begin{array}{r} 57 \\ -32 \\ \hline 25 \end{array}$$

Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21100076**

ID профиля: **892436**

Вариант 23

№4 Чистовик Вар 23.

$НОД(a;b;c) = 2 \cdot 11$ $\quad a = 2^{\alpha_1} \cdot 11^{\alpha_2}$

$НОК(a;b;c) = 2^{16} \cdot 11^{19}$ $\Rightarrow$ $b = 2^{\beta_1} \cdot 11^{\beta_2}$

$c = 2^{\gamma_1} \cdot 11^{\gamma_2}$

Причем все

$\alpha_1; \beta_1; \gamma_1 \in \{1, 2, 3 \dots 15, 16\}$

$\alpha_2; \beta_2; \gamma_2 \in \{1, 2, 3 \dots 18, 19\}$

и хоть 1 из $\alpha_1; \beta_1; \gamma_1 = 1$ и хоть 1 из $\alpha_1; \beta_1; \gamma_1 = 16$

и хоть 1 из $\alpha_2; \beta_2; \gamma_2 = 1$ и хоть 1 из $\alpha_2; \beta_2; \gamma_2 = 19$

Случаев, когда третий не 1 и не 16(19):

$3 \cdot 2 \cdot 14$ для $\alpha_1; \beta_1; \gamma_1$ и $3 \cdot 2 \cdot 17$ для $\alpha_2; \beta_2; \gamma_2$

Ещё 3 варианта $\{1; 1, 16\}$ 3 вар. $\{1, 1, 19\}$

и 3 варианта $\{16, 16, 1\}$ 3 вар $\{1, 19, 19\}$

Ответ: $(6 \cdot 14 + 6)(6 \cdot 17 + 6) = 9720$

1

21100076 (U892436 M1298064)

№5 Чистовик Вар. 23

Сделаем замену:

$a = \sqrt{x+34} \Rightarrow x > 34$

$b = -x-4 \Rightarrow x < -4$

$c = \sqrt{2x+23} \Rightarrow x > -\frac{23}{2} = -11,5$

$\Rightarrow -11,5 < x < -4$

$\sqrt{22,5} < a < \sqrt{30}$

$0 < b < 7,5$

$0 < c < \sqrt{15}$

$c < a$

При $x = -9$

$a = 5$

$b = 5$

$c = \sqrt{5}$

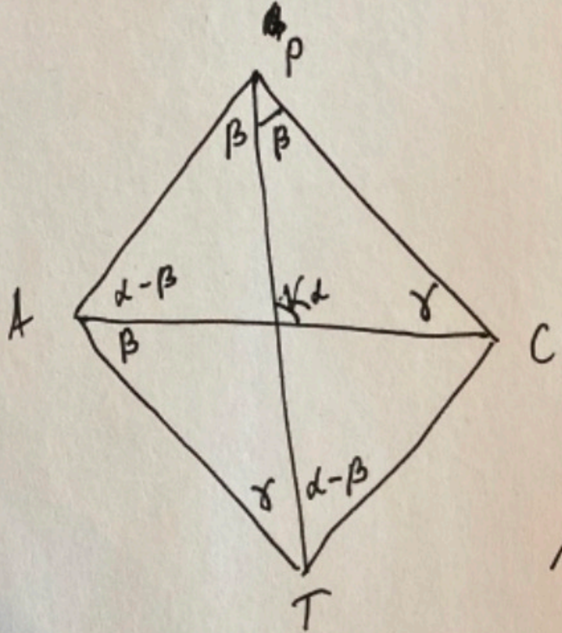
$\log_a c^2 = 1 \quad \log_{b^2} c^2 = 1 \quad \log_c b = 2$

Подходит

②

№6 Чистовик Вар. 23

Заметим, что APCT вписан.



$$S_{PKC} = \frac{PK \cdot KC \cdot \sin\alpha}{2} =$$

$$= \frac{\sin\gamma \sin\alpha \cdot KC^2}{2\sin\beta} = 13$$

$$KC = \frac{13}{28} AC$$

$$AK = \frac{15}{28} AC$$

$$AC^2 = \frac{28^2 \cdot 2 \cdot \sin\beta}{13\sin\gamma \cdot \sin\alpha}$$

$$S_{\triangle ABC} = \frac{AC \cdot BC \cdot \sin\alpha}{2} = \frac{AC^2 \cdot \sin\gamma \cdot \sin\alpha}{2\sin\beta} = \frac{28^2}{13}$$

Ответ: $60\frac{4}{13}$

3

№5

Сделаем замену:

$a = \sqrt{x+34} \Rightarrow x > 34$

$b = -x-4 \Rightarrow x < -4$

$c = \sqrt{2x+23} \Rightarrow x > -\frac{23}{2} = -11,5$

$-11,5 < x < -4$

$\sqrt{22,5} < a < \sqrt{30}$

$0 < b < 7,5$

$0 < c < \sqrt{15}$

$c < a$

При $x = -9$

$a = 5$

$b = 5$

$c = \sqrt{5}$

$\log_a b^2 = 1$; $\log_{b^2} a^2 = 1$ $\log_c b = 2$

Подходит

Остальные не подходят

$784 \,|\, 13$

$$\begin{array}{r} 28 \\ \times\ 28 \\ \hline 224 \\ 56 \\ \hline 784 \end{array}$$

$$\begin{array}{r|l} 784 & 13 \\ -75 & 52 \\ \hline 34 & \\ -26 & \\ \hline 8 & \end{array}$$

$$\begin{array}{r} 13 \\ \times\ 6 \\ \hline 78 \end{array}$$

$$\begin{array}{r} 13 \\ \times\ 60 \\ \hline 00 \\ 78 \\ \hline 780 \end{array}$$

$$\begin{array}{r|l} 784 & 13 \\ -78 & 6 \\ \hline 4 & \end{array}$$

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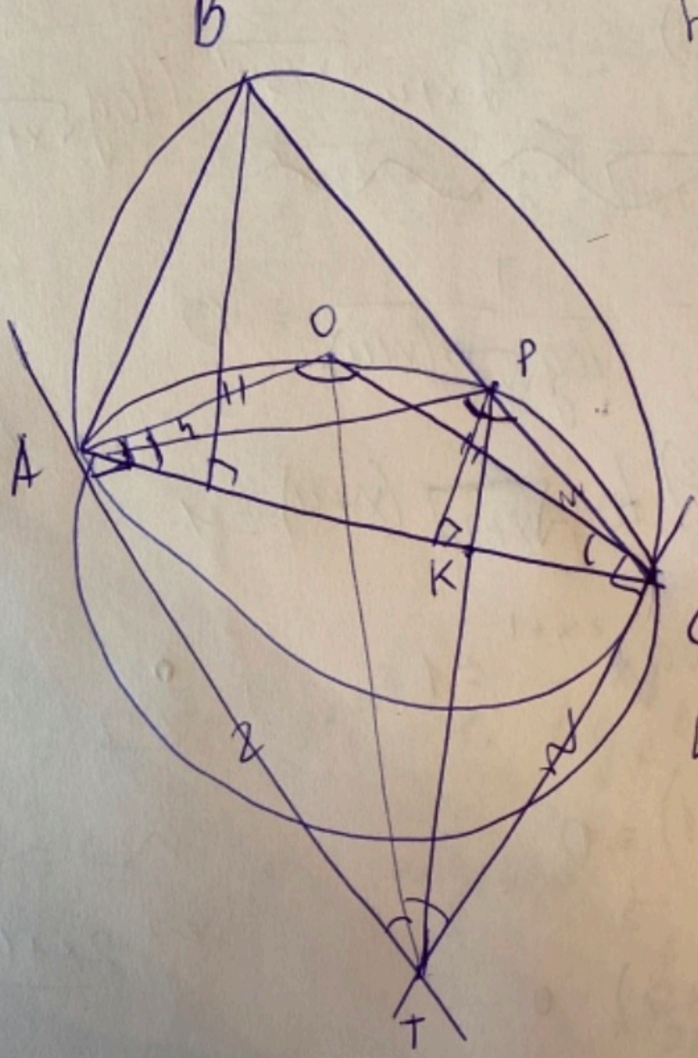
~~$BC \cdot BR = AB^2$~~



$h \cdot KC = 26$

$h \cdot AK = 30$

hAC

$h(AK + KC) = 56$

$\angle CAP = $

$b = 6 \qquad x = -10$

$НОД(a;b;c) = 2 \cdot 11$ $\qquad a = 2^{\alpha_1} \cdot 11^{\alpha_2}$

$НОК(a;b;c) = 2^{16} \cdot 11^{19} \Rightarrow b = 2^{\beta_1} \cdot 11^{\beta_2}$

$c = 2^{\gamma_1} \cdot 11^{\gamma_2}$

Причем все

$\alpha_1, \beta_1, \gamma_1 \in \{1, 2, 3 \dots 16\}$

$\alpha_2; \beta_2\ \gamma_2 \in \{1, 2, 3 \dots 19\}$

и хоть 1 из $\alpha_1; \beta_1, \gamma_1 = 1$ и хоть 1 из $\alpha_1; \beta_1; \gamma_1 = 16$
и хоть 1 из $\alpha_2; \beta_2; \gamma_2 = 1$ и хоть 1 из $\alpha_2; \beta_2; \gamma_2 = 19$
Случаи, когда третий не 1 не 16(19)

$3 \cdot 2 \cdot 14$ для $\alpha_1; \beta_1; \gamma_1$ и $3 \cdot 2 \cdot 17$ для $\alpha_2; \beta_2; \gamma_2$

Ещё 3 вар. $\{1, 1, 16\}$ $\qquad$ 3 вар $\{1, 1, 19\}$

и 3 вар $\{16, 16, 1\}$ $\qquad$ 3 вар $\{1, 19, 19\}$

Ответ: $(6 \cdot 14 + 6)(6 \cdot 17 + 6) = 9720$

21100076 (U892436 M1298064)

$\log \sqrt{x+\cancel{20}}$

$\log_{\sqrt{x+34}}(2x+23) = \log_{\sqrt{2x+23}}(-x-4)$

$\log_{\sqrt{x+34}}(2x+23) = \log_{x+4}\sqrt{x+34}\ (\log_{\sqrt{x+34}}{}^{\sqrt{x+34}}(\log_{x+4}{}^{\sqrt{x+34}})$

$\cancel{2x+23 = \sqrt{x+34}\ \log_{x+4}\sqrt{x+34}}$

$\log_{\sqrt{x+34}}(2x+23) - \dfrac{1}{\log_{\sqrt{x+34}}(x+4)} = 0$

$\log$

$\log_{\sqrt{x+34}}(2x+23)\ \cancel{\log}_{\sqrt{x+34}}(x+4) = 1$

$\log_x{}^{x+2}\ \log_x{}^{2x+1} = 1$

$\dfrac{x+2}{x} = \dfrac{2x+1}{x}$

$\cancel{x+2 = 2x+1}$

$(x+2)(2x+1) = 0$

$x=-2 \quad x=-\frac{1}{2}$

$\cancel{x+}\ \dfrac{2x+1-x+2}{x} = 0$

$(x-3)(x-7) = 0$

$\dfrac{x+3}{x} = 0$

$\log_x{}^{2x+3}\ \log_x{}^{x+7} = 1$

$x=-3$

$\cancel{\log}\ 1 \log$

$\dfrac{2x+3}{x} = \dfrac{x+7}{x}$

$\log$

$\log_{10}{}^{17}\ \log_{10}{}^{17} = 1$

$\dfrac{x-10}{x} = 0$

$x=10$