Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21104540**

ID профиля: **382099**

Вариант 22

ЧИСТОВИК

страница 2 лист 1.

① $\begin{cases} a_7 a_{16} > S-24 \\ a_{11} a_{12} < S+4 \end{cases}$ $m > 0$ $a_1 = ?$

$S = 15a_1 + 15 \cdot 7m = 15(a_1 + 7m)$

$a_7 = a_1 + 6m$
$a_{16} = a_1 + 15m$
$a_{11} = a_1 + 10m$
$a_{12} = a_1 + 11m$

$\begin{cases} (a_1 + 6m)(a_1 + 15m) > 15(a_1 + 7m) - 24 \\ (a_1 + 10m)(a_1 + 11m) < 15(a_1 + 7m) + 4 \end{cases}$ $+$

$(a_1 + 6m)(a_1 + 15m) + \cancel{S} + 4 > (a_1 + 10m)(a_2 + 11m) + \cancel{S} - 24$

$\cancel{a_1^2} + \cancel{21a_1m} + 90m^2 + 4 > \cancel{a_1^2} + \cancel{21a_1m} + 110m^2 - 24$

$20m^2 < 28$
$10m^2 < 14$

$5m^2 < 7$

$\begin{cases} m^2 < \frac{7}{5} \\ m > 0 \\ m \in \mathbb{Z} \end{cases} \Rightarrow$ $\boxed{m = 1}$.

$(a_1 + 6)(a_1 + 15) > 15a_1 + 105 - 24$

$a_1^2 + 6a_1 + \cancel{15a_1} + 90 > \cancel{15a_1} + 81$

$a_1^2 + 6a_1 + 9 > 0 \Rightarrow (a_1 + 3)^2 > 0$ $a_1 \neq -3$

$(a_1 + 10)(a_1 + 11) < 15a_1 + 105 + 4$

$a_1^2 + 10a_1 + 11a_1 + 110 < 15a_1 + 105 + 4$

$a_1^2 + 6a_1 + 1 < 0 \Rightarrow (a_1 + 3)^2 - 8 < 0 \Rightarrow$

$0 < |a_1 + 3| < \sqrt{8}$

(!)

$\Rightarrow$ $|a_1 + 3| = \left[\begin{array}{l} 0 \\ 1 \\ 2 \end{array}\right. \Rightarrow \left|\begin{array}{l} a_1 = \left[\begin{array}{l} -3 \\ -4 \\ -5 \\ -1 \\ -2 \end{array}\right. \\ a_1 \neq -3 \end{array}\right. \Rightarrow$

ответ.

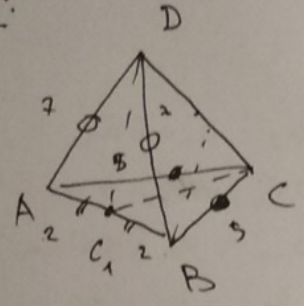
$\boxed{a_1 = -1; -2; -4; -5}$

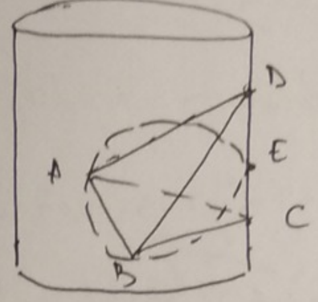
21104540 (U382099 M1295439)

Чистовик. страница 1 лист 1

~~Дано~~:

② $CD = ?$





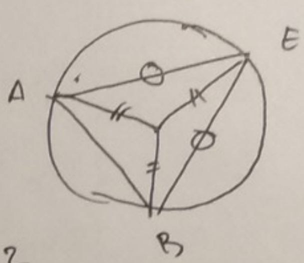
1) CC_1 - медиана

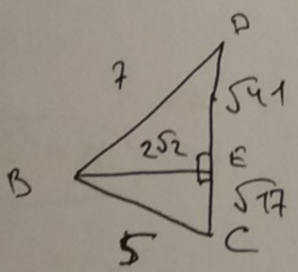
$\triangle ADC = \triangle DBC$ (по $3^м$ сторонам) $\Rightarrow AB \perp CD$

плоск $ABE \perp CD$

r min, когда $AB = 2r$

$AE = \sqrt{2} r$





$r = 2$

$DE = \sqrt{49-8} = \sqrt{41}$

$CE = \sqrt{25-8} = \sqrt{17} \Rightarrow$

$$\boxed{DC = \sqrt{41} + \sqrt{17}}$$

Док-во:

$\triangle ABC$ и $\triangle ABD$ - р/б

$DC_1 \perp AB$ и $CC_1 \perp AB$ $\Rightarrow$ $AB \perp CC_1D$ (по Т о ~~...~~ плоскости и $\perp$ прямой)

$CD \subset CC_1D \Rightarrow$ $\boxed{AB \perp CD}$

Черновик страница 3 лист 2

3)

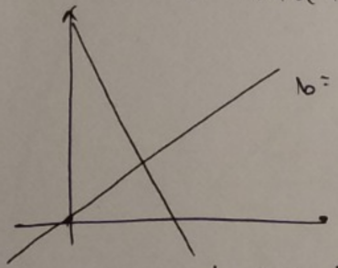
$$\begin{cases} (x-a)^2+(y-b)^2 \le 50 & (1) \\ a^2+b^2 \le \min(14a+2b, 50) & (2) \end{cases}$$

(2) $$\left[\begin{array}{l} \begin{cases} a^2+b^2 \le 50 \\ 14a+2b > 50 \end{cases} (3) \\ \begin{cases} a^2+b^2 \le 14a+2b \\ 14a+2b < 50 \end{cases} (4) \end{array}\right.$$

(3) $$\left[\begin{array}{l} \begin{cases} a^2+b^2 \le 50 \\ b > 25-7a \end{cases} \\ \begin{cases} (a-7)^2+(b-1)^2 \le 50 \\ b \le 25-7a \end{cases} \end{array}\right.$$

(4) $$\begin{cases} a^2-14a+49+b^2-2b+1 \le 50 \\ 14a+2b \le 50 \end{cases}$$

$$\begin{cases} (a-7)^2+(b-1)^2 \le 50 \\ 14a+2b \le 50 \end{cases} \qquad \begin{cases} (a-7)^2+(b-1)^2 \le 50 \\ b \le 25-7a \end{cases}$$

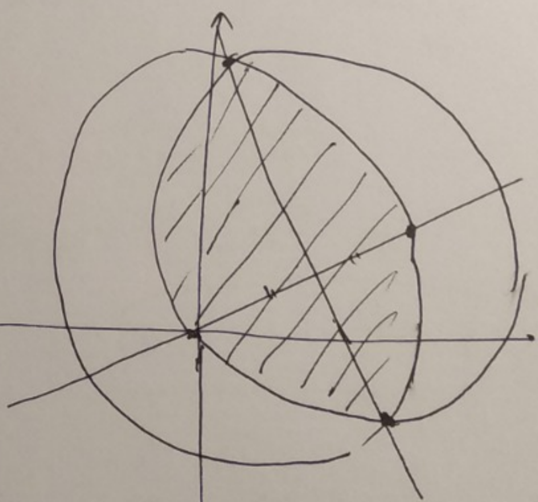


$b = \frac{1}{7} \cdot a$

$$\begin{cases} 7b-a=0 \\ b=25-7a \end{cases}$$

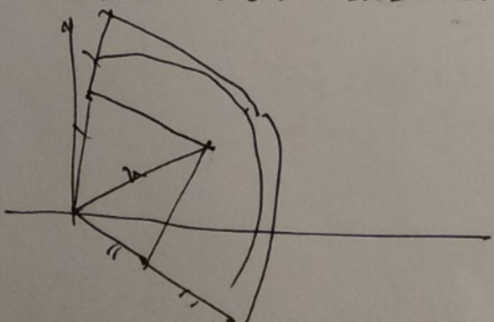
$b = \frac{1}{2}$

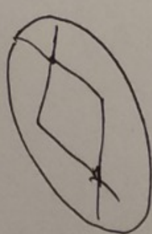
$b = 25-7a$



$(a;b)$ подходит ТОЛЬКО из зарисованной области

M - окружность радиуса $5\sqrt{2}$ вокруг каждой точки заштрихованной области





$S = \pi(2r)^2 \frac{1}{3} + \pi(2r)^2 \cdot \frac{1}{3} + 2\left[\pi r^2 \cdot \frac{1}{6}\right] - ...$

$= \pi r^2 \frac{2}{3} + \pi r^2 \frac{1}{3} - \frac{\sqrt{3}}{2} r^2$ при $r = 5\sqrt{2}$

$\boxed{S = 150\pi - 25\sqrt{3}}$

Черновик   страница 2   лист 1

$a^2 + 21a_1m + 110m^2 < 15a_1 + 105m + 4$

$a^2 + a_1(21m - 15) + 110m^2 - 105m - 4 < 0$

$a = 1 \quad b = 21m - 15 \quad c = 110m^2 - 105m - 4$

$D = b^2 - 4ac = (21m - 15)^2 - 4(110m^2 - 105m - 4) =$

$= 441m^2 - 630m + 225 - 440m^2 + 420m + 16 =$

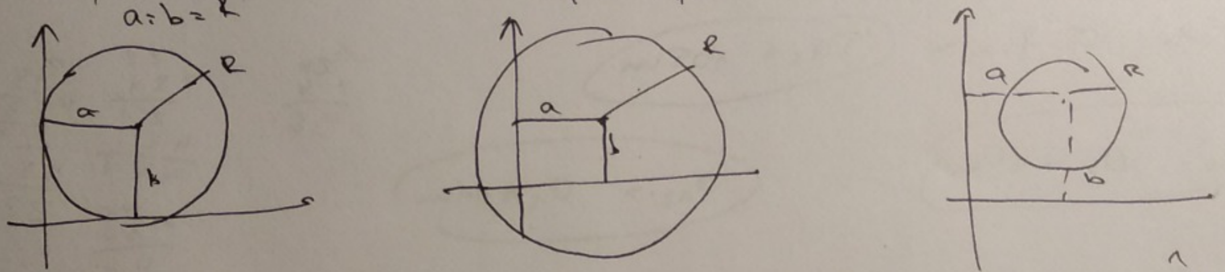
$= m^2 - 210m + 241.$

$21 \cdot 21 = 441 \qquad 15 \cdot 2 = 30 \qquad 21 \cdot 30 = 630 \qquad 630 - 420 = 210 \qquad 225 + 16 = 241$

окружность

3) $\begin{cases} (x-a)^2 + (y-b)^2 \le 50 \\ a^2 + b^2 \le \min(14a + 2b, 50) \end{cases}$ $\quad S = ?$

$a = b = R \qquad a \ne b \quad a, b < R \qquad a \ne b, \; a, b > R$



$\begin{cases} (x-a)^2 + (y-b)^2 \le 50 \\ a^2 + b^2 \le 14a + 2b \; (-) \end{cases}$ или $\begin{cases} (x-a)^2 + (y-b)^2 \le 50 \\ a^2 + b^2 \le 50 \end{cases}$

$4b^2 - 8b + 196 \qquad b^2 - 2b + \qquad b^2 + 2b$

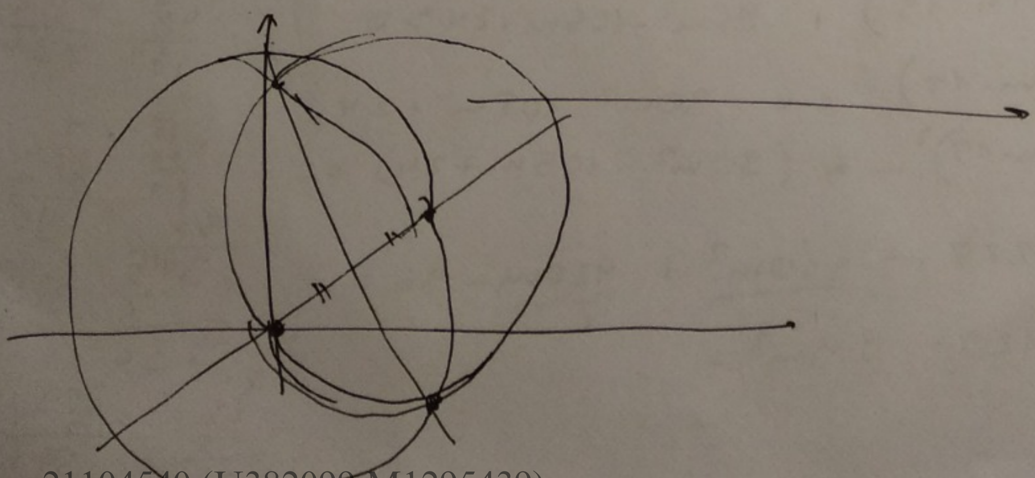
$14 \cdot 14 = 196$

$a^2 + b^2 \le 14a + 2b$

$a^2 - 14a + b^2 - 2b \le 0$ - кв. нер. отн-но a $\quad a = 1 \quad b = -14 \quad c = (b^2 - 2b)$

$a^2 - a \cdot 14 + (b^2 - 2b) \le 0 \qquad D = b^2 - 4ac = 196 + 4b^2 - 8b$

$\sqrt{D} = 2\sqrt{b^2 - 2b + 49}$



① S – 15^{th} первых Черновик Страница 1 лист 1.

арифметическая прогрессия

$a_1, a_2, a_3 \ldots\ a_n$

$$\begin{cases} a_7 a_{16} > S-24 \\ a_{11} a_{12} < S+4 \end{cases}$$

$a_1 = ?$

$a_1 = a_1$

$a_2 = a_1 + m$

$a_3 = a_2 + m = a_1 + 2m \Rightarrow$

$a_7 = a_6 + m = a_5 + 2m = a_4 + 3m =$

$= a_3 + 4m = a_2 + 5m = a_1 + 6m$

$a_{16} = a_1 + 15m$

$S = a_1 + a_2 + a_3 + a_4 \ldots + a_{15} = a_1 + a_1 + m + a_1 + 2m \ldots + a_1 + 14m =$

$= 15a_1 + (m + 2m + 3m + 4m + 5m + 6m + 7m + 8m + 9m + 10m \ldots + 14m) =$

$= 15a_1 + 15 \cdot 7m = 15a_1 + 105m$

$a_{11} = a_1 + 10m$ $\qquad a_{12} = a_1 + 11m$

$$\begin{cases} (a_1 + 6m)(a_1 + 15m) > 15a_1 + 105m - 24 \\ (a_1 + 10m)(a_1 + 11m) < 15a_1 + 105m + 4 \end{cases}$$

$$\begin{cases} a_1^2 + 15a_1 m + 6a_1 m + 90m^2 > 15a_1 + 105m - 24 \\ a_1^2 + 11a_1 m + 10a_1 m + 110m^2 < 15a_1 + 105m + 4 \end{cases}$$

$a_1^2 + 21a_1 m + 90m^2 - 15a_1 - 105m + 24 > 0$

$a_1^2 + a_1(21m - 15) + 90m^2 - 105m + 24 > 0$

$a = 1 \quad b = (21m - 15) \quad c = 90m^2 - 105m + 24$

$D = b^2 - 4ac = (21m - 15)^2 - 4(90m^2 - 105m + 24) =$

$= 441m^2 - 630m + 225 - 360m^2 + 420m - 96 =$

$= 81m^2 - 210m + 129 = 9 \cdot 9m^2 -$

$15 \times 7 = 105$ $\quad 15 \times 7 = 105$ $\quad 24 \times 4 = 96$ $\quad 21 \times 21 = 441$ $\quad 15 \times 15 = 225$ $\quad 21 \times 30 = 630$

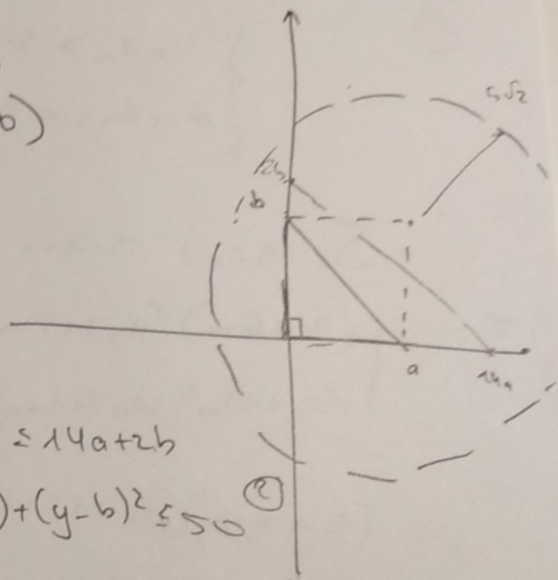
$630 - 420 = 210$ $\quad 441 - 360 = 81$ $\quad 225 - 96 = 129$

$21 \times 21 = 441$ $\quad 105 \times 4 = 420$ $\quad 15 \times 21 = 315$ $\quad 24 \times 4 = 96$ $\quad 315 \times 2 = 630$ $\quad 360$

15×15 $\quad 129 \,|\, 7$ $\quad 59$

3). $\begin{cases}(x-a)^2+(y-b)^2\le 50 \quad (1)\\ a^2+b^2\le \min(14a+2b,\ 50)\end{cases}$

(1) окружност $(O(a;b),\ R\le 5\sqrt{2})$



$\begin{cases}(x-a)^2+(y-b)^2\le 50\\ a^2+b^2\le 50\end{cases}$ ①   или $\begin{cases}a^2+b^2\le 14a+2b\\ (x-a^2)+(y-b)^2\le 50\end{cases}$ ②

② ~~$a^2\le 250$~~

$\begin{cases}a^2+b^2-50\le 0\\ x^2-2ax+a^2+y^2-2yb+b^2-50\le 0\end{cases}$ ⊖

$x^2-2ax+\cancel{a^2}+y^2-2yb+\cancel{b^2}-\cancel{50}-\cancel{a^2}-\cancel{b^2}+\cancel{50}\le 0$

$x^2-2ax+y^2-2yb\le 0$ — кв ур. отн-но x

$x^2-x\cdot 2a+y^2-2yb\le 0.$ $a=1$ $b=-2a$ $c=y^2-2yb$

$D=b^2-4ac=4a^2-4\cdot(y^2-2yb)=4a^2-4y^2-8yb$

$=4(a^2-y^2-2yb)$

25^2: 25 × 25 = 125, 50, 625

① $R\le 5\sqrt{2}$

$14a+2b \vee 50$

$7a+b\vee 25 \uparrow^2$

$49a^2+14ab+b^2\vee 625$

черновик

страница 1 лист 2

(1)
$$\begin{cases} a_7 a_{16} > S-24 \\ a_{11} a_{12} < S+4 \end{cases} \qquad m>0 \qquad a_1 = ?$$

$$\begin{array}{r} 15 \\ \times\ 7 \\ \hline 105 \end{array}$$

$$S = 15a_1 + 15\cdot 7m = 15(a_1+7m)$$

$$\begin{cases} (a_1+6m)(a_1+15m) > 15(a_1+7m)-24 \\ (a_1+10m)(a_1+11m) < 15(a_1+7m)+4 \end{cases} \quad (+)$$

$$(a_1+6m)(a_1+15m) + 15(a_1+7m)+4 > (a_1+10m)(a_1+11m) + 15(a_1+7m)-24$$

$$\begin{array}{r} 15 \\ \times\ 6 \\ \hline 90 \end{array} \qquad \begin{array}{r} \cdot 10 \\ 105 \\ -\ 24 \\ \hline 81 \end{array}$$

Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21104540**

ID профиля: **382099**

Вариант 22

Чистовик страница 1 лист 1.

(4) Каждое из (a, b, c) имеет вид $z_i = 2^{x_i} 7^{y_i}$, причем $x_i, y_i \geq 1$

т.к. НОК $(a,b,c) = 2^{17} 7^{18}$, то хотя бы одно из $x_i = 17$ а $y_i = 18$

т.к. НОД $(a,b,c) = 14$, то хотя бы одно из $x_i = 1$, а $y_i = 1$

Набор (a,b,c) однозначно задается набором

$$(x_1, x_2, x_3, y_1, y_2, y_3)$$

Посчитаем по ф-ле включений, исключений

$+17 \cdot 18 \cdot 3! \cdot 3!$ ← всевозможные x_3 и y_3, если $x_1 = 1$, $x_2 = 17$, $y_1 = 1$, $y_2 = 18$

перестановки

$-2 \cdot 3 \cdot 3! \cdot 17$ $x_3 = x_1 = 1$ и $x_3 = x_2 = 17$

$-2 \cdot 3 \cdot 3! \cdot 18$ $y_3 = y_1 = 1$ и $y_3 = y_2 = 18$

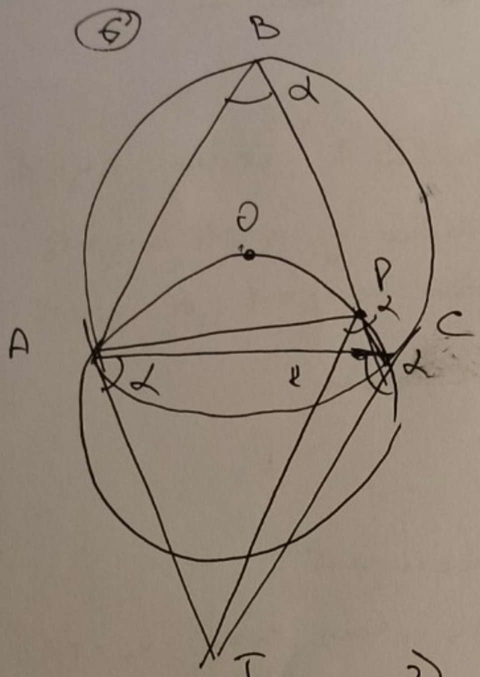
$+2 \cdot 3 \cdot 3$ $x_3 = x_1 = 1$ и $y_3 = y_1 = 1$; $x_3 = x_2 = 17$ и $y_3 = y_2 = 18$

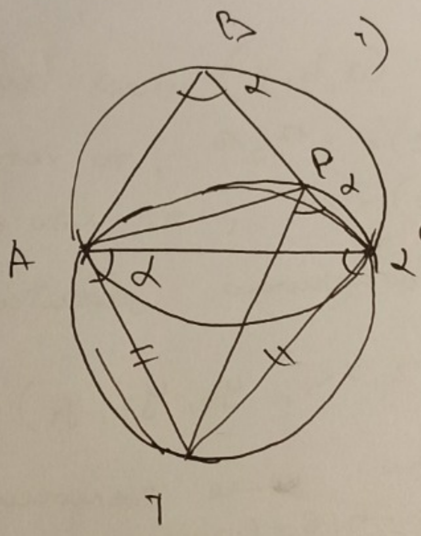
$+2 \cdot 3 \cdot 3$ $x_3 = x_1 = 1$ и $y_3 = y_2 = 18$; $x_3 = x_2 = 17$ и $y_3 = y_1 = 1$

$$\sum = 17 \cdot 18 \cdot 6^2 - 2 \cdot 3 \cdot 6 [17 + 18] + 9 \cdot 4 = 6^2 [17 \cdot 18 + 35 + 1] =$$

$\boxed{= 9792}$ ← ответ

⑥ Чистовик страница 2 лист 1





1) $AOCT$ – впис т.к. $\angle OAT + \angle OCT = \pi$ т.к. касат

2) пусть $\angle B = \alpha \Rightarrow \angle CAT = \alpha$ на касат и т.д.

$\angle CPT = \angle CAT = \alpha$, т.к. $\cup TC$

$PK \parallel AB \Rightarrow \triangle ABC \sim \triangle KPC$

$$\frac{S(\triangle APK)}{S(\triangle PKC)} = \frac{7}{5} \Rightarrow \frac{AK}{KC} = \frac{7}{5} \Rightarrow \frac{AC}{KC} = \frac{12}{5}$$

$$S(\triangle ABC) = \left(\frac{12}{5}\right)^2 S(\triangle CPK) = \frac{12^2}{5} = [\text{illegible}]$$

$= \boxed{28{,}8}$ ответ.

(2)

тепловик

$\log_{(\frac{x}{2}+1)^2}\left(\frac{7x}{2}-\frac{17}{4}\right)$; $\log_{\sqrt{\frac{7x}{2}-\frac{17}{4}}}\left(\frac{3x}{2}-6\right)^2$, больше 50%.

$\log_{\sqrt{\frac{3x}{2}-6}}\left(\frac{x}{2}+1\right)$

$5+6+7=18$

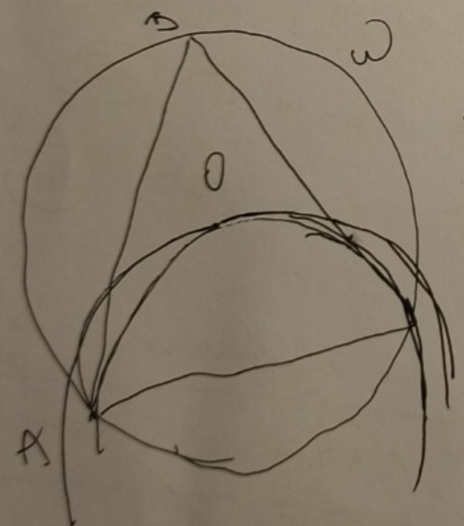
17 меньше 50%.

- Если равны $\log_{(x/2+1)^2}\left(\frac{7x}{2}+\frac{17}{4}\right)=\log_{\sqrt{\frac{7x}{2}+\frac{17}{4}}}\left(\frac{3x}{2}-6\right)^2$

$$\log_{(x/2+1)^2}\left(\frac{7x}{2}-\frac{17}{4}\right)=2\log_{\left(\frac{7x}{2}+\frac{17}{4}\right)}\left(\frac{3x}{2}-6\right)^2$$

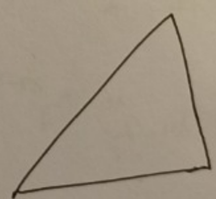
$$\frac{1}{\log_{\left(\frac{7x}{2}-\frac{17}{4}\right)}(x/2+1)^2}=2\log_{\left(\frac{7x}{2}+\frac{17}{4}\right)}\left(\frac{3x}{2}-6\right)^2$$

$$2\log_{\left(\frac{7x}{2}-\frac{17}{4}\right)}\left(\frac{x}{2}+1\right)^2\log_{\left(\frac{7x}{2}-\frac{17}{4}\right)}\left(\frac{3x}{2}-6\right)^2=1$$



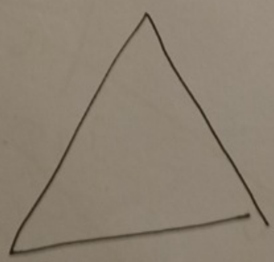
$$2\log_{\left(\frac{7x}{2}+\frac{17}{4}\right)}\left(\frac{x}{2}+1\right)^2\log_{\left(\frac{7x}{2}+\frac{17}{4}\right)}\left(\frac{3x}{2}-6\right)^2=$$

$$=\log_{\left(\frac{7x}{2}+\frac{17}{4}\right)}\left(\frac{7x}{2}+\frac{17}{4}\right)$$



$$2\left(\frac{x}{2}+1\right)^2\cdot\left(\frac{3x}{2}-6\right)^2=1$$

$$\frac{3}{\cancel{2}}\cdot 6\cdot\cancel{x}=18.$$



$$\left(\frac{x}{2}+1\right)^2\left(\frac{3x}{2}-6\right)^2=\frac{1}{2}$$

$$\left(\frac{x^2}{4}+x+1\right)\left(\frac{9x^2}{4}-18x+36\right)=\frac{1}{2}$$

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черновик

(4) $\begin{cases} НОД(a;b;c) = 14 \\ НОК(a;b;c) = 2^{17} \cdot 7^{17} \end{cases}$ наибольший общий делитель, наименьшее общее кратное

$\frac{23}{35} \cdot 100\% = $ приблизительно $= \frac{23 \cdot 20}{7} \approx 80\%$

$a = 2 \cdot 7 \cdot N_a \quad b = 2 \cdot 7 \cdot N_b \quad c = 2 \cdot 7 \cdot N_c$.

$НОД(N_a, N_b, N_c) = 1$.

тогда $НОК(a;b;c) = 14 \cdot N_a \cdot N_b \cdot N_c$

$N_a \cdot N_b \cdot N_c = 2^{16} \cdot 7^{17}$

$5+6+7+5$, 23

одна из N должна быть равна 1 т.к. в противном случае будут меняться НОК и НОД. =>

$a = 14 \quad b = 2 \cdot 7 \cdot 7^{17}$, $b = 14 \cdot 7^{17} \quad c = 2 \cdot 7 \cdot 2^{16}$, $c = 14 \cdot 2^{16}$ 1

~~Если мы~~

$a = 14 \cdot 7^{17-n} \quad b = 14 \cdot 7^{n} \quad c = 14 \cdot 2^{16}$ 16

$a = 14 \cdot 2^{16-m} \quad c = 14 \cdot 2^{m} \quad b = 14 \cdot 7^{17n}$ 15

$a = 14 \cdot 2^{16-m} 7^{17-n} \quad b = 14 \cdot 7^{n} \quad c = 14 \cdot 2^{m}$ 240 =>

$5+6+7+5+5+7$, 14, 10, 11

$10+11+14$

35.

таких a, b и c существует $17 \cdot 16 \cdot 1$

~~16 + 15 + 1 + 240 = 32 + 240 = 272~~?

6.

ответ.

???

упорядоченные

123
231
312
321
132
213

$\times \begin{array}{r} 15 \\ 16 \end{array}$

90
15
240

$\times \begin{array}{r} 272 \\ 6 \end{array}$

1622

ответ ????

$\frac{9^{\log}}{7}$ = $(ac)^{\log}$

Черновик

$$\log_{(\frac{x}{2}+1)^2}\left(\frac{7x}{2}-\frac{17}{4}\right) = \log_{\sqrt{\frac{3x}{2}-6}}\left(\frac{x}{2}+1\right)$$ тогда

$$2\log_{(\frac{x}{2}+1)}\left(\frac{7x}{2}-\frac{17}{4}\right) = \frac{1}{\frac{1}{2}\log_{(\frac{x}{2}+1)}\left(\frac{3x}{2}-6\right)}$$ ч

$$\cancel{\log_{(\frac{x}{2}+1)}x}\qquad \log_{\sqrt{\frac{7x}{2}-\frac{17}{4}}}\left(\frac{3x}{2}-6\right)^2 = 1.$$

$$\sqrt{\frac{7x}{2}-\frac{17}{4}} = \left(\frac{3x}{2}-6\right)^2$$

$$\frac{7x}{2}-\frac{17}{4} = \left(\frac{3x}{2}-6\right)^4 = \left(\frac{9x^2}{4}-18x+36\right)\left(\frac{9x^2}{4}-18x+36\right) =$$

$$= \frac{81x^4}{16} - \frac{18x^3\cdot 9}{4} + \frac{9\cdot 36x^2}{4} - \frac{18x^3\cdot 9}{4} + 18^2x^2 - 36\cdot 18x + \frac{9\cdot 36x^2}{4} - 18\cdot 36x + 36^2 =$$

$$\cancel{= \frac{81x^4}{16} - 81x^3 + 2\cdot 81x^2 + 4\cdot 81x^2 - 2\cdot 8\cdot 81x + 36^2}$$

$$\frac{7x}{2}-\frac{17}{4} =$$

Замена $\frac{x}{2}+1 = a$ $\qquad \frac{7x}{2}-\frac{17}{4} = b$ $\qquad \frac{3x}{2}-6 = c$

$\log_{a^2} b$, $\log_{\sqrt{b}} c^2$ $\quad \log_{\sqrt{c}} a$

$\frac{1}{2}\log_a b$ $\quad 4\log_b c$, $2\log_c a$

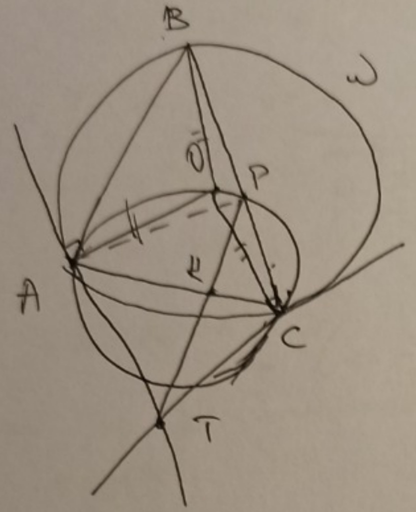
$$\begin{cases} \frac{1}{2}\log_a b = 4\log_b c \\ 2\log_c a + 1 = \frac{1}{2}\log_a b \end{cases}$$

$2\log_c a + 1 = 4\log_b c$

$\frac{1}{2\log_c b}$

$\log_c a + \log_a c = \frac{1}{2}\log_c b$

черновик.



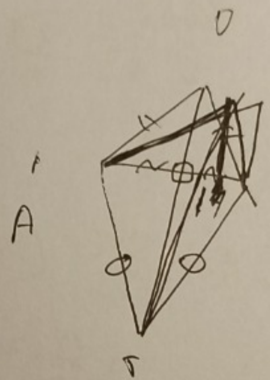
$S(\triangle APK) = 2$

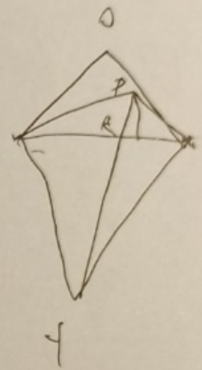
$S(\triangle CPK) = 5$

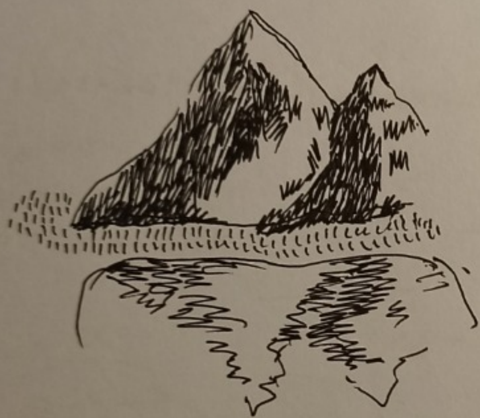
$S(\triangle ABC) = ?$

$S(\triangle APC) = 12.$

AC d:>







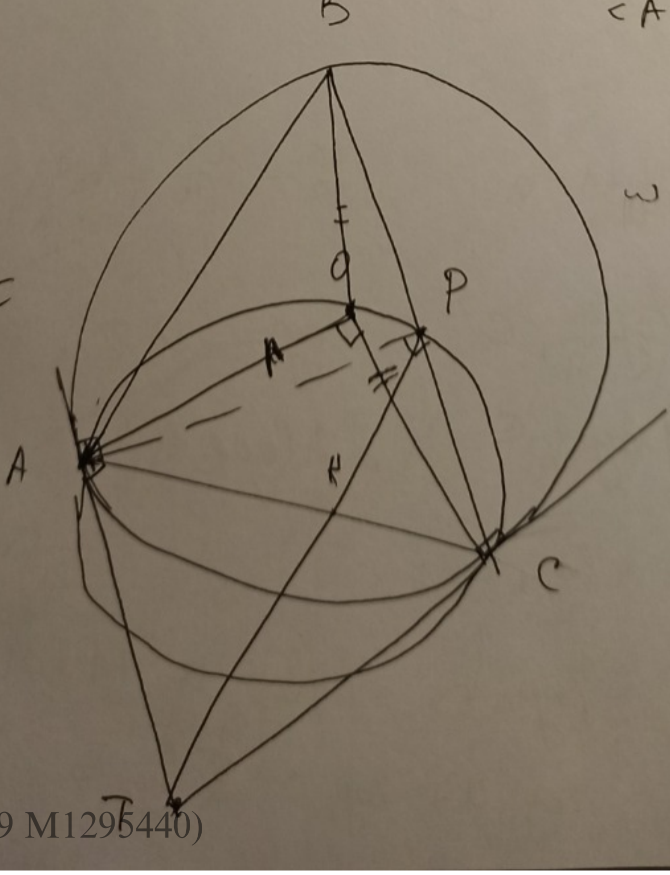
AC - диаметр =>

$\angle AOC = \angle APC = 90°$

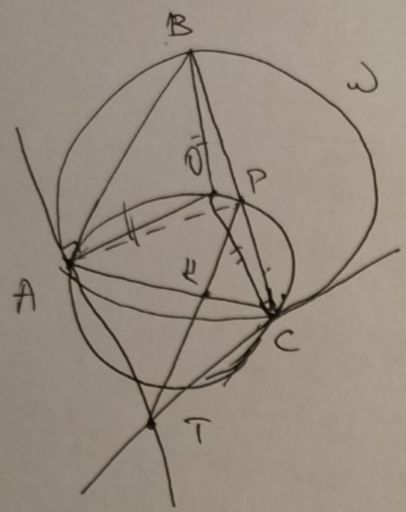
12^2

144

5



Черновик.



$S(\triangle APK) = 2$

$S(\triangle CPK) = 5$

$S(\triangle ABC) = ?$

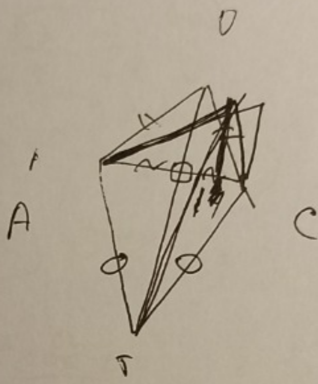
$S(\triangle APC) = 12.$

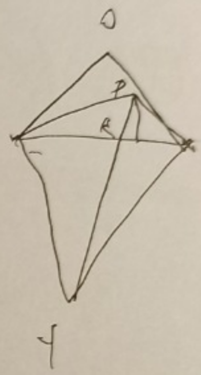
AC d =>

100%

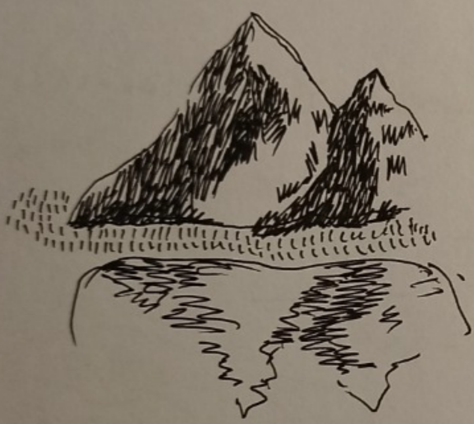
$\frac{\cdot 20}{7}$

80%





где



C

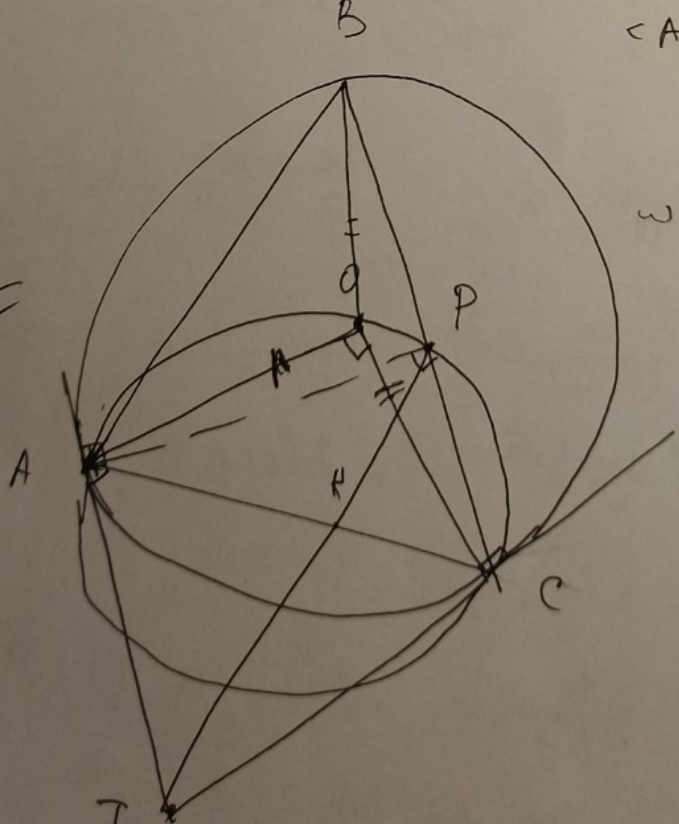
AC - диаметр =>

$\angle AOC = \angle APC = 90°$

12^2

144

5



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