

Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21104462**

ID профиля: **369655**

Вариант 22

B 22

Числовик

№ 1

Задача № 1

$$S = a_1 + \dots + a_{15} = \frac{a_1 + a_{15} + 14d}{2} \cdot 15 = 15(a_1 + 7d)$$

$$a_7 \cdot a_{16} = (a_1 + 6d)(a_1 + 15d) > 15(a_1 + 7d) - 24$$

$$a_{11} \cdot a_{12} = (a_1 + 10d)(a_1 + 11d) < 15(a_1 + 7d) + 4$$

$$\begin{cases} a_1^2 + (21d - 15)a_1 + (90d^2 - 105d + 24) > 0 \\ a_1^2 + (21d - 15)a_1 + (110d^2 - 105d - 4) < 0 \end{cases} \quad \ominus$$

$$105d - 90d^2 - 24 < 4 + 105d - 110d^2$$

$$20d^2 < 28$$

$$d^2 < \frac{7}{5}$$

$$d \in \mathbb{Z}$$

$$\Rightarrow \begin{cases} d = 0 \\ d = +1 \\ d = -1 \end{cases}$$

нормально
возвращаем
 $d = 1$

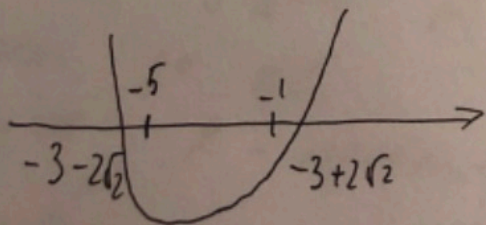
$$\begin{cases} a_1^2 + 6a_1 + 9 > 0 \\ a_1^2 + 6a_1 + 1 < 0 \quad (2) \end{cases}$$

$$\begin{cases} a_1 \neq -3 \\ a_1 \in [-5; -1] \end{cases} \Rightarrow$$

$$(2): D = 32$$

$$\Rightarrow a_1 = -5; -4; -2; -1$$

$$a_1 = -3 \pm 2\sqrt{2}$$



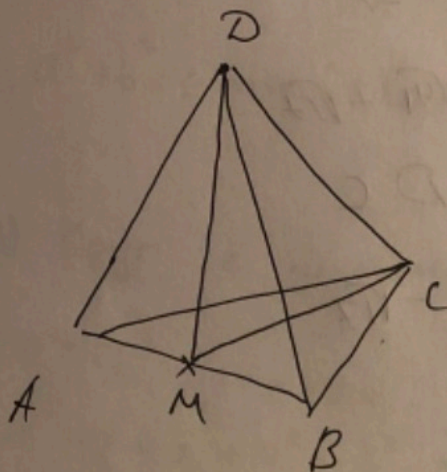
Ответ:

$$a_1 = -5$$

$$a_1 = -4$$

$$a_1 = -2$$

$$a_1 = -1$$



1) Пусть M - середина $AB \Rightarrow$

DM, CM - медианы, т.к. $\triangle ADB$ и $\triangle ACB$ - р.т.,
то DM, CM - высоты

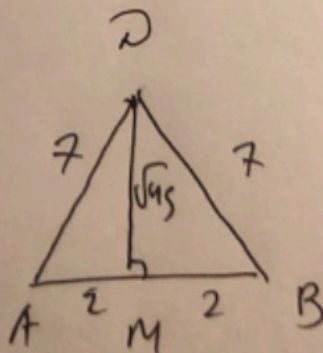
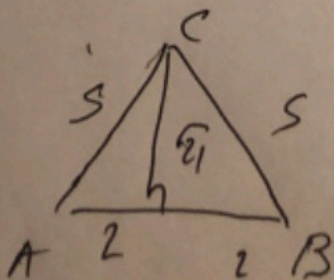
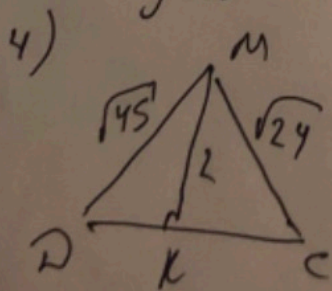
по т. о. 3 $\perp AB \perp (DMC) \Rightarrow$

$\Rightarrow AB \perp DC \Rightarrow AB \parallel$ осн. цилиндра

2) AB - хорда окружности \Rightarrow радиус перпендикулярен
если AB - диаметр $\Rightarrow R_{\text{окр}} = \frac{AB}{2} = 2$

3) AB - диаметр $\Rightarrow M$ лежит на основании
Значит $\angle M$ 90° DC равно $\angle OB$
осн 90° DC

Пусть K - проекция



K нити на DC:

Ученик В 22

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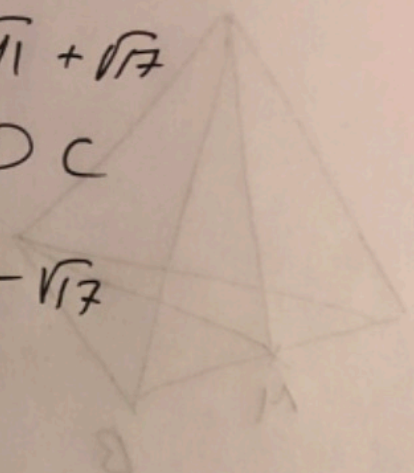
$$DK = \sqrt{41} ; KC = \sqrt{17} \Rightarrow$$

$$DC = DK + KC = \sqrt{41} + \sqrt{17}$$

K не нити на DC

$$DC = DK - KC = \sqrt{41} - \sqrt{17}$$

ответ: $\sqrt{41} \pm \sqrt{17}$



$$\begin{cases} (x-a)^2 + (y-b)^2 \leq 50 & (1) \\ a^2 + b^2 \leq \min(14a+2b; 50) & (2) \end{cases}$$

(1) Круг с центром $(a; b)$ радиусом $\sqrt{50}$

(2) $a^2 + b^2 \leq 14a + 2b$

$$a^2 - 2 \cdot 7 \cdot a + 7^2 + b^2 - 2 \cdot b + 1^2 \leq 50$$

$$(a-7)^2 + (b-1)^2 \leq 50$$

$$\begin{cases} (x-a)^2 + (y-b)^2 \leq 50 \end{cases}$$

$$(a-7)^2 + (b-1)^2 \leq 50, \quad 14a + 2b < 50 \quad (*)$$

$$a^2 + b^2 \leq 50, \quad 14a + 2b > 50 \quad (**)$$

(*) $a=0 \quad b=0$
 $a=1 \quad b \in [-3; 4]$
 $a=2 \quad b \in [-5; 6]$
 $a=3 \quad b \in [-3; 4]$

$$a_1^2 + 11d a_1 + 10d^2 < 15a_1 + 105d + 4$$

$$a_1^2 + a_1(21d - 15) + (10d^2 - 105d - 4) < 0$$

$$D = 441d^2 -$$

$$110 - 105 - 4 = 5 - 4 = 1$$

$$\begin{array}{r} \cdot 10 \\ 24 \\ -15 \\ \hline 9 \end{array}$$

x

$$\begin{array}{r} \cdot 10 \\ 21 \\ -15 \\ \hline 6 \\ 105 \\ 90 \\ \hline 15 \end{array}$$

$$a_1^2 + (21d - 15)a_1 + (90d^2 - 105d + 24) \geq 0$$

$$a_1^2 + (2d - 15)a_1 + (110d^2 - 105d - 4) < 0$$

$$x > (-24 - 90d^2 + 105d)$$

$$x < (4 + 105d - 110d^2)$$

$$-24 - 90d^2 + 105d < x < 4 + 105d - 110d^2$$

$$\begin{array}{r} -110 \\ 90 \\ \hline 20 \\ 28 \frac{1}{2} \end{array}$$

$$-24 - 90d^2 + 105d < 4 + 105d - 110d^2$$

$$20d^2 < 28 \frac{1}{2}$$

$$d^2 < \frac{7}{5}$$

возможны варианты
 $d \geq 1$
 $d = 1$

$$S_1 = 111$$

$$a_1^2 + 15a_1d + 6a_1d + 90d^2 > 15a_1 + 105d - 24$$

$$a_1^2 + 21d \cdot a_1 + 90d^2 > 15a_1 + 105d - 24$$

$$a_1^2 + a_1(21d - 15) + 90d^2 - 105d + 24 > 0$$

$$D = (21d - 15)^2 - 4(90d^2 - 105d + 24) =$$

$$= 21^2 d^2 - 2 \cdot 21 \cdot 15d + 15^2 - 4 \cdot 90d^2 + 4 \cdot 105d - 4 \cdot 24$$

$$= 21^2 d^2 + 15 \cdot 38d + 129$$

$$15(4 - 2 \cdot 21)$$

$$= 15 \cdot 38$$

$$a_1^2 + (21d - 15)a_1 + 90d^2 - 105d + 24 > 0$$

$$D = (21d - 15)^2 - 4(90d^2 - 105d + 24) =$$

$$= 441d^2 - 630d + 225 - 360d^2 +$$

$$+ 420d - 96 =$$

$$= (9d)^2 - 210d + 129$$

$$(9d)^2 - 2 \cdot 35 \cdot 3d$$

$$\begin{array}{r} 129 \\ + 96 \\ \hline 225 \end{array}$$

$$\begin{array}{r} 24 \\ \times 4 \\ \hline 96 \end{array}$$

$$\begin{array}{r} 15 \\ \times 15 \\ \hline 225 \end{array}$$

$$\begin{array}{r} 15 \\ \times 15 \\ \hline 225 \\ - 96 \\ \hline 129 \end{array}$$

$$\begin{array}{r} 10 \\ \cdot 42 \\ \hline 42 \\ - 4 \\ \hline 58 \end{array}$$

$$\begin{array}{r} 21 \\ \times 21 \\ \hline 21 \\ 42 \\ \hline 441 \end{array}$$

$$\begin{array}{r} 21 \\ \times 21 \\ \hline 21 \\ 42 \\ \hline 441 \end{array}$$

$$\begin{array}{r} 21 \\ \times 15 \\ \hline 105 \\ 21 \\ \hline 315 \\ \times 2 \\ \hline 630 \end{array}$$

$$\begin{array}{r} 90 \\ \times 4 \\ \hline 360 \end{array}$$

$$\begin{array}{r} 105 \\ \times 4 \\ \hline 420 \end{array}$$

$$\begin{array}{r} 24 \\ \times 4 \\ \hline 96 \end{array}$$

$$\begin{array}{r} 10 \\ 441 \\ - 360 \\ \hline 81 \end{array}$$

$$\begin{array}{r} 630 \\ - 420 \\ \hline 210 \end{array}$$

$$\begin{array}{r} 10 \\ 225 \\ - 96 \\ \hline 129 \end{array}$$

$$\begin{array}{r} 1 \\ 129 \\ + 96 \\ \hline 225 \end{array}$$

$$\begin{array}{r} 210 \overline{) 2} \\ \underline{105} \\ 115 \end{array}$$

$$\begin{array}{r} 105 \overline{) 3} \\ \underline{75} \\ 35 \end{array}$$

$$\begin{array}{r} 2 \\ 35 \\ \times 35 \\ \hline 175 \end{array}$$

$$\begin{array}{r} 105 \\ \hline 1225 \end{array}$$

$$\begin{array}{r} 35 \\ \times 6 \\ \hline 210 \end{array}$$

$$\begin{array}{r} 129 \overline{) 3} \\ \underline{129} \\ \hline 43 \end{array}$$

$$S_1 = 15(a_1 + 7d)$$

$$a_7 a_{16} > 5 - 24$$

$$a_{11} a_{12} < 5 + 4$$

~~$$a_7 a_{16} > a_{11} a_{12}$$~~

$$a_7 a_{16} + 8 + 4 > a_{11} a_{12} + 9 - 24$$

$$a_7 a_{16} - a_{11} a_{12} > -28$$

$$a_{11} a_{12} - a_7 a_{16} > 28$$

$$(a_1 + 10d)(a_1 + 11d) - (a_1 + 6d)(a_1 + 15d) > 28$$

$$a_1^2 + 21a_1 d + 110d^2 - (a_1^2 + 21a_1 d + 90d^2) > 28$$

$$20d^2 > 28 \quad d^2 > \frac{28}{20}$$

$$d = 0$$

$$d = \pm 1$$

$$\Rightarrow d = 1$$

$$-5, -4, -3, -2, -1$$

$$\begin{cases} a_1^2 + 6a_1 + 9 > 0 \\ a_1^2 + 6a_1 + 1 < 0 \end{cases} \quad (a+3)^2 > 0 \quad a+3 \neq 0$$

N°1

$$S = a_1 + \dots + a_{15}$$

$$a_7 \cdot a_{16} > S - 24$$

$$a_{11} \cdot a_{12} < S + 4$$

$a_1 = ?$

	d.	2d	3d	4d
3	5d	6d	7d	8d
x 15	9d	10d	11d	12d
105	13d	14d		

$$S = a_1 + a_1 + d + \dots + a_1 + 14d =$$

$$= \frac{a_1 + a_1 + 14d}{2} \cdot 15 = \frac{2a_1 + 14d}{2} \cdot 15 =$$

$$= 15a_1 + 7 \cdot 15 \cdot d = 15a_1 + 105d$$

$$a_7 \cdot a_{16} = (a_1 + 6d)(a_1 + 15d) > 15a_1 + 105d - 24$$

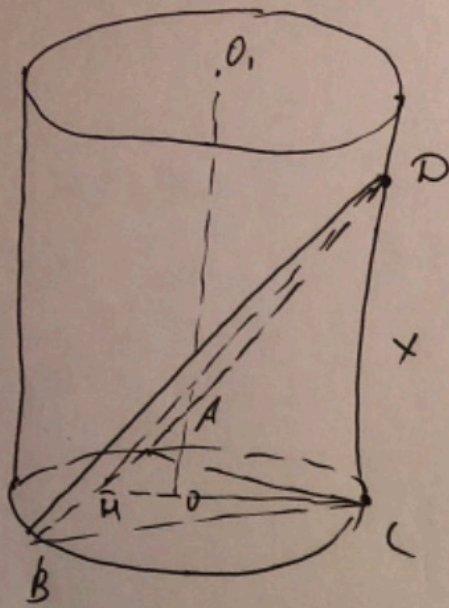
a_1, d - yegone

$$\begin{array}{r} 3 \\ \times 15 \\ \hline 45 \\ 90 \\ \hline \end{array}$$

$$a_{11} \cdot a_{12} = (a_1 + 10d)(a_1 + 11d) < 15a_1 + 105d + 4$$

$$\begin{array}{r} 21 \\ \times 21 \\ \hline 42 \\ 42 \\ \hline 441 \end{array}$$

$$\begin{array}{r} 21 \\ \times 15 \\ \hline 105 \\ 21 \\ \hline 315 \end{array}$$



$$AC = CB = 5$$

$$AB = 4$$

$$AD = BD = 7$$

$$CD \parallel OO_1$$

$$1) \left. \begin{array}{l} OO_1 \perp (ABC) \\ CD \parallel OO_1 \end{array} \right\} \Rightarrow CD \perp (ABC)$$

$$2) \triangle ACH$$

$$CH = \sqrt{AC^2 - AH^2} = \sqrt{25 - 4} = \sqrt{21}$$

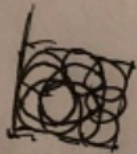
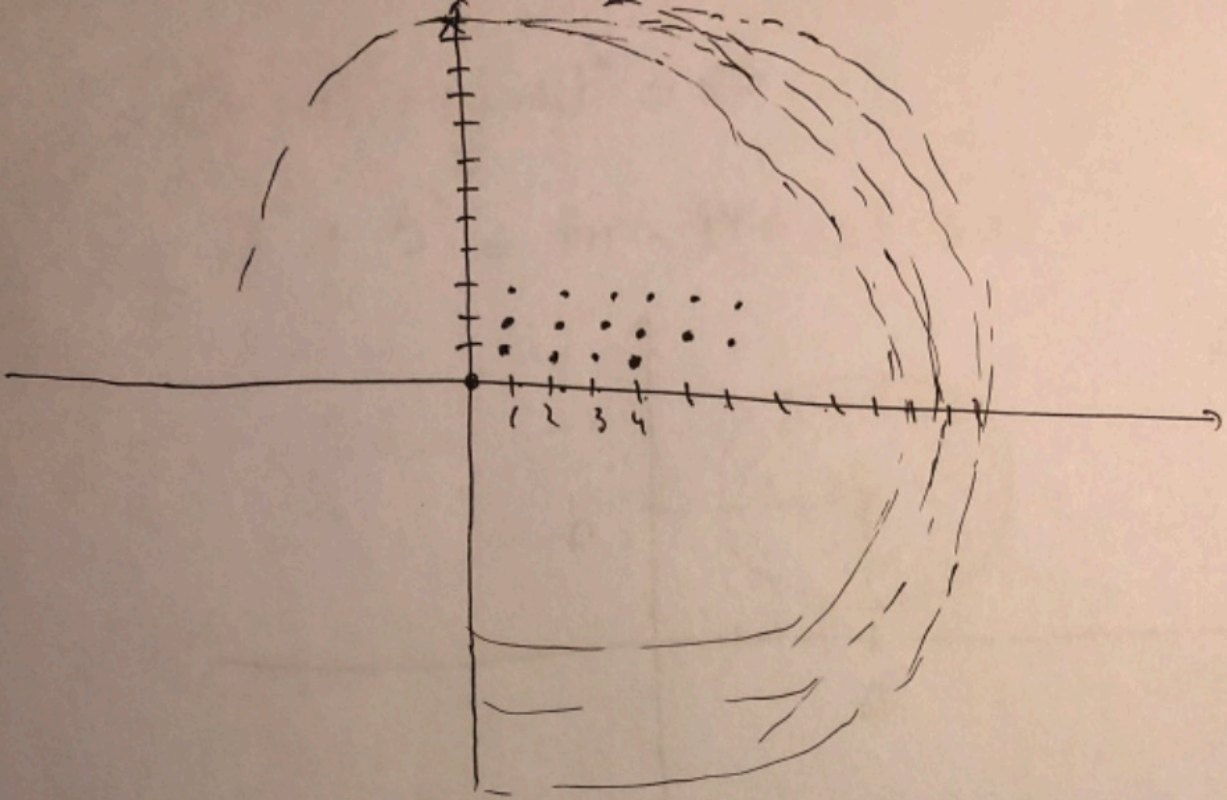
$$3) \triangle DCH$$

$$DH = \sqrt{AD^2 - AH^2} = \sqrt{49 - 4} = \sqrt{45}$$

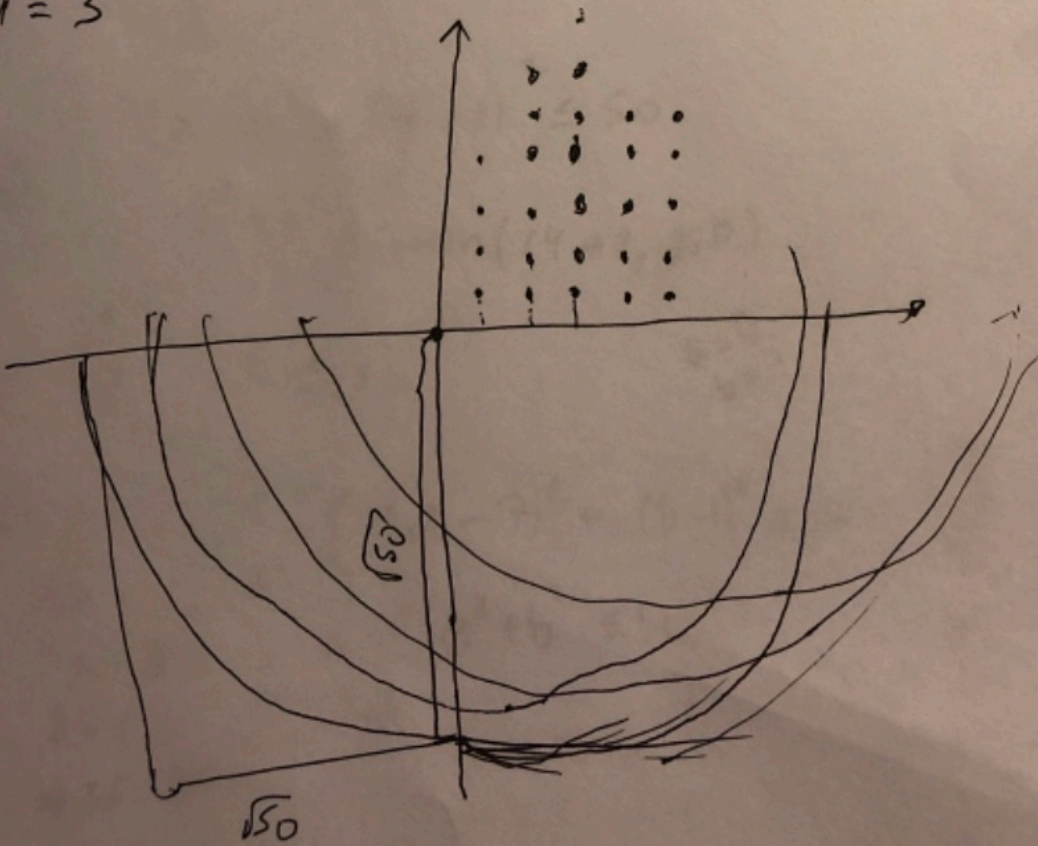
$$4) \triangle DCH$$

$$DC = \sqrt{DH^2 - HC^2} = \sqrt{45 - 21} = \sqrt{24} = 2\sqrt{6}$$

$$\text{Ответ: } 2\sqrt{6}$$

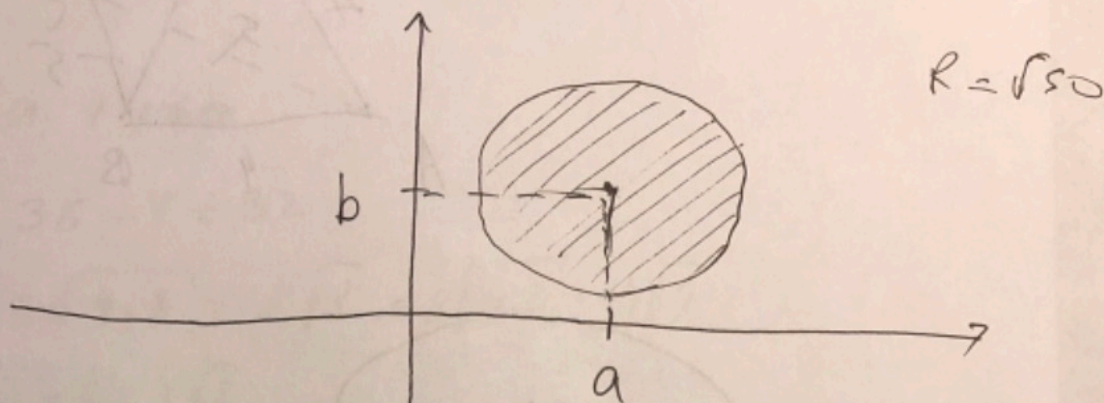


$a = 3$



N3

$$\begin{cases} (x-a)^2 + (y-b)^2 \leq 50 \\ a^2 + b^2 \leq \min(14a + 2b; 50) \end{cases}$$



$$\begin{matrix} 14 \\ + \\ 36 \\ \hline -23 \end{matrix}$$

$$\sqrt{2 \cdot 25} = 5\sqrt{2}$$

$$\begin{matrix} 2 \\ 14 \\ 5 \\ \hline 12,0 \end{matrix}$$

$$\begin{matrix} a=1 \\ b=1 \end{matrix}$$

$$(x-1)^2 + (y-1)^2 \leq 50$$

$$1^2 + 1^2 \leq \min(14 + 2; 50)$$

$$2 \leq 16$$

$$\begin{matrix} a=0 \\ b=0 \end{matrix}$$

$$a^2 + b^2 > 14a + 2b$$

$$a^2 - 2 \cdot 7 \cdot a + 7^2 + b^2 - 2 \cdot b + 1^2 > 50$$

$$(a-7)^2 + (b-1)^2 \geq 50$$

$$\begin{matrix} 50 \\ -36 \\ \hline 14 \\ - \\ 1 \\ \hline 13 \end{matrix}$$

$$\begin{matrix} 50 \\ -36 \\ \hline 14 \\ - \\ 1 \\ \hline 13 \end{matrix}$$

$$(a-7)^2 + (b-1)^2 \leq 50$$

$$a^2 + b^2 \leq 50$$

$$\begin{matrix} 14 \\ \times \\ 3 \\ \hline 42 \end{matrix}$$

$$a^2 + b^2 \geq 50$$

$$28 + 2 \cdot 6 \quad 28 + 12 = 40 \quad 42$$

$$36 \quad 16$$

- 25 0 1
- 1 2
- 4 3
- 9 4
- 16 5
- 25 6

- a=1 b=1
- a=1 b=2
- a=1 b=3
- a=1 b=4

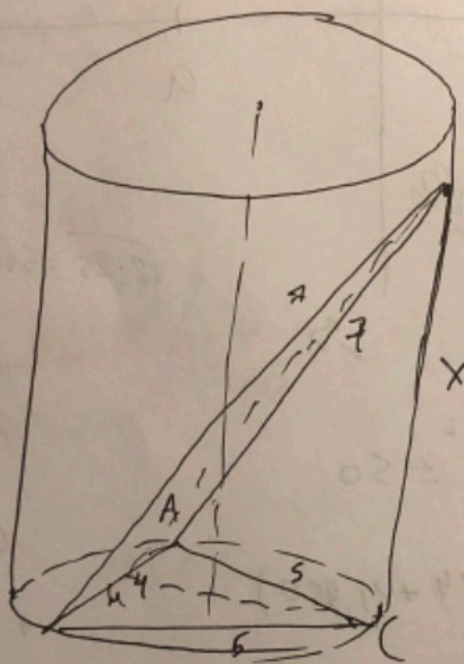
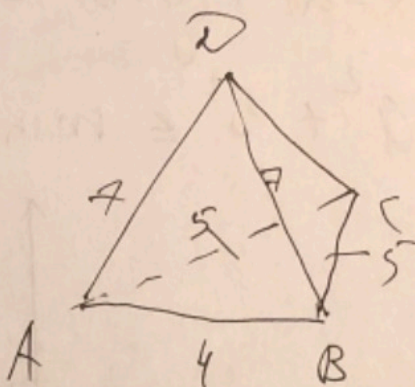
- a=2 b=1
- a=2 b=2
- a=2 b=3
- a=2 b=4
- a=2 b=5
- a=2 b=6

- a=3 b=1
- a=3 b=2
- a=3 b=3
- a=3 b=4

$AB = 4$

$AC = CB = 5$

$AD = DB = 7$



$CD \parallel O_1O_2$

$CH = \sqrt{21}$

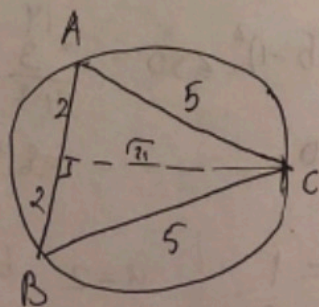
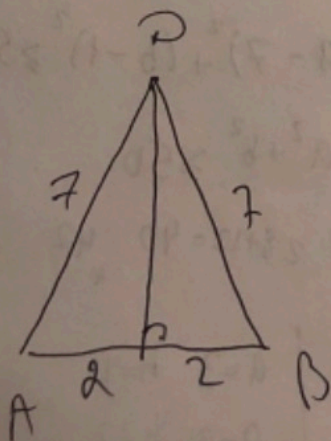
$DH = \sqrt{45}$

$DC = \sqrt{DH^2 - CH^2} =$

$= \sqrt{45 - 21} =$

$= \sqrt{24} = \sqrt{4 \cdot 6} = 2\sqrt{6}$

$\frac{45 - 21}{24}$



$S = \frac{abc}{4R} = \frac{5 \cdot 5 \cdot X}{4R} = 2\sqrt{21}$

$\Rightarrow R = \frac{25}{2\sqrt{21}}$

$S = \frac{1}{2} \cdot 4 \cdot \sqrt{21} = 2\sqrt{21}$

$\sqrt{49 - 4} = \sqrt{45}$

$= \sqrt{45}$

$\sqrt{25 - 4} = \sqrt{21}$

N°1

$$S = a_1 + \dots + a_{15}$$

$$a_7 \cdot a_{16} > S - 24$$

$$a_{11} \cdot a_{12} < S + 4$$

$a_1 = ?$

3	d.	2d	3d	4d
15	5d	6d	7d	8d
x 7	9d	10d	11d	12d
105	13d	14d		

$$S = a_1 + a_1 + d + \dots + a_1 + 14d =$$

$$= \frac{a_1 + a_1 + 14d}{2} \cdot 15 = \frac{2a_1 + 14d}{2} \cdot 15 =$$

$$= 15a_1 + 7 \cdot 15 \cdot d = 15a_1 + 105d$$

$$a_7 \cdot a_{16} = (a_1 + 6d)(a_1 + 15d) > 15a_1 + 105d -$$

a_1, d - yegone - 24

$$\begin{array}{r} 3 \\ \times 15 \\ \hline 45 \\ \times 6 \\ \hline 90 \end{array}$$

$$a_{11} \cdot a_{12} = (a_1 + 10d)(a_1 + 11d) < 15a_1 + 105d + 4$$

$$\begin{array}{r} 21 \\ \times 21 \\ \hline 21 \\ \times 21 \\ \hline 42 \\ \hline 441 \end{array}$$

$$\begin{array}{r} 21 \\ \times 15 \\ \hline 105 \\ \times 21 \\ \hline 315 \end{array}$$

$$a_1^2 + 6a_1 + 9 > 0$$

$$a_1^2 + 2 \cdot 3 \cdot a_1 + 3^2 > 0$$

$$(a_1 + 3)^2 > 0 \Rightarrow a_1 - \text{not } 0$$

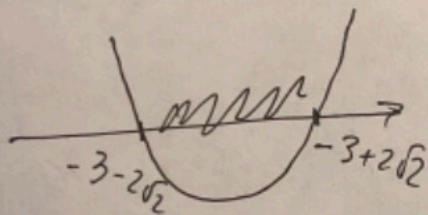
$$a_1^2 + 6a_1 + 1 < 0$$

$$D = 36 - 4 = 32$$

$$\sqrt{D} = \sqrt{4 \cdot 8} = 2\sqrt{8} = 2\sqrt{4 \cdot 2} = 4\sqrt{2}$$

$$a_1 = \frac{-6 \pm 4\sqrt{2}}{2}$$

$$a_1 = -3 \pm 2\sqrt{2}$$



$$a_1 = -6$$

1,4

$$\frac{1,4}{2,8}$$

$$-3 - 2,8 = -5,8$$

$$-3 + 2,8 = -0,8$$

0
a_1 \in (-5,8; -0,8)

-5 ; -4 ; -2 ; -1

Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21104462**

ID профиля: **369655**

Вариант 22

Задача № 4

$$\begin{cases} \text{НОД}(a; b; c) = 14 \\ \text{НОК}(a; b; c) = 2^{17} \cdot 7^{18} \end{cases}$$

Обозначим $a = 2^{x_1} \cdot 7^{x_2}$

$b = 2^{y_1} \cdot 7^{y_2}$

$c = 2^{z_1} \cdot 7^{z_2}$

Условия из НОД и НОК:

$$\begin{cases} \min(x_1, y_1, z_1) = 1 \\ \max(x_1, y_1, z_1) = 17 \end{cases}$$

Значит какие-то два числа равны 1 и 17, а

третье $\in [1; 17]$

третье степеней

всего 6 троек

Тогда с учетом повторяющихся троек степеней

 $(17; 1; 1); (1; 17; 1); \dots (17; 17; 1)$

Получим кол-во уникальных вариантов

$17 \cdot 3! - 6 = 96$

Таким же образом находится кол-во троек для 7 степеней: $3! \cdot 18 - 6 = 102$ При этом заметим, что пара троек степеней 2 и 7 однозначно задает тройку $(a; b; c)$ Значит всего троек $(a; b; c)$ будет

$96 \cdot 102 = 9792$

Ответ: 9792 троек

№5
1) $\log_{\left(\frac{x}{2}+1\right)^2} \left(\frac{7x}{2} - \frac{17}{1}\right)$

Пусть

$a = \frac{x}{2} + 1$

2) $\log_{\sqrt{\frac{7x}{2} - \frac{17}{4}}} \left(\frac{3x}{2} - 6\right)^2$

$b = \frac{7x}{2} - \frac{17}{4}$

3) $\log_{\sqrt{\frac{3x}{2} - 6}} \left(\frac{x}{2} + 1\right)$

$c = \frac{3x}{2} - 6$
попробуем найти

I) $\log_{a^2} b \cdot \log_{b^2} c^2 \cdot \log_{c^2} a =$
 $= \frac{1}{2} \log_a b \cdot 4 \log_b c \cdot 2 \log_c a = 4$

II) Из условия;

$t^2(t-1) = 4 \Rightarrow$

$t^3 - t^2 - 4 = 0$

$(t-2)/(t^2+t+2) = 0$
D ≠ 0

$t = 2$

$\Rightarrow 2 \log_c a = 2$

$\Rightarrow \begin{cases} \frac{x}{2} + 1 = \frac{3x}{2} - 6 \\ \frac{x}{2} + 1 = \sqrt{\frac{3x}{2} - 6} \end{cases}$

$\begin{cases} x = 7 \\ y^2 + 2y - 3y + 7 = 0 \end{cases}$

$x = 7$

Проверим что наш ответ

$a = \frac{7}{2} + 1 = \frac{9}{2} > 0$

$b = \frac{7 \cdot 7}{2} - \frac{17}{4} = \frac{81}{4} > 0$

$c = \frac{3 \cdot 7}{2} - 6 = \frac{9}{2} > 0$

Ответ: $x = 7$

$$1) \log_{\left(\frac{x}{2}+1\right)^2} \left(\frac{7x}{2} - \frac{17}{4}\right)$$

$$(1) = (3) \quad (2) = (1) - 1$$

$$(1) = (2) \quad (3) = (1) - 1$$

$$2) \log_{\sqrt{\frac{7x}{2} - \frac{17}{4}}} \left(\frac{3x}{2} - 6\right)^2 = \log_{\left(\frac{7x}{2} - \frac{17}{4}\right)^{\frac{1}{2}}} \left(\frac{3x}{2} - 6\right)^2$$

$$(2) = (3) \quad (1) = (2) - 1$$

$$3) \log_{\sqrt{\frac{3x}{2} - 6}} \left(\frac{x}{2} + 1\right)$$

$$\log_a b = \frac{1}{\log_b a}$$

$$1) \frac{1}{2} \log_{\left(\frac{x}{2}+1\right)} \left(\frac{7x}{2} - \frac{17}{4}\right)$$

$$\log_a x + \log_a y = \log_a(xy)$$

$$2) 4 \log_{\left(\frac{7x}{2} - \frac{17}{4}\right)} \left(\frac{3x}{2} - 6\right)$$

$$\frac{7x}{2} - \frac{17}{4} = a$$

$$3) 2 \log_{\frac{3x}{2} - 6} \left(\frac{x}{2} + 1\right)$$

$$\frac{7}{2}x = a + \frac{17}{4}$$

$$x = \frac{2}{7}a + \frac{17}{7}$$

$$\frac{x}{2} = \frac{17}{14} + \frac{a}{7}$$

$$(1) = (2) \quad \log_a b \quad \log_b a$$

$$\frac{1}{2} \cdot \frac{1}{\log_{\left(\frac{7x}{2} - \frac{17}{4}\right)} \left(\frac{x}{2} + 1\right)} = 4 \log_{\left(\frac{7x}{2} - \frac{17}{4}\right)} \left(\frac{3x}{2} - 6\right)$$

$$\frac{1}{8} = \log_{\left(\frac{7x}{2} - \frac{17}{4}\right)} \left(\frac{x}{2} + 1\right) \log_{\left(\frac{7x}{2} - \frac{17}{4}\right)} \left(\frac{3x}{2} - 6\right)$$

$$\frac{1}{8} = \log_a \left(\frac{a}{7} + \frac{17}{8} + 1\right) \cdot \log_a \left(\frac{3a}{7} + \frac{51}{8} - 6\right)$$

$$\frac{1}{8} = \log_a \left(\frac{a}{7} + \frac{25}{8}\right) \log_a \left(\frac{3a}{7} + \frac{1}{4}\right)$$

$$\log_a b = (a-1)(b-1) \quad \log$$

$$\begin{array}{r} 17 \\ + 8 \\ \hline 25 \\ \times 2 \\ \hline 50 \\ + 17 \\ \hline 67 \end{array}$$

$$\frac{48}{8}$$

$$\frac{2}{8} = \frac{1}{4}$$

№4

Условия на x_1, x_2, z_1, z_2

$$a = 2^{x_1} \cdot 7^{x_2}$$

$$\min(x_1, y_1, z_1) = 1$$

$$b = 2^{y_1} \cdot 7^{y_2}$$

$$\max(x_1, y_1, z_1) = 17$$

$$c = 2^{z_1} \cdot 7^{z_2}$$

Знаем \downarrow какне \rightarrow

2 числа равни 1 и 17,

а 3-тото число равно 1 или 17 в промежутке $[1, 17]$

Тогда в учете повторения трех степеней

всего 6 троек $(17, 1, 1), (1, 17, 1), \dots, (17, 17, 1)$
получим только 6 троек $(17, 1, 1), (1, 17, 1), \dots, (17, 17, 1)$
или 17 троек $(17, 1, 1), (1, 17, 1), \dots, (17, 17, 1)$

$$3! = 1 \cdot 2 \cdot 3 = 6$$

$$17 \cdot 6 - 6 = 96$$

$$17 \cdot 3! - 6 = 96$$

$$\begin{array}{r} 16 \\ \times 6 \\ \hline 96 \end{array}$$

Таким же образом находим количество троек где степени 7: $3! \cdot 18 - 16 = 102$

При этом заметим, что пара троек степеней 2 и 7 однозначно задает третью (a, b, c) и наоборот. Значит всего троек (a, b, c)

$$\text{Всего: } 96 \cdot 102 = 9600 + 192 = 9792$$

$$\text{Ответ: } \underline{9792}$$

$$\begin{array}{r} 6 \cdot 18 - 6 \\ 6 \cdot 17 \\ \hline 102 \end{array}$$

2 · 7 · 2 · 9

N 4

(a; b; c) - !

16	18
2	2
2	9
2	
2	
2	

НОД (a; b; c) = 14

НОК (a; b; c) = 2¹⁷ · 7¹⁸

$$\frac{2^{17} \cdot 7^{18}}{a} = t$$

14	14	14
14	14 ⁷	28

a = 14k₁

b = 14k₂

c = 14k₃

k₁ k₂ k₃ - гармоническая прогрессия

2¹⁹ · 7¹⁸ = at₁

2¹⁷ · 7¹⁸ = bt₂

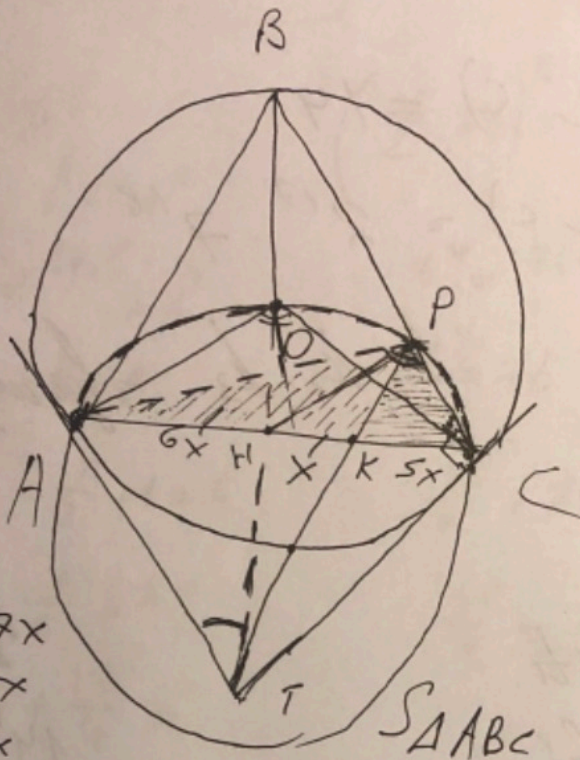
2¹⁷ · 7¹⁸ = ct₂

2¹⁷ · 7¹⁸ = 27k₁ · t₁

2¹⁶ · 7¹⁷ = k₁ · t₁

14

14¹⁷ · 7



$AK = 7x$
 $KC = 5x$
 $AH = 6x$

S_{ABC}

$S_{\Delta APK} = 7$

$S_{\Delta CPK} = 5$

$7 = S_{\Delta APK} = \frac{1}{2} h \cdot AK$

$5 = S_{\Delta CPK} = \frac{1}{2} h \cdot KC$

$\frac{7}{5} = \frac{AK}{KC}$

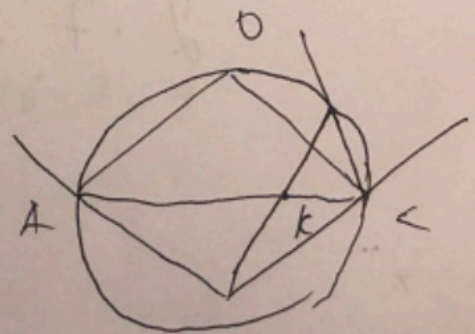
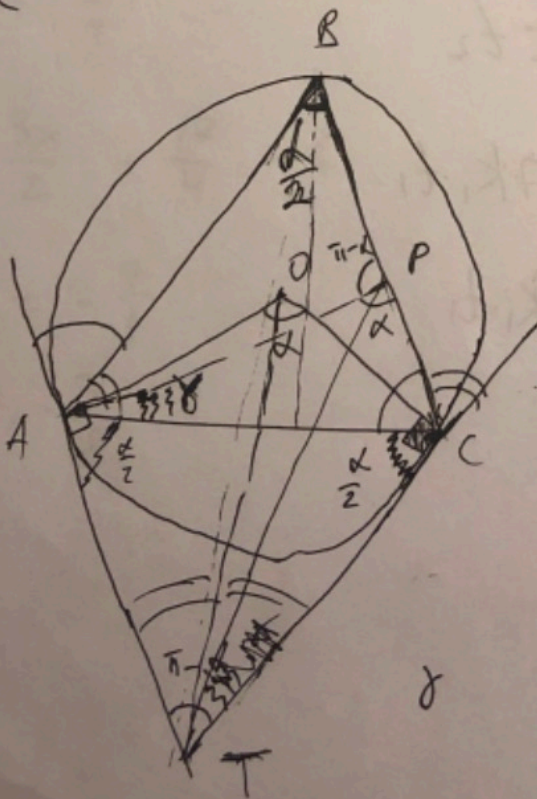
$AB \parallel PK?$

$AK = 7x$

$KC = 5x$

$AC = 12x$

$S_{APC} = \frac{1}{2} h_A \cdot PK = 12$



$$\frac{1}{x} = \frac{1}{4} \cdot \frac{1}{2}$$

$$\log\left(\frac{7x}{2} - \frac{17}{4}\right) \left(\frac{x}{2} + 1\right) = \frac{1}{4}$$

$$\log_a b = c \quad \left(\frac{x}{2} + 1\right)^4 = \frac{7x}{2} - \frac{17}{4}$$

$$b = a^c$$

$$a^{\log_a b} = b$$

$$a^c = b$$

$$t^4 = 7(t-1) - \frac{17}{4}$$

$$t^4 = 7t - \frac{45}{4}$$

$$\frac{45}{4} = 7t - t^4$$

$$\frac{45}{4} = t(7-t^3)$$

~~4~~

$$\begin{array}{r} 10 \\ 24 \\ -17 \\ \hline 7 \end{array}$$

$$\frac{17}{4} - \frac{24}{4} = \frac{17-24}{4} = -\frac{7}{4}$$

$$\log_a b \log_a c = \log_a c$$

$$\begin{array}{r} 1 \\ 28 \\ +17 \\ \hline 45 \end{array}$$
$$\frac{x}{2} = t-1$$
$$7 + \frac{17}{4} =$$
$$= \frac{28+17}{4} = \frac{45}{4}$$

$$\frac{7x}{2} - \frac{3x}{2} = \frac{17}{4} - 6$$

$$2x = -\frac{7}{4}$$

$$x = -\frac{7}{8}$$

$$16-7=9$$

$$\left(1 - \frac{7}{16}\right)^2 = \frac{3}{2} \cdot \frac{-7}{8} - 6$$

$$\left(\frac{9}{16}\right)^2 = -$$

$$\log_a b$$

$$\log_b c$$

$$a = b = c$$

$$a = \left(\frac{x}{2} + 1\right)^2$$

$$\frac{3x}{2} - 6 = t$$

$$b = \left(\frac{3x}{2} - \frac{17}{4}\right)$$

$$\frac{3x}{2} - 6 = t$$

$$c = \left(\frac{3x}{2} - 6\right)^4$$

$$\frac{x}{2} = \frac{t+6}{3}$$

$$a = b = c$$

$$\begin{array}{r} 42 \overline{) 3} \\ \underline{3} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

$$a =$$

$$a = \left(\frac{t+6}{3} + 1\right)^2 = \left(\frac{t}{3} + 3\right)^2$$

$$14 - \frac{17}{4} =$$

$$= \frac{14 \cdot 4 - 17}{4} =$$

$$b = \left(\frac{7t+42}{3} - \frac{17}{4}\right) = \frac{7t}{3} + \frac{39}{4}$$

$$c = t^4$$

$$a = c$$

$$\left(\frac{t+9}{3}\right)^2 = t^4$$

$$3^4 = 81$$

$$(t+9)^2 = 9t^4$$

$$t^2 + 18t + 81 = 9t^4$$

$$9(t^4 - 9)$$

$$9 + 18 \cdot 3 + 81 \neq 9 \cdot 81$$

$$9(t^2 - 3)^2 =$$

$$9 + 54 + 81 \neq 729$$

$$= 9(t^2 - 3)(t^2 + 3)$$

$$t(t+18) = 9(t^2 - 3)(t^2 + 3)$$

$$\begin{array}{r} 14 \\ \times 4 \\ \hline 56 \end{array}$$

$$\begin{array}{r} 56 \\ -17 \\ \hline 39 \end{array}$$

$$\begin{array}{r} 18 \\ \times 3 \\ \hline 54 \end{array}$$

$$\begin{array}{r} 81 \\ \times 9 \\ \hline 729 \end{array}$$

$$1) \log\left(\frac{x}{2} + 1\right)^2 \left(\frac{7x}{2} - \frac{17}{4}\right) = \frac{1}{2} \log\left(\frac{x}{2} + 1\right) \left(\frac{7x}{2} - \frac{17}{4}\right)$$

$$2) \log\left(\frac{7x}{2} - \frac{17}{4}\right)^{\frac{1}{2}} \left(\frac{3x}{2} - 6\right)^2 = 4 \log\left(\frac{7x}{2} - \frac{17}{4}\right) \left(\frac{3x}{2} - 6\right)$$

$$3) \log\left(\frac{3x}{2} - 6\right)^{\frac{1}{2}} \left(\frac{x}{2} + 1\right) = 2 \log\left(\frac{3x}{2} - 6\right) \left(\frac{x}{2} + 1\right)$$

$$\text{I) } (1) = (3) \Rightarrow (2) = (1) - 1$$

$$(1) = (2) \Rightarrow (3) = (1) - 1$$

$$\text{II) } (2) = (3) \Rightarrow (1) = (2) - 1$$

$$\frac{1}{2} \log\left(\frac{x}{2} + 1\right) \left(\frac{7x}{2} - \frac{17}{4}\right) = 2 \log\left(\frac{3x}{2} - 6\right) \left(\frac{x}{2} + 1\right)$$

$$\log\left(\frac{x}{2} + 1\right) \sqrt{\frac{7x}{2} - \frac{17}{4}} = \log\sqrt{\frac{3x}{2} - 6} \left(\frac{x}{2} + 1\right)^2$$

$$\log\left(\frac{x}{2} + 1\right) \sqrt{\frac{7x}{2} - \frac{17}{4}} = \frac{1}{\log\left(\frac{x}{2} + 1\right) \sqrt{\frac{3x}{2} - 6}}$$

$$\log\left(\frac{x}{2} + 1\right) \sqrt{\frac{7x}{2} - \frac{17}{4}} \cdot \log\left(\frac{x}{2} + 1\right) \sqrt{\frac{3x}{2} - 6} = 1$$

$$\begin{cases} \left(\frac{x}{2} + 1\right) = \sqrt{\frac{7x}{2} - \frac{17}{4}} \\ \left(\frac{x}{2} + 1\right) = \sqrt{\frac{3x}{2} - 6} \end{cases}$$

$$\begin{cases} \frac{7x}{2} - \frac{17}{4} = \frac{3x}{2} - 6 \\ \left(\frac{x}{2} + 1\right)^2 = \frac{3x}{2} - 6 \end{cases} \quad X = -\frac{7}{8}$$

$$t^2 + t + 2 = 0$$

$$D = 1 - 4$$