Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21102299**

ID профиля: **865925**

Вариант 22

1. S - сумма первых 15 ч. возр. арифм. прогр. Чистовик стр. 1

$a_1, a_2, a_3, \ldots \in \mathbb{Z}$

$a_7 a_{16} > S - 24$

$a_{11} a_{12} < S + 4$

a_1 - ?

$a_7 = a_1 + 6d$

$a_{16} = a_1 + 15d$

$a_{11} = a_1 + 10d$

$a_{12} = a_1 + 11d$

$S = \frac{a_1 + a_{15}}{2} \cdot 15 = \frac{a_1 + a_1 + 14d}{2} \cdot 15 = (a_1 + 7d) \cdot 15$

$a_7 \cdot a_{16} = (a_1 + 6d)(a_1 + 15d) = a_1^2 + 21a_1 d + 90d^2$

$a_{11} \cdot a_{12} = (a_1 + 10d)(a_1 + 11d) = a_1^2 + 21a_1 d + 110d^2$

$a_1^2 + 21a_1 d + 90d^2 > 15a_1 + 105d - 24$

$a_7 a_{16} > S - 24 > (a_{11} a_{12} - 4) - 24$

$90d^2 > 110d^2 - 28$

$\begin{cases} 20d^2 < 28 \\ d > 0 \end{cases}$

$0 < d < \sqrt{\frac{28}{20}} < 2$

$a_1 \in \mathbb{Z};\ a_2 \in \mathbb{Z}$

$d = a_2 - a_1 \in \mathbb{Z}$

$1 \overset{<}{\vee} \sqrt{\frac{28}{20}}$

$1 \overset{<}{\vee} \frac{28}{20}$

$2 \overset{>}{\vee} \sqrt{\frac{28}{20}}$

$4 \overset{>}{\vee} \frac{28}{20}$

$\begin{cases} 0 < d < 2 \\ d \in \mathbb{Z} \end{cases} \Leftrightarrow \boxed{d = 1}$

$\times\ 14$
14
$+\ 56$
14
196

~~$a_7 a_{16} = a_1^2 + 21a_1 + 90 > (a_1 + 7) \cdot 15 - 24$~~

~~$a_1^2 + 6a_1 + 90 - 105 + 24 > 0$~~

~~$a_1^2 + 6a_1 - (129 - 90) > 0$~~

~~$a_1^2 + 6a_1 - 39 > 0$~~

~~$a_1 = \frac{-6 \pm \sqrt{192}}{2}$~~

~~$36 + 39 \cdot 4 = 36 + (40 - 1) \cdot 4 = 160 + 36 - 4 =$~~

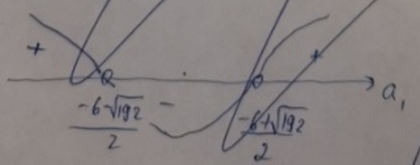
~~$= 192$~~

~~$\sqrt{192} < \sqrt{196} = 14$~~

~~$\frac{-6 + \sqrt{192}}{2} < \frac{14 - 6}{2} = 4$~~

~~$-\sqrt{192} > -14$~~

~~$\frac{-6 - \sqrt{192}}{2} > \frac{-6 - 14}{2} = -10$~~



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$a_{11} a_{12} < S + 4$

$a_1^2 + 21a_1 d + 110d^2 < (a_1 + 7d) \cdot 15 + 4$

$a_1^2 + 21a_1 + 110 < (a_1 + 7) 15 + 4$

$a_1^2 + 6a_1 + 110 - 105 - 4 < 0$

$a_1^2 + 6a_1 + 1 < 0$

$D = 36 - 4 = 32$

$a_1 = \dfrac{-6 \pm 4\sqrt{2}}{2} = -3 \pm 2\sqrt{2}$

$2\sqrt{2} - 3 \overset{<}{\vee} 0$ $\qquad 2\sqrt{2} < 3$

$2\sqrt{2} - 3 \overset{>}{\vee} -1$ $\qquad -2\sqrt{2} > -3$

$2\sqrt{2} \overset{>}{\vee} 2$

$-3 - 2\sqrt{2} > -6$

$-3 - 2\sqrt{2} \overset{>}{\vee} -7$

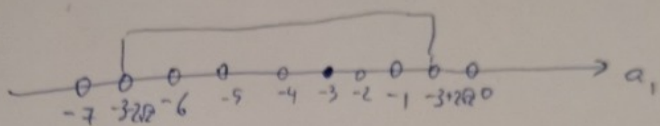
$4 \overset{>}{\vee} 2\sqrt{2}$

$a_1^2 + 6a_1 + 90 + 24 - 105 > 0$

$a_1^2 + 6a_1 + 9 > 0$

$(a_1 + 3)^2 > 0$

$a_1 \neq -3$



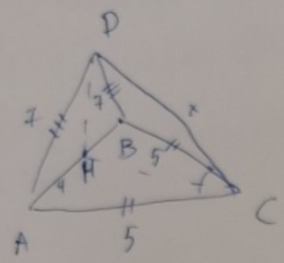
$\begin{cases} a_1 = \{-6; -5; -4; -3; -2; -1\} \\ a_1 \neq -3 \end{cases} \Leftrightarrow a_1 = \{-6; -5; -4; -2; -1\}$

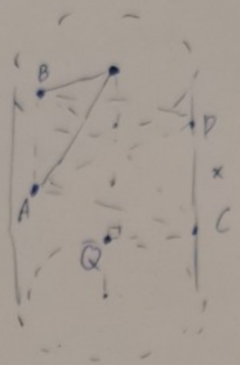
Ответ: $-6; -5; -4; -2; -1$.

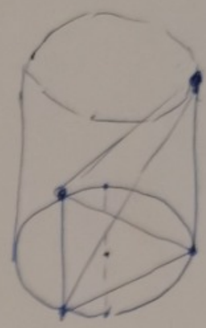
$a_1 = -3$
$d = 1$
$a_{11} = a_1 + 10d = -3 + 10 = 7$
$a_{12} = a_1 + 11d = -3 + 11 = 8$
$S = (-3 + 7) \cdot 15 = 60$
$a_7 = a_1 + 6d = -3 + 6 = 3$
$a_{16} = a_1 + 15d = -3 + 15 = 12$

$a_1 = -1$
$d = 1$
$a_{11} = -1 + 10 = 9$
$a_{12} = 10$
$S = (-1 + 7) \cdot 15 = 90$
$a_7 = -1 + 6 = 5$
$a_{16} = -1 + 15 = 14$

2. $AB=4$; $AC=CB=5$; $AD=DB=7$ r_{min} $CD-?$ Чистовик стр. 3.

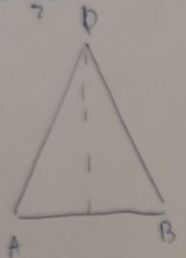






$p_1=\frac{5+5+4}{2}=5+2=7$

$p_2=\frac{7+7+4}{2}=7+2=9$



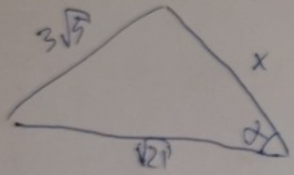
$S_{ABC}=\sqrt{p_1(p_1-5)^2(p_1-4)}=\sqrt{7\cdot(7-5)^2\cdot 3}=2\sqrt{21}$

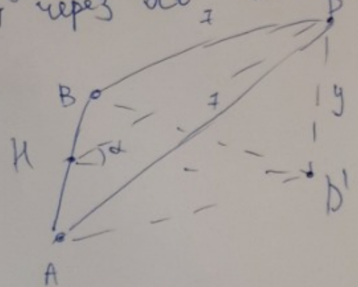
$S_{ABD}=\sqrt{p_2(p_2-7)^2(p_2-4)}=\sqrt{9\cdot(9-7)^2\cdot 5}=6\sqrt{5}$

$CH=\frac{S_{ABC}}{\frac{1}{2}AB}=\frac{2\sqrt{21}}{2}=\sqrt{21}$

$DH=\frac{S_{ABD}}{\frac{1}{2}\cdot AB}=\frac{6\sqrt{5}}{2}=3\sqrt{5}$

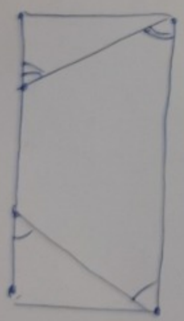
Из ~~...~~ середины AB опустим перпен-р на CD, он пройдёт через ось цилиндра, $\Delta ABD'$ – впис.





$r=\frac{abc}{4S}$

$r=\frac{(49-y^2)\cdot 4}{4\cdot S_{ABD'}}$



Выразим радиус и найдём производную:

$r=\frac{(49-(\sin\alpha\cdot DH)^2)}{\cos\alpha\cdot S_{ABD}}$

$r=\frac{(49-\sin^2\alpha\cdot 45)}{\cos\alpha\cdot 2\sqrt{21}}$

$\left(\frac{(49-\sin^2x\cdot 45)}{\cos x\cdot 2\sqrt{21}}\right)'=\frac{\cos x\cdot 2\sqrt{21}\cdot(-45)\,2\cos x\cdot\sin x-}{\cos^2x\cdot 4\cdot 21}$

$(\sin^2x)'=(\sin x\cdot\sin x)'=$

$-2\cos x\cdot\sin x$

$\frac{-(49-\sin^2x\cdot 45)\cdot(2\sqrt{21}\cdot(-\sin x))}{}=0$

$\sin x\cdot\left(\left(2\sqrt{21}(49-\sin^2x\cdot 45)\right)-4\sqrt{21}\cos^2x\cdot 45\right)=0$

$\left[\begin{array}{l}\sin x=0\\ 49-\sin^2x\cdot 45-90\cos^2x=0\end{array}\right.$

$49-45(\sin^2x+\cos^2x)-45\cos^2x=0$

$\cos^2x=\frac{4}{45}$

$\sin^2x=1-\frac{4}{45}=\frac{41}{45}$

$r_1=\frac{49}{2\sqrt{21}}=49\left(\frac{1}{2\sqrt{21}}\right)$

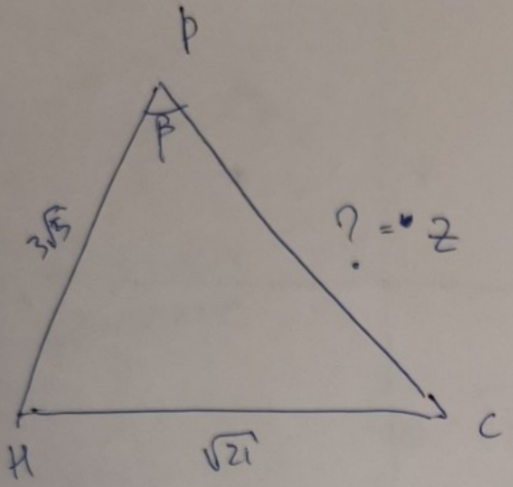
$r_2=\frac{49-\frac{41}{45}\cdot 45}{2\sqrt{21}\cdot\frac{2}{\sqrt{45}}}=\frac{1}{2\sqrt{21}}\cdot\left(8\cdot\frac{\sqrt{45}}{2}\right)$

$r_2=r_{min}$

Чистовик стр. 4.

$\cos\alpha = \frac{2}{\sqrt{45}}$

$\cos\angle HDD' = \sin\alpha = \sqrt{1 - \frac{4}{45}} = \sqrt{\frac{41}{45}}$



По th cos-в:

$21 = 45 + z^2 - 2 \cdot \cancel{3\sqrt{5}} \cdot z \cdot \frac{\sqrt{41}}{\cancel{3\sqrt{5}}}$

$z^2 - 2\sqrt{41}z + 24 = 0$

$D = 4 \cdot 41 - 4 \cdot 24 = 4 \cdot 17$

$z = \frac{2\sqrt{41} \pm 2\sqrt{17}}{2} = \sqrt{41} \pm \sqrt{17}$

Ответ: $\sqrt{41} \pm \sqrt{17}$.

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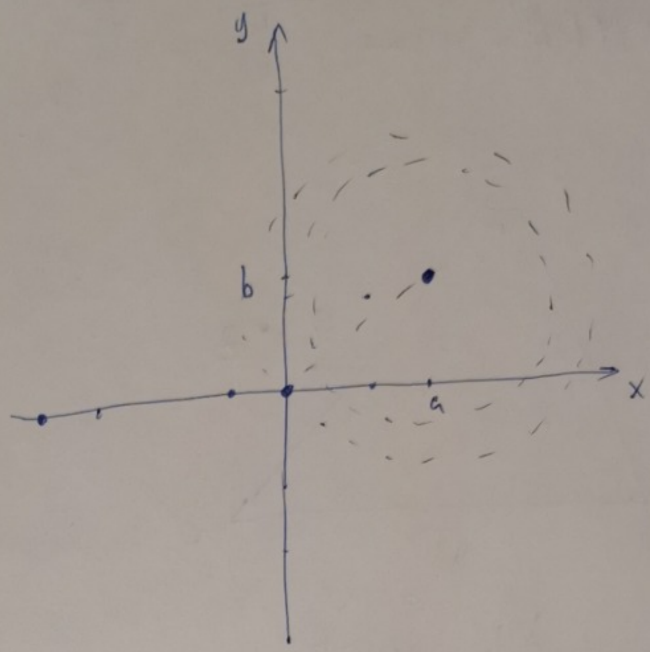
3. (x,y) $a, b \in \mathbb{R}$ Черновик.

$$M: \begin{cases} (x-a)^2+(y-b)^2 \le 50 \\ a^2+b^2 \le \min(14a+2b, 50) \end{cases}$$

S_M - ?



1. $14a+2b \le 50$: $7a+b \le 25$

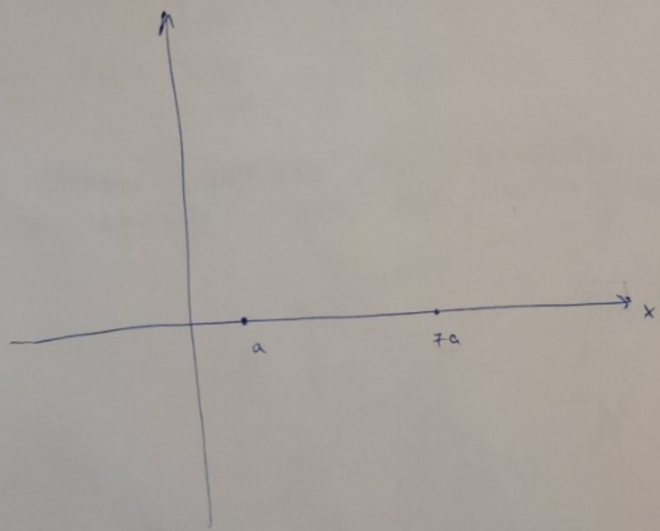
$a^2+b^2 \le 14a+2b$ $b \le 25-7a$

$a^2+b^2 \le 50$

$(a-7)^2+(b-1)^2 \le 50$

2. $14a+2b > 50$:

$a^2+b^2 \le 50$



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Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

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Вариант 22

Чистовик стр. 1

4. кол-во троек (a, b, c):

$$\begin{cases} \text{НОД}(a; b; c) = 14 \\ \text{НОК}(a; b; c) = 2^{17} \cdot 7^{18} \end{cases}$$

$a = 2^x \cdot 7^y$; $b = 2^z 7^d$; $c = 2^e 7^f$

$\text{НОД}(a; b; c) = 2 \cdot 7$

$x, y, z, d, e, f \geq 1$

одно равно $2^{17} \cdot 7^{18}$, другое $2 \cdot 7$

третее $2^x \cdot 2^y$ $x = 1, 2, \ldots, 17$ $y = 1, 2, \ldots, 18$.

кол-во: $17 \cdot 18 \cdot 3! = 17 \cdot 18 \cdot 6 = 1836$.

Ответ: 1836.

Чистовик стр. 2.

5. $\log_{(\frac{x}{2}+1)^2}\left(\frac{7x}{2}-\frac{17}{4}\right)$, $\log_{\sqrt{\frac{7x}{2}-\frac{17}{4}}}\left(\frac{3x}{2}-6\right)^2$, $\log_{\sqrt{\frac{3x}{2}-6}}\left(\frac{x}{2}+1\right)$

$$\frac{1}{2}\cdot\frac{\lg\left(\frac{7x}{2}-\frac{17}{4}\right)}{\lg\left(\frac{x}{2}+1\right)}\cdot 4\cdot\frac{\lg\left(\frac{3x}{2}-6\right)}{\lg\left(\frac{7x}{2}-\frac{17}{4}\right)}\cdot 2\cdot\frac{\lg\left(\frac{x}{2}+1\right)}{\lg\left(\frac{3x}{2}-6\right)}=4$$

обозначим одно из равных чисел за y, тогда:

$y\cdot y\cdot(y-1)=4$

$y^3-y^2-4=0$

~~при~~ $y=2$: $8-4-4=0$ - верно, $y=2$ - корень

$(y-2)(y^2+y+2)=0 \Leftrightarrow y=2$

$y^2+y+2=0$

$D=1-4<0$

1. $\log_{\sqrt{\frac{3x}{2}-6}}\left(\frac{x}{2}+1\right)=2$

$\log_{\frac{3x}{2}-6}\left(\frac{x}{2}+1\right)=1$

$\frac{x}{2}+1=\frac{3x}{2}-6$

$x=7$: ~~подходит~~

$\log_{(\frac{x}{2}+1)^2}\left(\frac{7x}{2}-\frac{17}{4}\right)=\frac{1}{2}\cdot\log_{\frac{9}{2}}\frac{81}{4}=1$

$4\log_{(\frac{7x}{2}-\frac{17}{4})}\left(\frac{3x}{2}-6\right)=4\cdot\log_{(\frac{81}{4})}\left(\frac{9}{2}\right)=4\cdot\frac{1}{2}=2$

$x=7$ - подходит

2. $4\log_{(\frac{7x}{2}-\frac{17}{4})}\left(\frac{3x}{2}-6\right)=2$

$\frac{3x}{2}-6=\sqrt{\frac{7x}{2}-\frac{17}{4}} \Leftrightarrow \begin{cases}\left(\frac{3x}{2}-6\right)^2=\left(\frac{7x}{2}-\frac{17}{4}\right)\\ \frac{3x}{2}-6\geq 0\end{cases}$

$(3x-12)^2=14x-17$

$9x^2-72x+144=14x-17$

$9x^2-86x+161=0$

$D=86^2-161\cdot 9\cdot 4=1600=40^2$

$x=\frac{86\pm 40}{18}=\left[\begin{array}{l}\frac{126}{18}=7\\ \frac{46}{18}\end{array}\right.$

$3x-12\geq 0$

$\frac{46}{6}-12\geq 0$ - неверно

3. $\log_{(\frac{x}{2}+1)^2}\left(\frac{7x}{2}-\frac{17}{4}\right)=2$,

тогда один из двух других $=2$, т.е. при $x=7$, этот пункт не м. расширить мн-во реш.

Ответ: 7

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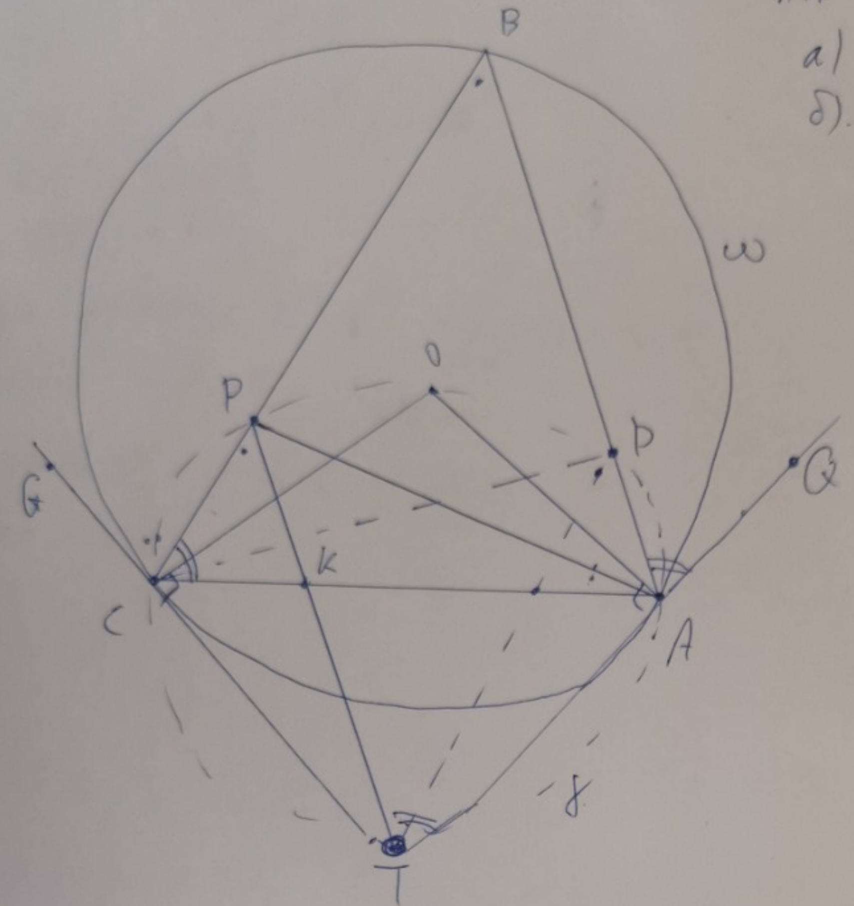
6. Чистовик стр. 3.

$S_{APK}=7$; $S_{CPK}=5$

а). S_{ABC}

б). $\angle ABC = arctg\frac{3}{4}$ Н! AC



$OC\perp CT$; $OA\perp AT$ (касательные)

$\angle OCT+\angle OAT=180^\circ \Leftrightarrow OCTA$ - впис.

T лежит на опис. окр. $\triangle OAC$

точки C, P, O, A, T лежат на одной окр. (обозначим её γ)

$\angle BAQ=\angle BCA$ (AT - касат.)

$\angle BCA=\angle ATP$ (опир. на одн. дугу) $\Big\} \Rightarrow \angle BAQ=\angle ATP \Leftrightarrow$

$\Leftrightarrow PT \parallel BA$, значит $\angle CKP=\angle CAB$, $\angle C$ - общ. $\Big\} \overset{\text{пр.}}{\Longleftrightarrow} \triangle CPK \sim \triangle CBA \Rightarrow$

$$\Rightarrow \frac{S_{ABC}}{S_{CPK}}=\left(\frac{CA}{CK}\right)^2=\left(\frac{S_{APC}}{S_{CPK}}\right)^2=\left(\frac{5+7}{5}\right)^2=\frac{12^2}{5^2}$$

$$S_{ABC}=\cancel{7}\cdot\frac{12^2}{5^2}=\frac{144}{5}=\boxed{28,8}$$

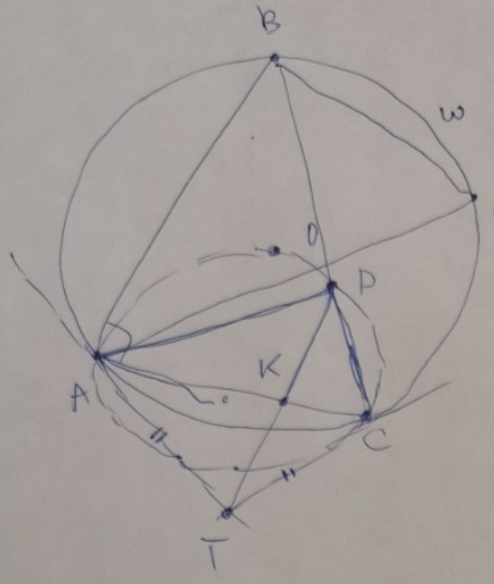
Обозначим точку пер. BA с γ: D

$\angle GCB=\angle CAB$ $\Big\} \Rightarrow CB \parallel TD$

$\angle CAD=\angle CTD$

6

Черновик

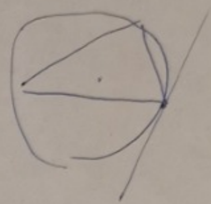


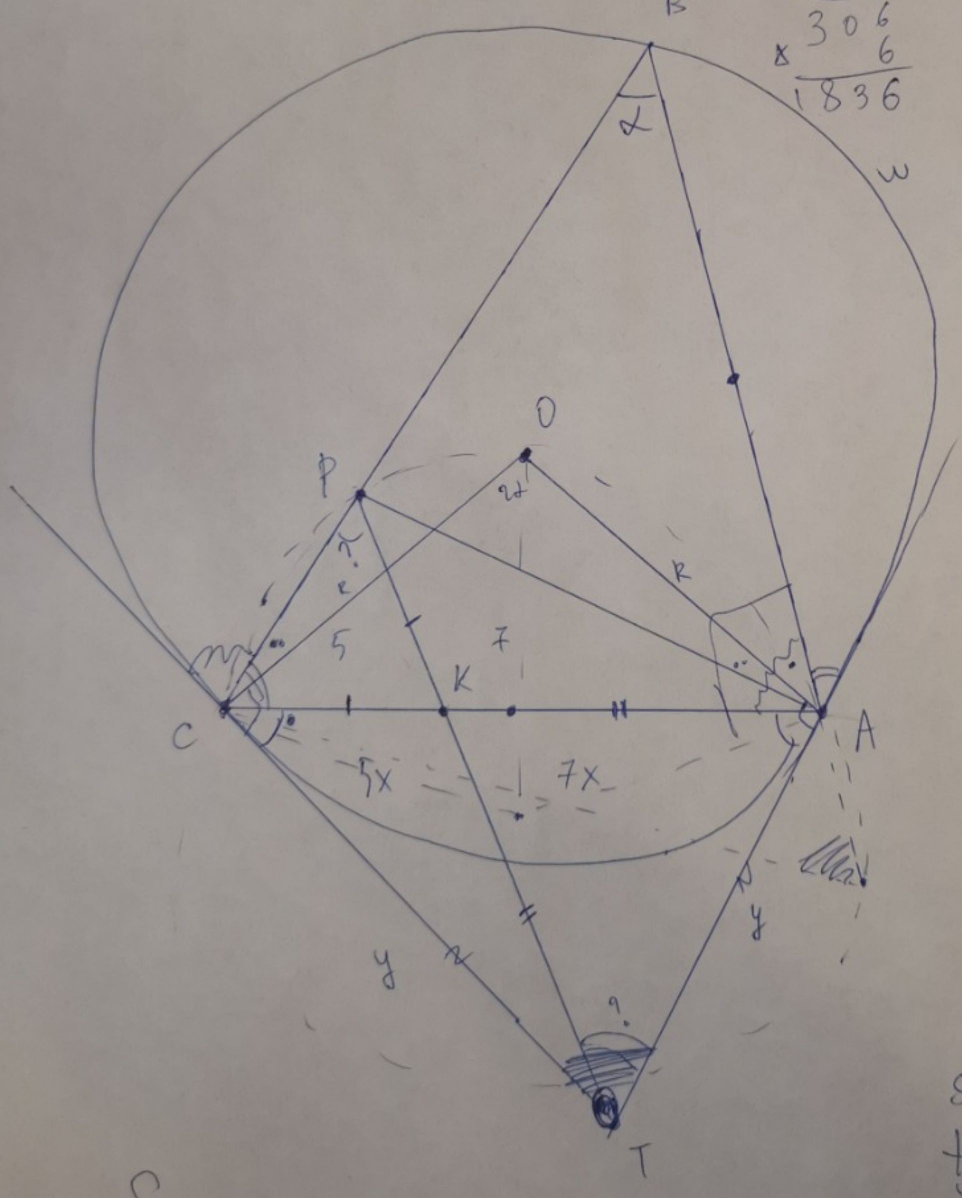
$S_{APK}=7$; $S_{CPK}=5$

а). S_{ABC}

б). $\angle ABC=\operatorname{arctg}\frac{3}{4}$. Н! AC

$$\begin{array}{r} 5 \\ 17 \\ \times\ 18 \\ \hline +\ 136 \\ 17 \\ \hline 306 \\ \times\ 6 \\ \hline 1836 \end{array}$$





$\frac{AC}{\sin\alpha}=2R$

S,α AC-?

$$\begin{array}{r|l} 144 & 5 \\ \underline{10} & \overline{28{,}8} \\ 44 & \\ \underline{40} & \\ 40 & \end{array}$$

$\frac{CK}{CA}=\frac{5}{12}$

$\frac{S_{CPK}}{S_{ABC}}=\left(\frac{5}{12}\right)^2$

$S_{ABC}=5\cdot\frac{12^2}{5^2}=\frac{144}{5}$

$\sin^2\alpha+\cos^2\alpha=1$ $:\cos^2\alpha$

$\operatorname{tg}\alpha=\frac{3}{4}$

$\operatorname{tg}^2\alpha+1=\frac{1}{\cos^2\alpha}$

$\cos^2\alpha=\frac{1}{\operatorname{tg}^2\alpha+1}=\frac{1}{\frac{9}{16}+\frac{16}{16}}=\frac{16}{25}$

$\sin^2=1-\frac{16}{25}=\frac{9}{25}=\frac{3}{5}$

$S=\frac{1}{2}x^2\cdot\sin\alpha$

$\frac{144}{5}=\frac{1}{2}\cdot x^2\cdot\frac{3}{5}$ $x^2=\frac{144\cdot2}{3}=\left(12\cdot\sqrt{\frac{2}{3}}\right)^2$

Черновик

$\frac{1}{2}\log_{\left(\frac{x}{2}+1\right)}\left(\frac{7x}{2}-\frac{17}{4}\right)=2$

$\cancel{2}\log_{\left(\frac{3x}{2}-6\right)}\left(\frac{x}{2}+1\right)=\cancel{2}$

$\frac{x}{2}+1=\frac{3x}{2}-6$

$7=x$

$\frac{1}{2}\cdot\log_{\left(\frac{9}{2}\right)}\left(\frac{81}{4}\right)=1$

$4\cdot\log_{\left(\frac{7x}{2}-\frac{17}{4}\right)}\left(\frac{3x}{2}-6\right)=4\cdot\log_{\left(\frac{81}{4}\right)}\left(\frac{9}{2}\right)=4\cdot\frac{1}{2}=2$

$4\cdot\log_{\left(\frac{7x}{2}-\frac{17}{4}\right)}\left(\frac{3x}{2}-6\right)=2$

$\frac{3x}{2}-6=\sqrt{\frac{7x}{2}-\frac{17}{4}}$

$3x-12=\sqrt{14x-17}$

161×36: $966 + 483 = 5796$

86×86: $516 + 688 = 7396$

$(3x-12)^2=9x^2-72x+144=14x-17$

$2300-700=1600$

$9x^2-86x+161=0$

$D=86^2-161\cdot 9\cdot 4=(90-4)^2-36\cdot 161$

$\frac{86\pm 40}{18}=\left[\begin{array}{l}\frac{126}{18}=7\\ \frac{46}{18}\end{array}\right.$

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5. $\log_{(\frac{x}{2}+1)^2}\left(\frac{7x}{2}-\frac{17}{4}\right)$, $\log_{\sqrt{\frac{7x}{2}-\frac{17}{4}}}\left(\frac{3x}{2}-6\right)^2$, $\log_{\sqrt{\frac{3x}{2}-6}}\left(\frac{x}{2}+1\right)$

Черновик

$\frac{1}{2}\cdot\log_{(\frac{x}{2}+1)}\left(\frac{7x}{2}-\frac{17}{4}\right)$ $\quad 4\log_{(\frac{7x}{2}-\frac{17}{4})}\left(\frac{3x}{2}-6\right)$ $\quad 2\log_{(\frac{3x}{2}-6)}\left(\frac{x}{2}+1\right)$

"a" "b" "c"

1 3 3 1
1 4 6 4 1

$\frac{1}{2}\cdot\frac{1}{\log_{(\frac{7x}{2}-\frac{17}{4})}\left(\frac{x}{2}+1\right)}$

$a=b$
$c=b-1$

$\cancel{\frac{1}{2}\log_a b \quad 4\log_b c \quad 4\log_c b \quad \log_c a}$

$\frac{1}{2}\frac{\lg b}{\lg a}\cdot 4\frac{\lg c}{\lg b}\cdot 2\frac{\lg a}{\lg c} =$
$= 4$

$49\cdot 2=98$

$\frac{21}{2}-\frac{12}{2}$

$y\cdot y\cdot(y-1)=4$

$y^3-y^2-4=0$

$(y-2)(y^2+y+2)=0$ $\quad 1-4<0$

$y^3+y^2+2y-2y^2-2y-4$ $\quad y=2$