

# Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21101207**

ID профиля: **297322**

Вариант 22

Чепуобук

$a_0$   
 $a_1 = a_0 + d$   
 $a_2 = a_0 + 2d$   
 $a_n = a_0 + nd$

$a_0$   
 $a_{15} = a_0 + 15d$

$S = \frac{a_0 + a_{15}}{2} \cdot 15 = \frac{a_0 + a_0 + 14d}{2} \cdot 15 = (a_0 + 7d) \cdot 15$

$a_1^2 + 21ad + 90d^2 = 15a_1 + 105d - 24$

$(a_1 + 6d)(a_1 + 15d) \geq (a_1 + 10d)(a_1 + 11d)$

$a_1^2 + 21a_1d + 90d^2 \geq 15a_1d + 105d^2 - 24$

$a_1^2 + 6a_1d \geq -24$

$a_1^2 + 21a_1d + 110d^2 \leq 15a_1d + 105d^2 + 24$

$a_1^2 + 6a_1d + 1 < 0$

$t - 1 > 0$   
 $t + 1 < 0$   
 $t > 1$

-6

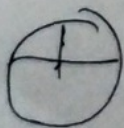
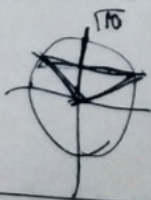
$\sqrt{2} = -3$   
 $-3 - 3$

$a^2 + b^2 \leq 14a + 2b$

$a^2 - 14a + b^2 - 2b \leq 0$

$(a-7)^2 + (b-1)^2 \leq 50$

$a^2 - 14a + 49 + b^2 - 2b + 1$



$a = 14 \neq 7$

$\frac{D}{6} + \frac{1}{6} = \frac{Q}{8} = \frac{3}{2}$



45 15+4



$a_1^2 + 21a_1d + 90d^2 = 15a_1 + 105d - 24$

$a_1^2 + 6a_1d + 24 - 15d = 0$

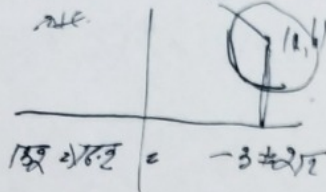
$a_1^2 + 6a_1d + 9 = 0$



$25 - 7a = 0$

$a = \frac{25}{7} = 3\frac{4}{7}$

$a_1^2 + 6a_1d$



$\frac{-6 \pm \sqrt{36-4}}{2}, \frac{-6 \pm \sqrt{32}}{2}$

$a^2 + b^2 \leq 50$

$b = 25 - 7a$

$a^2 + (25 - 7a)^2 \leq 50$

$14a + 2b \geq 50$

$a^2 + b^2 \leq 50$

$14a + 2b \geq 50$

$b \geq 25 - 7a$

$x^2 + y^2 \leq 50$

$y \geq 25 - 7x$

$\frac{25}{7} \geq x$

$14a + 2b \leq 50$

$\frac{\sqrt{50}}{2}$

$\frac{115}{15} / \frac{15}{25}$

$25 - 7a + 1$

$400 + 160b + 25$

$\frac{580}{16}$

$\frac{20+4}{8} = \frac{400+16}{80}$

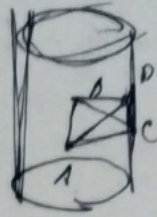
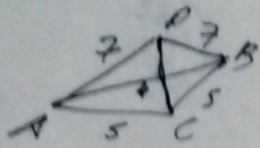
$\frac{160}{45}$

$\frac{160}{45}$

$\frac{160}{45}$

$\frac{160}{45}$

$\frac{160}{45}$



$$50 \frac{\sqrt{3}}{8} a^2$$

$$K = a \cdot \frac{\sqrt{3}}{2} \cdot \frac{2}{3} = \frac{a}{\sqrt{3}}$$

$$\sqrt{7^2 - k^2} + \sqrt{5^2 - k^2} = k$$

$$\frac{A \cdot C}{2R} = 5$$

49 + 25

$\{a_n\}$  - арифметична прогресія,  $a_n \in \mathbb{Z}$

~~$a_n = a_0 + nd$   
 $S_{15} = a_0 + a_1 + a_2 + \dots + a_{14} = \frac{a_0 + a_{14}}{2} \cdot 15 = \frac{(a_0 + 14d)}{2} \cdot 15$   
 $a_7 = a_0 + 7d$   
 $a_{16} = (a_0 + 16d)$   
 $a_7 a_{16} = 5 - 24$   
 $(a_0 + 7d)(a_0 + 16d) > 15(a_0 + 7d) - 24$   
 $a_{11} = a_0 + 11d$   
 $a_{12} = a_0 + 12d$   
 $a_{11} a_{12} = 5 + 4$~~

$a_n = a_1 + (n-1)d$   
 $S_{15} = a_1 + a_2 + a_3 + \dots + a_{15} = \frac{a_1 + a_{15}}{2} \cdot 15 = \frac{a_1 + a_1 + 14d}{2} \cdot 15 = \frac{(a_1 + 7d)}{2} \cdot 15$   
 $a_7 a_{16} = 5 - 24$

$(a_1 + 6d)(a_1 + 15d) > 15(a_1 + 7d) - 24$   
 $a_{11} a_{12} = 5 + 4$

$(a_1 + 10d)(a_1 + 11d) < 15(a_1 + 7d) + 4$   
 $(a_1 + 6d)(a_1 + 15d) > 15(a_1 + 7d) - 24$

$(a_1 + 10d)(a_1 + 11d) < 15(a_1 + 7d) + 4$   
 $24 + (a_1 + 6d)(a_1 + 15d) > (a_1 + 10d)(a_1 + 11d) - 4$

$24 + a_1^2 + 21a_1d + 20d^2 > a_1^2 + 21a_1d + 110d^2 - 4$   
 $24 > 20d^2 - 4$   
 $28 > 20d^2$   
 $d^2 < 1,4$

$d \in (-\sqrt{1,4}; \sqrt{1,4})$

$d = 0$

$d \in (0; \sqrt{1,4})$

$d \in \mathbb{Z}$

$d = 1$

$(a_1 + 6)(a_1 + 15) > 15(a_1 + 7) - 24$   
 $(a_1 + 10)(a_1 + 11) < 15(a_1 + 7) + 4$

$a_1^2 + 6a_1 + 9 > 0$   
 $a_1^2 + 6a_1 + 1 < 0$

$a_1 \neq -3$   
 $a_1 \in \mathbb{Z}$

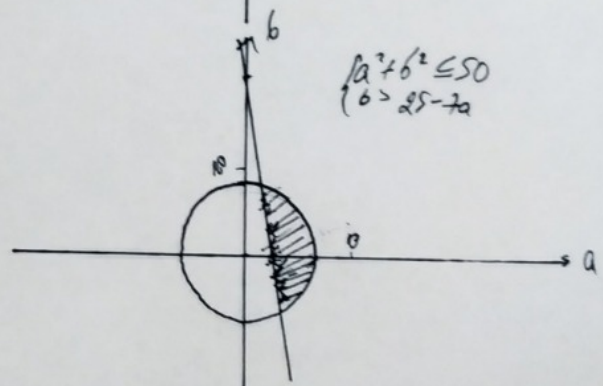
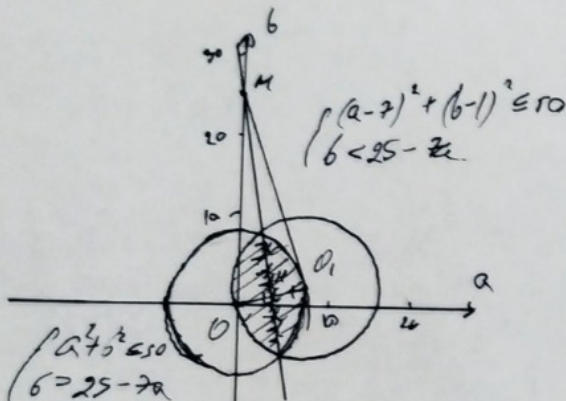
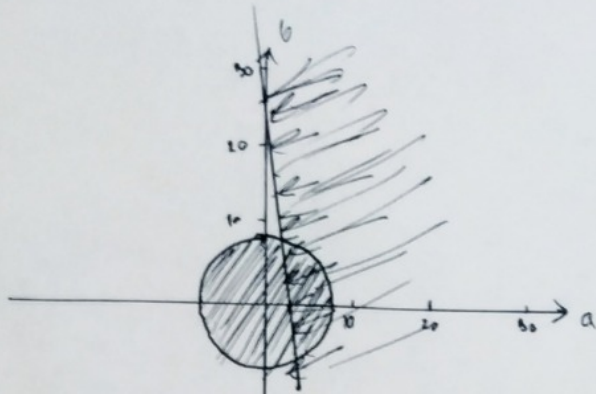
$(a_1 + 3)^2 > 0$   
 $a_1 \in (-3 - 2\sqrt{2}; -3 + 2\sqrt{2})$

- Отже: ~~5; -4; -2; 1; 0; 1; 2; 3; 4; 5~~  
~~-5; -4; -2; -1~~

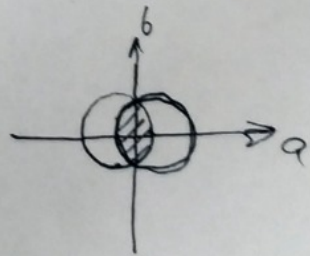
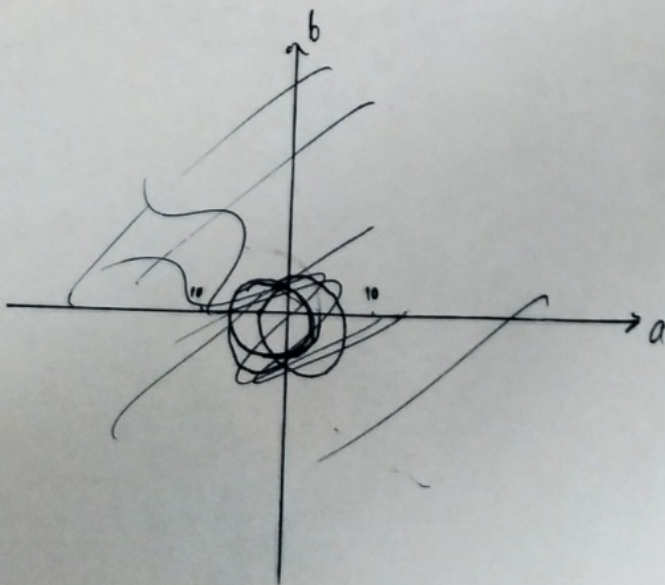
$$\begin{cases} (x-a)^2 + (y-b)^2 \leq 50 \\ a^2 + b^2 \leq \min(14a+2b, 50) \end{cases}$$

1)  $14a+2b > 50$   
 $a^2 + b^2 \leq 50$   
 $\begin{cases} a^2 + b^2 \leq 50 \\ b > 25 - 7a \end{cases}$

2)  $14a+2b < 50$   
 $a^2 + b^2 \leq 14a+2b$   
 $\begin{cases} (a-7)^2 + (b-1)^2 \leq 50 \\ b < 25 - 7a \end{cases}$



Трапеция  $b = 25 - 7a \perp$  гипотенузу  $b = \frac{a}{7}$ .  $OM = MQ = 25$ . Тогда  $OM > OQ$ .  
 множество точек  $(a, b)$  будет ~~субанквисно~~ ~~интервал~~ ~~интервалом~~  
~~интервалом~~.)

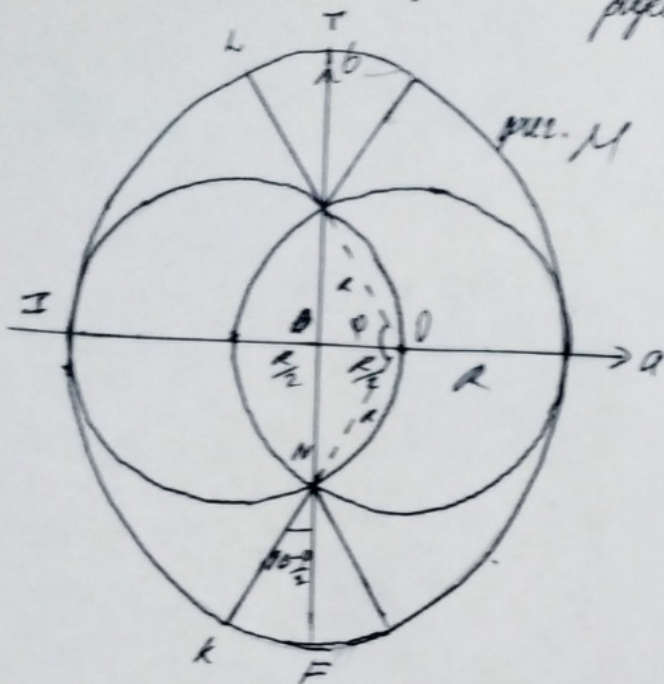


$$R = \sqrt{50}$$

каждый тонкий (об) элемент вырежем из  $\sigma$  поперечным сечением;

$$(x-a)^2 + (y-b)^2 \leq 50$$

Учебник.  
Математика, 11кл.



$$\cos \frac{\varphi}{2} = \frac{1}{2}$$

$$\frac{\varphi}{2} = 60^\circ$$

$$\varphi = 120^\circ = \frac{2\pi}{3}$$

$$S_{\text{окл}} = \frac{2\pi}{3} \cdot (2R)^2 \cdot \frac{1}{3} = \frac{8\pi R^2}{3}$$

$$S_{\text{FIT}} = S_{\text{окл}} - 2S_{\text{онд}} + 2S_{\text{нкф}}$$

$$S_{\text{онд}} = \frac{R}{2} \cdot R \cdot \frac{\pi \cdot 60}{2} = \frac{R^2}{4} \cdot \frac{\pi}{2} = \frac{\pi R^2}{8}$$

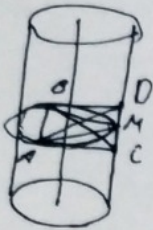
$$S_{\text{нкф}} = \frac{\pi}{6} \cdot R^2 = \frac{\pi R^2}{6}$$

$$S_{\text{FIT}} = \frac{8\pi R^2}{3} - 2 \cdot \frac{\pi R^2}{8} + 2 \cdot \frac{\pi R^2}{6} = \frac{4\pi R^2}{3} - \frac{\pi R^2}{4} + \frac{\pi R^2}{3} = \frac{5\pi R^2}{2} - \frac{\pi R^2}{4}$$

$$S_{\text{M}} = 2S_{\text{FIT}} = 5\pi R^2 - \frac{\pi R^2}{2} = 5\pi \cdot 50 - \frac{\pi \cdot 50}{2} = 150\pi - 25\pi$$

$$\text{Ответ: } 150\pi - 25\pi$$

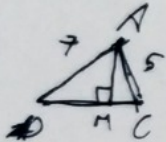
ABCD - трапеция.  
 AB = 4  
 AC = CB = 5  
 AD = DB = 7.



рассматривать.

Сечение цилиндра, проходящее через T. A и T. B является  
 хордой. CD. Рассматривать сечение цилиндра будет. хордой. CD.  
 T, M - точка хорды CD. Точка M - точка хорды CD.  
 $\triangle ABM$  вписан в  $\odot$  с центром O.

Рассмотрим  $\triangle AOC$



Пусть  $OC = x$ . Тогда  $7^2 + 5^2 - 2 \cdot 5 \cdot 7 \cos \angle A = x^2$

$74 - 70 \cos A = x^2$

$\frac{74 - x^2}{70} = \cos A$

$\frac{\sin^2 A}{x^2} \rightarrow \max$

$1 - \frac{(74 - x^2)^2}{70^2} \rightarrow \max$

$\sin A = \sqrt{1 - \left(\frac{74 - x^2}{70}\right)^2}$

$S_{AOC} = \frac{7 \cdot 5}{2} \cdot \sin A = \frac{AM \cdot x}{2}$

$\frac{\sin A}{x} \rightarrow \max$

$AM = \frac{35 \sin A}{x}$

$AM \rightarrow \max$

$AM = BM$ , т.к.  $\triangle AOC \cong \triangle BOC$ .

Рассмотрим  $\triangle AMB$

$AM \cdot \sin \frac{\angle M}{2} \geq R$



$\sin \angle M \rightarrow \max$

$\sin \frac{\angle M}{2} = \frac{R}{AM}$

$\sin \frac{\angle M}{2} \rightarrow \max$

$AM \leq 2R$

Пусть  $R$  - радиус описанной окружности

$\frac{(AM)^2 \cdot R}{AR} = S_{AMB}$

$R = \frac{AM^2}{S_{AMB}} = \frac{AM^2}{AM^2 \cdot \frac{\sin \angle M}{2}} = \frac{2}{\sin \angle M}$

Т.к.  $R \rightarrow \min$ , то  $\sin \angle M \rightarrow \max$

$$\frac{1 - \left(\frac{74-x^2}{70}\right)^2}{x^2} \rightarrow \text{max mit } 12 \quad \text{Umsatz}$$

$$x^2 = t, \quad t \geq 0 \quad t > 0$$

$$\frac{1 - \left(\frac{74-t}{70}\right)^2}{t} \rightarrow \text{max mit}$$

$$\frac{1 - \left(\frac{74-t}{70}\right)^2}{t} \geq \frac{70^2 - (74-t)^2}{70^2 t} \geq \frac{(70-74+t)/(70+74-t)}{70^2 t} \geq \frac{(t-4)(144-t)}{70^2 t}$$

$$\frac{(t-4)(144-t)}{t} \rightarrow \text{max mit } 1402t \geq 4$$

$$\frac{144t - t^2 + 4t - 4 \cdot 144}{t} = 148 - t + \frac{4 \cdot 144}{t} = 148 - t - \frac{4 \cdot 144}{t}$$

$$\left(148 - t - \frac{4 \cdot 144}{t}\right)' \geq -1 + \frac{4 \cdot 144}{t^2} \geq 0$$

$$t^2 \geq 4 \cdot 144 - \text{max}$$

$$t \geq 12$$

$$x^2 = 24$$

$$CD = 12$$

$$CD = 12 \Rightarrow 2\sqrt{6}$$

$$\text{Ober: } 2\sqrt{6}$$

$$\text{Ober: } 2; 12$$



$$t = 4 \text{ mit } 144$$

$$x^2 = 4/144$$

$$x = 2/12$$

$$CD = 2; 12$$

$$148 - 4 - \frac{4 \cdot 144}{4} = 148 - 4 - 144 = 0$$

$$148 - 144 - \frac{4 \cdot 144}{144} = 0$$



# Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21101207**

ID профиля: **297322**

Вариант 22

2)  $\log_7 c = \log_7 2^a \cdot 7^b = a \log_7 2 + b$

$\log_7(a, b, c) = 14$

$\log_7(a, b, c) = 2^{17} \cdot 7^{16}$

$a = 16x, b = 16y, c = 16z$

$\log_7(x, y, z) = 1$   
 $xyz = 2^{16} \cdot 7^{17}$

1) $(2^m \cdot 7^k, 2^k \cdot 7^k, 7)$ $m+k=16$ $k=1, 2, \dots, 16$ $m=15, 14, \dots, 0$	2) $(2^m, 7^k, 7^k)$ $m=16$ $k=1, 2, 3, \dots, 16$ $16 \cdot 3$	3) $(2^m, 2^k, 7^k)$ $m+k=16$ $m=1, 2, 3, \dots, 15$ $k=1, 2, 3, \dots, 16$ $16 \cdot 17 \cdot 3$
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$3691$   
 $3$   
 $11073$   
 $16(16+1) = 272$   
 $271$   
 $13$   
 $3523$   
 $813$   
 $68$   
 $3691$   
 $271$   
 $3523$

Како намери такви  $x, y, z \in \mathbb{N}$ , што  $\log_7(x, y, z) = 1$ ,  $xyz = 2^{16} \cdot 7^{17}$  то е возможно, кога  $x, y, z$  се прости!

- $(2 \cdot 7, 2 \cdot 7, 7)$
- $(2, 7, 7) (2, 7, 7)$
- $(2, 2, 7)$
- ~~$(2, 7, 7, 7)$~~
- $(2 \cdot 7, 7, 7)$
- $(2 \cdot 7, 2 \cdot 7, 2 \cdot 7)$

$3(4(16 \cdot 17 - 1) + 2 \cdot 17 \cdot 2 + 2 \cdot 16 + 1 + 1 + 9(16 \cdot 17 - 1)) =$   
 $= 3(13(16 \cdot 17 - 1) + 2 \cdot 33 + 2) = 3(13(16 \cdot 17 - 1) + 68)$   
 $= 3(13 \cdot 271 + 68) = 11073$

Одгов: 11073

$x = 2^a \cdot 7^b$   
 $y = 2^c \cdot 7^d$   
 $z = 2^e \cdot 7^f$   
 $a+b+c+d+e+f = 16$   
 $a, b, c, d, e, f \in \{1, 2, 3, \dots, 16\}$

- 1)  $(2 \cdot 7, 2 \cdot 7, 7)$
- 2)  $(2, 7, 7)$
- 3)  $(2, 2, 7)$
- 4)  $(2^m \cdot 7^k, 7^k, 7^k)$
- 5)  $(2, 7, 7)$
- 6)  $(2 \cdot 7, 2 \cdot 7, 2 \cdot 7)$

$$\frac{x}{2} + 1 \neq 1 \quad \frac{7x}{2} - \frac{17}{4} > 0$$

$$\frac{x}{2} + 1 \neq -1 \quad \frac{7x}{2} - \frac{17}{4} \neq 1$$

$$\frac{3x}{2} - 6 > 0$$

$$\frac{3x}{2} - 6 \neq 1$$

Пуск  $\frac{x}{2} + 1 = a, \quad \frac{7x}{2} - \frac{17}{4} = b, \quad \frac{3x}{2} - 6 = c$

$$\log_a b, \log_{\sqrt[3]{b}} c^2, \log_{\sqrt{c}} a$$

$$3a - 9 = c$$

$$7a - \frac{45}{4} = b$$

$$a > 0, b > 0, c > 0$$

$$a \neq \pm 1, b \neq 1, c \neq 1$$

$$\frac{\log_a b}{2} \quad 4 \log_b c \quad 2 \log_c a$$

$$1) \frac{\log_a b}{2} = 4 \log_b c = 2 \log_c a + 1$$

$$2) \frac{\log_a b}{2} = 2 \log_c a = 4 (\log_b c + 1)$$

$$\log_a b = 8 \log_b c$$

$$\log_a b = 4 \log_c a + 2$$

$$\log_a b = 8 \frac{\log_a c}{\log_a b}$$

$$\log_a b = \frac{4 \log_a a}{\log_a c} + 2$$

$$\log_a^2 b = 8 \log_a c$$

$$\log_a b = \frac{4}{\log_a c} + 2$$

$$\log_a^2 b = 8 \log_a c$$

$$\log_a c = \frac{4}{\log_a b - 2}$$

$$\log_a^2 b = \frac{32}{\log_a b - 2}$$

$$\log_a b = t$$

$$t^2 = \frac{32}{t-2}$$

$$t^3 - 2t^2 - 32 = 0$$

$$t = 4$$

$$t^2 + 2t + 8 = 0$$

$$t \in \emptyset$$

$$\log_a b = 4$$

$$\log_b c = \frac{1}{2}$$

$$\log_c a = \frac{1}{2}$$

$$b = c^2$$

$$c = a^2$$

$$a^4 = b$$

$$3a - 9 = a^2$$

$$a^2 - 3a + 9 = 0$$

$$a \in \emptyset$$

$$\log_a b = 4 \log_c a$$

$$\log_a b = 8 \log_b c + 2$$

$$\log_a b = \frac{4 \log_a a}{\log_a c}$$

$$\log_a b = \frac{8 \log_a c}{\log_a b} + 2$$

$$\log_a b = \frac{4}{\log_a c}$$

$$\log_a b = 8 \frac{4}{\log_a b} + 2$$

$$\log_a b = t$$

$$t = \frac{32}{t-2}$$

$$t^2 - 2t - 32 = 0$$

$$t = 4$$

$$t^3 - 2t^2 - 32 = 0$$

$$t^2 + 2t + 8 = 0$$

$$t \in \emptyset$$

$$\log_a b = 4$$

$$a^2 - 3a + 9 = 0$$

$$a \in \emptyset$$

Ucradbeti

$$3) \sqrt{\log_6 c} = 2 \log_c a = \frac{\log_a^6 + 1}{2}$$

$$\sqrt{\log_c a} = 2 \log_6 c$$

$$\log_a^6 = \sqrt{\log_c a} - 2$$

$$\sqrt{\log_a^6} = \frac{2 \log_a^6}{\log_a^6}$$

$$\log_a^6 = \frac{4}{\log_a^6} - 2$$

$$\log_a^6 = 2 \log_a^2 c$$

$$\log_a^6 = \frac{4}{\log_a^6} - 2$$

$$\log_a^6 = t$$

$$2t^2 = \frac{4}{t} - 2$$

$$t = 1$$

$$2t^3 + 2t - 4 = 0$$

$$t^3 + t - 2 = 0$$

$$t^3 - 1 + t - 1 = 0$$

$$(t-1)(t^2+t+1) + t-1 = 0$$

$$(t-1)(t^2+t+2) = 0$$

$$t^2 + t + 2 = 0$$

$$t \leq 0$$

$$t = 1$$

$$\log_a^6 = 1$$

$$a = c$$

$$2a - 9 = a$$

$$2a = 9$$

$$a = 4,5$$

$$\frac{x}{2} + 1 = 4,5$$

$$x = 7$$

Order: 7.

# Чепробек

$$\frac{x}{2} + 7 - y = \frac{3x}{2} - \frac{17}{4}$$

$$y = 7 + \frac{17}{4} = \frac{28+17}{4} = 11,75 = \frac{47}{4}$$

$$\log_a \frac{7a-45}{4} = 8 \log \left( \frac{3a-9}{4} \right)$$

$$\log_a b = \log_c b^r$$

$$\log_a b = \frac{1}{\log_c b^r}$$

$$\log_a b \cdot \log_c b = 8$$

$$\log_a b = \frac{4}{\log_a c} + 2$$

$$\log_a b \log_a c = 4 + 2 \log_a c$$

$$2 \log_a c \log_a c = 4 + 2 \log_a c$$

$$\frac{1 \log_a c}{\log_a c} = 4 + 2 \log_a c$$

$$8 \log_a c = 4 + 2 \log_a c + 2 \log_a c \log_a c$$

$$8 \log_a c = 4 \log_a c + 2 \log_a c$$

$$4 \log_a c = 2 \log_a c + \log_a b$$

$$4 \log_a c \log_a c$$

- (2, 2, 7, 1)
- (7, 2, 7, 1)
- (2, 7, 2, 7, 1)
- (2, 7, 1, 3)

- a b c 14
- 10 15 20
- 5
- a = 4x b = 14y z = 1/2

$$\text{gcd}(x, y, z) = 1$$

$$14xyz = 2^{17} \cdot 7^{18}$$

$$xyz = 2^{16} \cdot 7^{17}$$

$$x = 2^8 \cdot 7^8$$

$$y = 2^8 \cdot 7^8$$

$$z = 2^8 \cdot 7^9$$

$$a = 14$$

$$b = 2^8 \cdot 7^8 \cdot 2^{17}$$

$$c = 2^8 \cdot 7^8 \cdot 2^{18}$$

$$2^9 \cdot 7^{10}$$

$$\text{gcd}(x, y, z) = 1$$

$$\text{НОК}(x, y, z) = 2^{16} \cdot 7^{17}$$

$$xyz = 2^{16} \cdot 7^{17}$$

$$3000$$

$$15 \cdot 20 \cdot 10$$

$$2 \cdot 7$$

$$2 \cdot 7$$

$$2 \cdot 7$$

$$2$$

$$2$$

- (2, 2, 7)
- (7, 2, 7)

- 10 15 20
- gcd = 5
- НОК = 60
- 5 \cdot 80 = 10 \cdot 15 \cdot 20

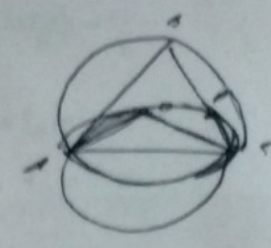
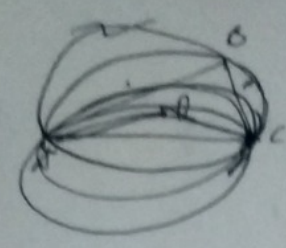
6-14

$$(a, b, c) = 14$$

$$2 \cdot 14 \cdot 3 \cdot 14 \cdot 14$$

$(2, 2, 7, 7)$   
 $(2, 2, 7, 7)$   
 $(2, 2, 7, 7)$   
 $(2, 2, 7, 7)$

$\log a = 2^{16} \cdot 7^{17}$   
 $\log(myz)$



$(2, 7, 2, 2, 2, 2)$   
 $(2, 7, 2, 7, 1)$   
 $(2, 7, 7)$   
 $(2, 2, 7)$   
 $(2, 7, 7, 2)$   
 $(2, 7, 1, 1)$

$2^9 \cdot 7^3, 7^9 \cdot 2^3, 7^5 \cdot 2^5$   
 $2^9 \cdot 7^3$

$(2, 7, 7, 2)$

$\frac{\log a^b}{2} = 4 \log_p c = 2 \log_p a$

$xyt = 2^{16} \cdot 7^{17}$   
 $\gcd(2^{16}, 7^{17}, 1)$   
 $2^6 \cdot 7^7$

$u(8u+1)^2 = 4$

$t = 8u+1$

$8ut + t^2 = \frac{4}{(8u+1)^2} + t$

$y = 2x^3 + x$   
 $2x^3 + x - 9 = 0$

$u = \frac{4}{(8u+1)^2}$

$u = \frac{1}{2}$

$t = 4$        $x = 2$   
 $x = 2$

$64 - 32 - 8$

$64 - 2 \cdot 16 - 32 = 0$

$\frac{4}{x} = \frac{32x^2 + 1}{16}$

$\log a^2 b = \log_p a$

$\frac{y}{x} = 2x^2 + 1$

$(t^2 + 2t + 1)(t-4) = t^3 - 2t^2 - 32$

$t^3 + 2t^2 + t - 4t^2 - 4t - 4a$

$a = \frac{9}{6}$

$a = 8$

$b = 4$

$3 \pm \sqrt{9 - 4 \cdot 9}$

$t = 5$   
 $b = 2$

$$\begin{array}{r|rr} 1 & 1 & -2 & 0 & -32 \\ -4 & 1 & -6 & 24 & \end{array}$$

$$\begin{array}{r|rr} t^3 - 2t^2 - 32 & t-4 \\ -t^3 + 4t^2 & \\ \hline & 2t^2 - 32 \\ & -2t^2 + 8t \\ \hline & & -8t \end{array}$$