Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

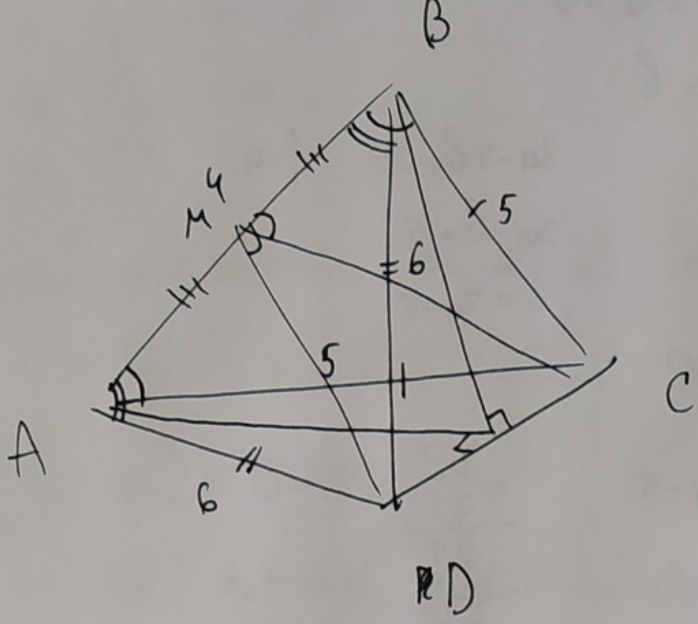
Шифр: **21103662**

ID профиля: **200049**

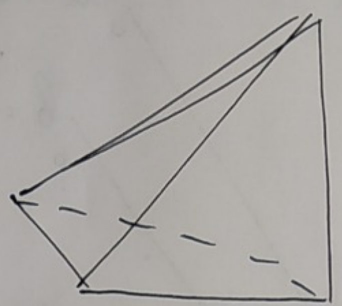
Вариант 21

Черновик

$AB \perp (CDM)$



$36-4=32$



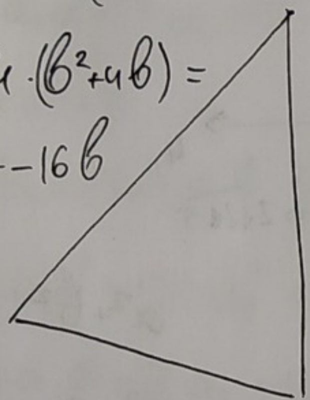
$DM=4\sqrt{2}.$

$a^2-8\cdot a+(b^2+4b)\le 0$

$D=(-8)^2-4\cdot(b^2+4b)=$

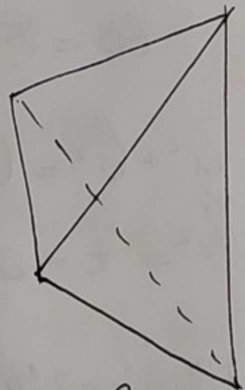
$=64-4b^2-16b$

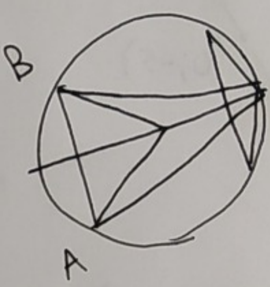
$b^2+4b-16.$



$DM^2=6^2-2^2=$

$=8\cdot 4=32=16\cdot 2$



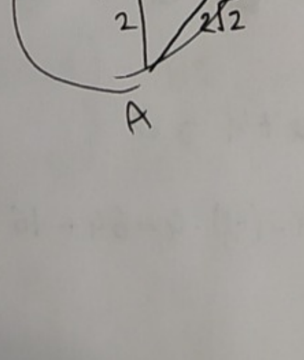


$(2\sqrt{2})^2=$

$=2\cdot 4$

$a^2+b^2\le 8a-4b$

$a^2+b^2-8a+4b\le 0.$



$b^2+4b+(a^2-8a)\le 0$

$D=4^2-4(a^2-8a)=16-4a^2+32a$

$-4(a^2-8a-4)=-4(a^2-8a+16-20)$

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Черновик

$$\begin{array}{r} -\,529 \\ 448 \\ \hline 81 \end{array}$$

$(9d)^2$

$8a-4b=0$

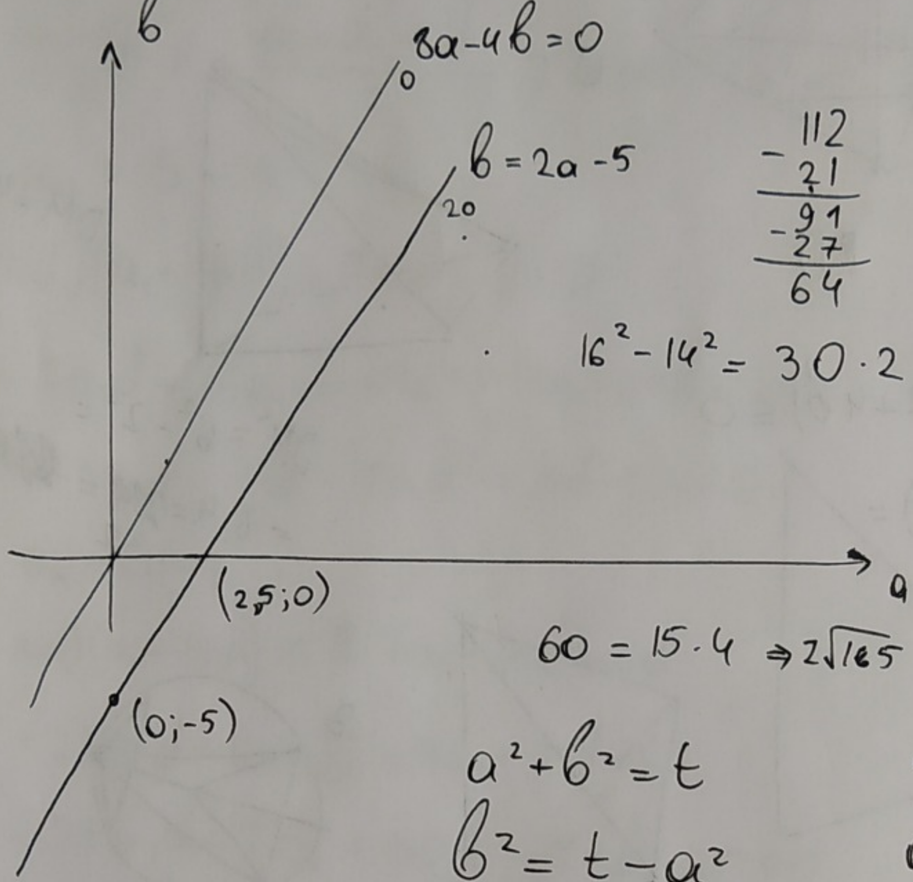
$2a=b$

$$\begin{array}{r} -\,327 \\ 87 \\ \hline 240 \end{array}$$

$8a-4b=20 \quad |:4$

$2a-b=5$

$b=2a-5$



$$\begin{array}{r} -\,112 \\ 21 \\ \hline -\,91 \\ 27 \\ \hline 64 \end{array}$$

$16^2-14^2=30\cdot 2=$

$60=15\cdot 4 \Rightarrow 2\sqrt{165}$

$a^2+b^2=t$

$b^2=t-a^2$

$a^2+b^2-8a+4b\leq 0$

$a^2-8a+(b^2+4b)$

$b^2+4b+(a^2-8a)$

$D-$

$a^2+23ad-7a+112d^2-21d-27$

$a^2+\cancel{ad}\ 23da\ -7d+130d^2-21d-60$

$a^2+b^2=\cancel{t}\ 0 \quad b^2=-a^2 \quad a(a-8)$

$-a^2+8a+4\geq 0$

$a_{1,2}=\dfrac{-8\pm\sqrt{80}}{-2}=4\mp\sqrt{20}$

$D=8^2-4\cdot(-1)\cdot 4=64+16=80=$

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1. $S = a_1 + a_2 + \ldots + a_7$ Чистовик. В21

$16 \cdot 7 = 112$

$a_8 \cdot a_{17} > S + 27$

$23 \cdot 23 = 529$ $\quad 23 \cdot 14 = 322$

$a_{11} \cdot a_{14} < 60 + S$

a_1 — ?

$27 \cdot 4 = 108$

$a_n = a_1 + d(n-1), \quad a_1 \in \mathbb{Z}; \ d \in \mathbb{N}$

$S = \frac{a_1 + a_7}{2} \cdot 7 = \frac{a_1 + a_1 + 6d}{2} \cdot 7 = (a_1 + 3d) \cdot 7 = 7a_1 + 21d.$

$a_8 \cdot a_{17} > S + 27$

$(a_1 + 7d)(a_1 + 16d) > 7a_1 + 21d + 27$

$a_1^2 + 7a_1d + 16a_1d + 112d^2 > 7a_1 + 21d + 27$

1) $a_1^2 + 23a_1d - 7a_1 + (112d^2 - 21d - 27) > 0$

$a_{11} \cdot a_{14} < 60 + S$

$(a_1 + 10d)(a_1 + 13d) < 7a_1 + 21d + 60$

$a_1^2 + 23a_1d + 130d^2 < 7a_1 + 21d + 60$

$a_1^2 + 23a_1d - 7a_1 + 130d^2 - 21d - 60 < 0$

2) $a_1^2 + a_1(23d - 7) + (130d^2 - 21d - 60) < 0$

① $a_1^2 + a_1(23d - 7) + (112d^2 - 21d - 27) > 0$

$D = (23d-7)^2 - 4 \cdot (112d^2 - 21d - 27) = 529d^2 - 322d + 49 - 448d^2 + 82d + 108 =$
$= 81d^2 - 240d + 157$

② $a^2 + a_1(23d - 7) + (130d^2 - 21d - 60) < 0 \quad | \cdot (-1)$

$-a_1^2 - a_1(23d-7) - (130d^2 - 21d - 60) > 0$

① + (−②) $= \cancel{a_1^2} + \cancel{a_1(23d-7)} + (112d^2 - 21d - 60) -$

$- \cancel{a_1^2} - \cancel{a_1(23d-7)} - (130d^2 - 21d - 60) > 0$

Чистовик. В21

$112d^2 - \cancel{21d} - 27 - 130d^2 + 60 + \cancel{21d} > 0$

$-18d^2 + 33 > 0 \Rightarrow 18d^2 < 33 \Rightarrow 6d^2 < 11 \Rightarrow d^2 < \frac{11}{6} \Rightarrow$

$d \in (0; \sqrt{\frac{11}{6}}) \cap \mathbb{N}$ $\left.\begin{array}{l} \sqrt{\frac{11}{6}} < \sqrt{\frac{12}{6}} = \sqrt{2} < 2 \\ \sqrt{\frac{11}{6}} > \sqrt{\frac{6}{6}} = \sqrt{1} = 1 \end{array}\right\} d = 1.$

1) $a_1^2 + 23a_1 - 7a_1 + 112 - 21 - 27 > 0$

$a_1^2 - 16a_1 + 64 > 0$

$(a_1 - 8)^2 > 0 \Rightarrow a_1 \neq 8$

2) $a_1^2 + 23a_1 - 7a + 130\cancel{d} - 21\cancel{d} - 60 < 0$

$a_1^2 + 16a_1 + 49 < 0$

$D = 16^2 - 4 \cdot 49 = 256 - 196 = 60$

$a_{12} = \frac{-16 \pm \sqrt{60}}{2} = -8 \pm \sqrt{15}$

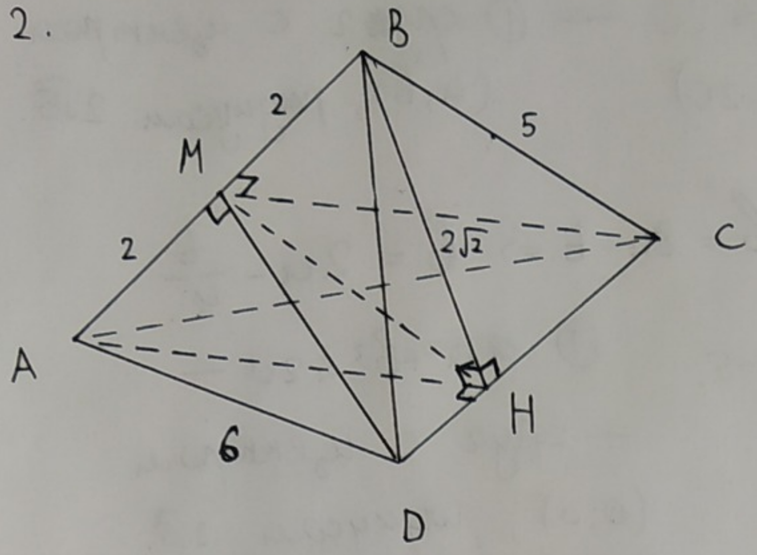
$\sqrt{15} < \sqrt{16} = 4$

$3 < \sqrt{15} < 4 \Rightarrow \left.\begin{array}{l} -8 - \sqrt{15} > -12 \\ -8 + \sqrt{15} < -4. \end{array}\right\} \Rightarrow a_1 \in [-11; -5] \cap \mathbb{Z}.$

Ответ: $a_1 \in [-11; -5] \cap \mathbb{Z}$

Чистовик. Вариант 21.

2.



$AB = 4$; $AC = BC = 5$;

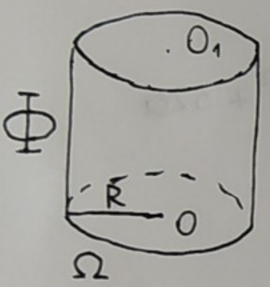
$AD = BD = 6$.

$\triangle ACD = \triangle BCD$:

$\left.\begin{array}{l} AD = BD; \\ AC = BC; \\ CD - \text{общ.} \end{array}\right\} \Rightarrow AH = BH$

$\left.\begin{array}{l} AH \perp CD \\ BH \perp CD \\ AH \cap BH = H \end{array}\right\} \Rightarrow CD \perp (ABH)$

$AB \subset (ABH) \Rightarrow AB \perp CD$



$\left.\begin{array}{l} \left.\begin{array}{l} (ABH) \perp CD \\ \Omega \perp CD \end{array}\right\} \Rightarrow \Omega \parallel (ABH) \\ A, B \in \Phi \end{array}\right\} \Rightarrow R$ цилиндра $\equiv$

$\equiv R$ опис. окр. $\triangle ABH$. (ω).

AB – хорда $\omega \Rightarrow R \to \min$, когда $AB \equiv d(\omega)$

$AB = 2R \Rightarrow \angle AHB$ опир. на диамет $\omega \Rightarrow$

$\Rightarrow AH \perp BH$; $AH = BH \Rightarrow AH = BH = \frac{AB}{\sqrt{2}} = 2\sqrt{2}$

$CH^2 = BC^2 - BH^2 = 5^2 - (2\sqrt{2})^2 = 25 - 8 = 17 \Rightarrow CH = \sqrt{17}$

$DH^2 = BD^2 - BH^2 = 6^2 - (2\sqrt{2})^2 = 36 - 8 = 28 \Rightarrow DH = 2\sqrt{7}$.

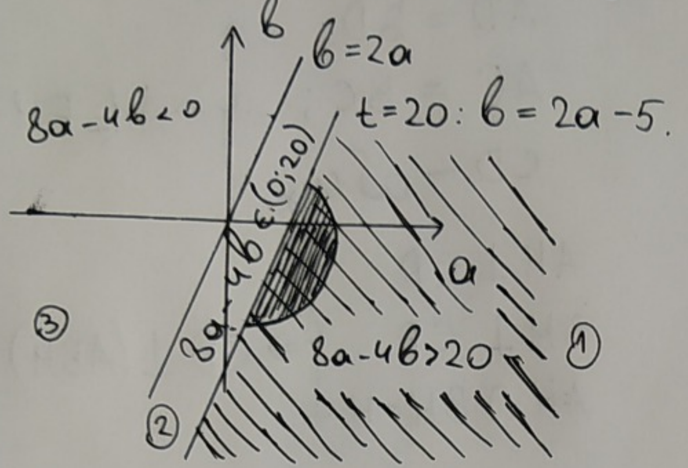
$\left[\begin{array}{l} CD = DH + CH = \sqrt{28} + \sqrt{17} \\ CD = DH - CH = \sqrt{28} - \sqrt{17}. \end{array}\right.$

Ответ: $\sqrt{28} + \sqrt{17}$; $\sqrt{28} - \sqrt{17}$.

Чистовик. Вариант 21.

3. $\begin{cases}(x-a)^2+(y-b)^2 \le 20 \\ a^2+b^2 \le \min(8a-4b;\ 20)\end{cases}$ — ① круг с центром $(a;b)$, радиусом $2\sqrt{5}$.

$8a-4b=t \Rightarrow 4b=8a-t \Rightarrow b=2a-\frac{t}{4}$.



① $a^2+b^2\le 20$ — круг с центром $(0;0)$, радиусом $2\sqrt{5}$.

② $a^2+b^2 \le 8a-4b$

$a^2+b^2-8a+4b\le 0$

$b^2+4b+(a^2-8a)\le 0$; $D=4^2-4\cdot(a^2-8a)=16-4a^2+32a$

$b_1=\frac{-4+\sqrt{-4a^2+32a+16}}{2}=-2+\sqrt{4+8a-a^2}$

$b_2=-2-\sqrt{4+8a-a^2}$

Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21103662**

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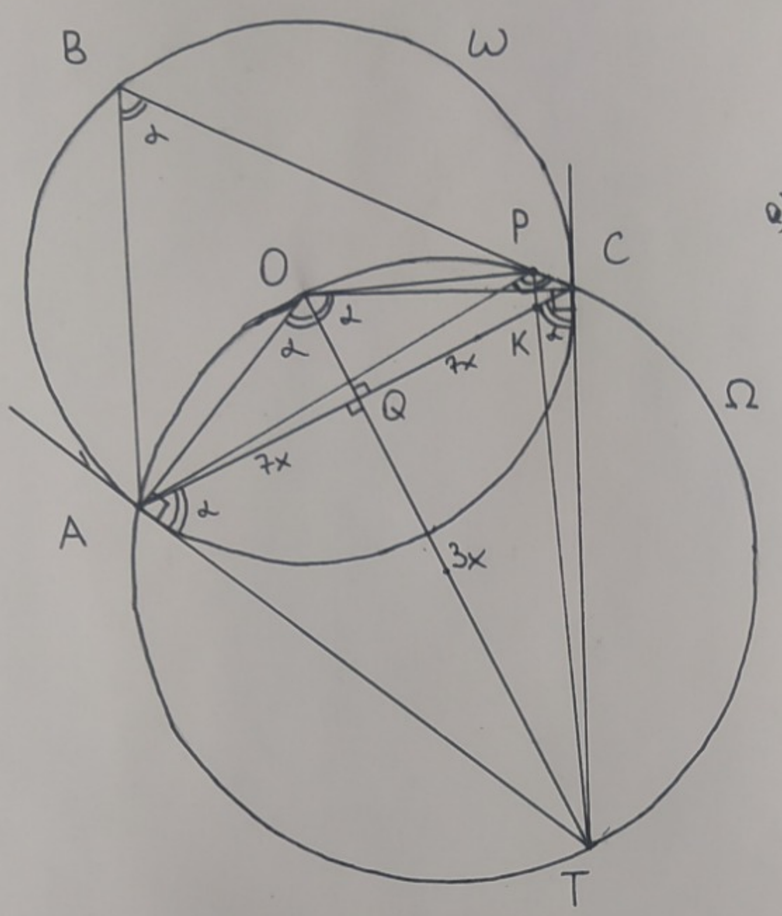
Вариант 21

Чистовик. Вариант 21. № 6



$S_{\triangle}APK = 12$

$S_{\triangle}CPK = 9$

а) $S_{\triangle}ABC$ — ?

$A, C \in \Omega$

$\angle TAO = \angle TCO = 90^\circ \Rightarrow$

$\Rightarrow OT$ — диаметр Ω

AT, CT — кас. к ω из T

$\Rightarrow AT = CT \Rightarrow \angle AOT = \angle COT$

$\angle AOT = \angle APT$ (опир. на $\smile AT$ Ω)

$\angle CPT = \angle CAT$ (опир на $\smile CT$ Ω)

$(\omega, \smile AC) \Rightarrow \angle ABC = \frac{1}{2} \cdot \angle AOC$. $\angle CAT = \angle ABC$; $\angle ACT = \angle ABC$.

$\angle ABC = \angle CPK \Rightarrow AB \parallel PK$ (соотв. при сек. BC) $\Rightarrow$

$\Rightarrow \triangle ABC \sim \triangle KPC$ по 2 углам ($\angle ABC = \angle KPC$; $\angle ACB$ — общ.)

$\Rightarrow S_{\triangle}ABC : S_{\triangle}KPC = (AC : KC)^2$. Пусть $\rho(P; AC) = h \Rightarrow$

$S_{\triangle}APK = \frac{1}{2} \cdot AK \cdot h$; $S_{\triangle}CPK = \frac{1}{2} \cdot CK \cdot h \Rightarrow \frac{AK}{CK} = \frac{S_{\triangle}APK}{S_{\triangle}CPK} = \frac{12}{9} = \frac{4}{3} \Rightarrow$

$\Rightarrow \frac{CK}{AC} = \frac{3}{7} \Rightarrow S_{\triangle}ABC = \left(\frac{7}{3}\right)^2 \cdot S_{\triangle}CPK = \frac{49}{9} \cdot 9 = 49$.

б) $\angle ABC = \operatorname{arctg} \frac{3}{7}$. AC — ?

$tg(\alpha) = \frac{3}{7} = \frac{CT}{CO} = \frac{TQ}{AQ} \Rightarrow TQ = 3x$; $AQ = 7x = CQ \Rightarrow AC = 14x$. $AK : CK = \frac{4}{3} \Rightarrow$

$\Rightarrow AK = 8x$; $CK = 6x$. $\triangle AKT \sim \triangle PKC$ ($\angle CAT = \angle TPC$; $\angle AKT = \angle PKC$) $\Rightarrow$

$\Rightarrow h(\triangle PKC) : h(\triangle AKT) = CK : AK = \frac{3}{4} \Rightarrow h(\triangle PKC) = \frac{3}{4} \cdot 3x = \frac{9}{4}x \Rightarrow S_{\triangle}PKT = \frac{1}{2} \cdot 6x \cdot \frac{9}{4}x =$

$= \frac{27}{4}x^2 = 9 \Rightarrow x^2 = \frac{4}{3} \Rightarrow x = \sqrt{\frac{4}{3}} \Rightarrow AC = 14x = 14\sqrt{\frac{4}{3}}$. Ответ: а) 49; б) $\frac{14\sqrt{12}}{3}$.

Черновик

НОД $(a; b; c)$

$2x^2 - 3x + 5 \cancel{\ } \geq 0$

$D = 3^2 - 4 \cdot 2 \cdot 5 = 9 - 40 < 0$

$2x^2 - 3x + \overset{4}{\cancel{5}} = \cancel{1}$

$2x^2 - 3x + 4 = 0$

$D = 9 - 4 \cdot 2 \cdot 4 =$

$a = 35k$

$b = 35n$

$c = 35m$

$(k, n, m) = 1.$

$a = 5^{\alpha} \cdot 7^{\beta}$

$b = 5^{n} \cdot 7^{m}$

$c = 5^{x} \cdot 7^{y}$

$\max(\beta, m, y) = 16$

$1 \quad [1 \dots 18] \quad 18$

$p \in [2; 17]: \quad 1, \underset{}{\overset{16}{\tilde{p}}}, 18$

$1 \quad p \quad 18$

$3! \cdot 16$

$p = 1: \; 3$

$\frac{3!}{2!1!}$

$1, 1, 18$

$1, 18, 1$

$18, 1, 1$

$2x - 3 > 0$

$2x > 3$

$x > \frac{3}{2} > -1$

$x > -1$

$2x - 3 \neq 1$

$2x \neq 4$

$x \neq 2.$

$\log_a(b)$

$\log_c a^4 = 4 \log_c a$

$\log_b c$

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Черновик

$tg(\angle ABC) = \frac{3}{7}$.

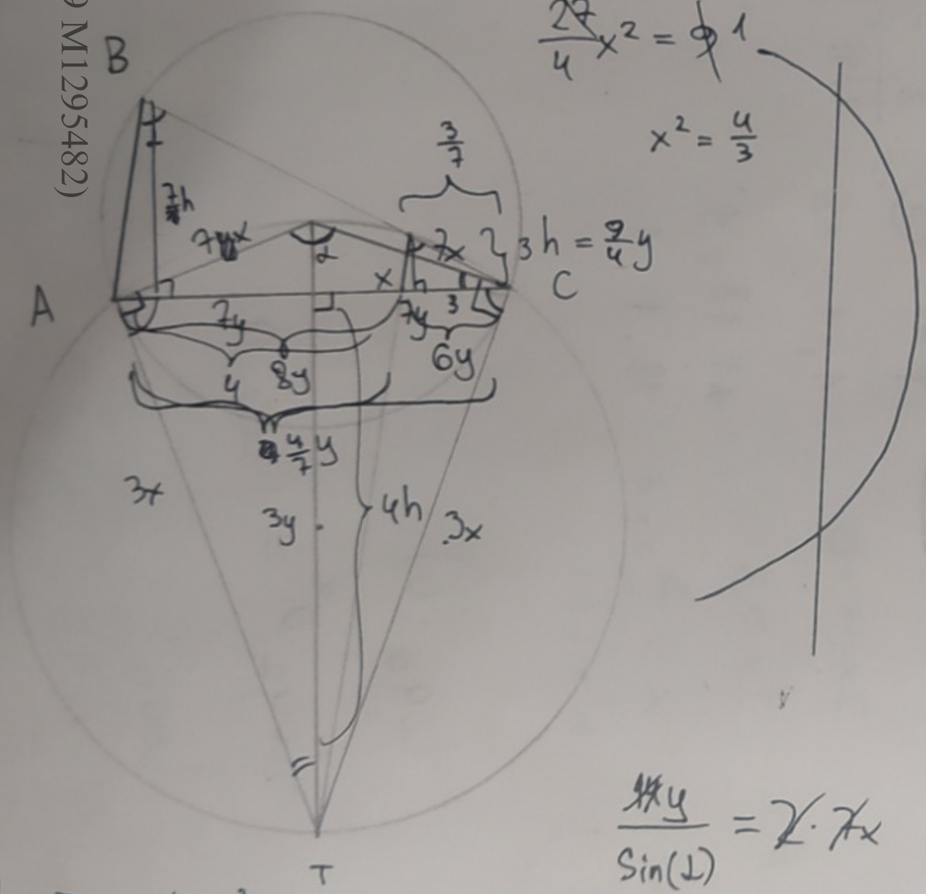
$tg(\alpha) = \frac{3}{7} = \frac{\sin(\alpha)}{\cos(\alpha)} \quad |\ ^2$

$\frac{\sin^2(\alpha)}{\cos^2(\alpha)} = \frac{9}{49} \Rightarrow \frac{1-\cos^2(\alpha)}{\cos^2(\alpha)} = \frac{9}{49}$

$\frac{1}{\cos^2(\alpha)} = 1 + \frac{9}{49} = \frac{58}{49}$

$\cos^2(\alpha) = \frac{49}{58}$

$\cos(\alpha) = \frac{7}{\sqrt{58}}$



$\frac{\cancel{27}^{3}}{4}x^2 = \cancel{9}\,1$

$x^2 = \frac{4}{3}$

$3h = \frac{9}{4}y$

$(7x)^2 + (3x)^2 = (3y)^2 \Rightarrow \boxed{58x^2 = 9y^2}$

$y = \frac{\sqrt{58}}{3}x$

$(7x)^2 - (7y)^2 =$

$= (7x-7y)(7x+7y) = 49(x^2-y^2)$

$\cancel{49}y^2 = (3y - 49(x^2-y^2)) \cdot \cancel{49}(x^2-y^2)$

$y^2 - (3y - 49x^2 + 49y^2)(x^2-y^2)$

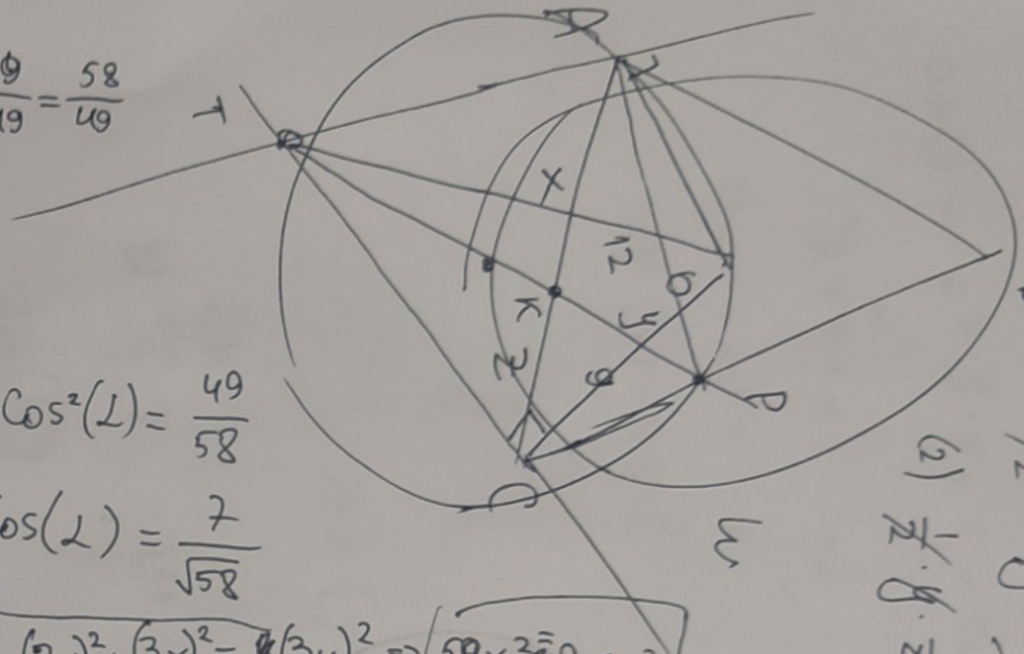
$\frac{\cancel{4}y}{\sin(\alpha)} = \cancel{2} \cdot \cancel{7}x$

$\frac{9}{58} = \sin^2(\alpha)$

$y^2 = x^2 \cdot \frac{9}{58}$

$\sin(\alpha) = \frac{7y}{7x} = \frac{y}{x} \Rightarrow y = x \cdot \sin(\alpha)$.

$4h = 3y \Rightarrow h = \frac{3}{4}y$



(1) $\cancel{\frac{1}{2}} \cdot x \cdot \cancel{y} \cdot \cancel{\sin(\pi-\alpha)} = 12$

(2) $\cancel{\frac{1}{2}} \cdot \cancel{y} \cdot z \cdot \cancel{\sin(\alpha)} = 9$

$14\sqrt{\frac{4 \cdot 3}{3 \cdot 3}} = \frac{14}{3}\sqrt{12}$

(1) : (2) $= \frac{x}{z} = \frac{4}{3}$

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$\frac{1}{2} \cdot 7x \cdot 7x \cdot \sin(2\alpha) = \frac{1}{2} \cdot 2 \cdot 7y \cdot \cancel{3x} \cdot \frac{7y}{tg(\alpha)}$

Черновик

$(a^{(b^2)}) (a^b)^b = c^b$

$\log_a c$

$tg(\alpha) = \frac{7y}{h} \Rightarrow h =$ $a^{b^2} = c$

$b = (\log_a c)^{1/2}$

$\frac{3x}{h} = \frac{4}{2} \Rightarrow h = \frac{9}{4}x$

$x^2 \cdot \sin(2\alpha) = 2 y^2 / tg(\alpha)$

$\frac{1}{2} \cdot \frac{9}{4}x \cdot 6x = 9$

$x^2 \cdot \cancel{2} \sin(\alpha) \cdot \cos(\alpha) = \cancel{2} \cdot y^2 \cdot \frac{\sin(\alpha)}{\cos(\alpha)}$

$\frac{3}{4} x^2 = 1$

$x^2 \cdot \cos^2(\alpha) = y^2$ $\quad p = \log_a b = \log_c a$ $\quad x^2 = \frac{4}{3}$

$(a)^p = b$

$x \cdot \cos(\alpha) = y$ $\quad c^p = a.$

$14x = 14\sqrt{\frac{4}{3}} = 14\sqrt{\frac{4 \cdot 3}{3^2}} = \frac{14}{3}\sqrt{12}.$

$(7x^2)^2 - (7y^2)^2 = 49(x^2 - y^2)$

$a^{\frac{b}{2}}$

$7\sqrt{x^2 - y^2} \cdot 3y$ $\quad (c^p)^p = b$ $\quad \log_a b \cdot \log_c a = \log_c b$

$c^{(p^2)} = b$ $\quad \sqrt{a}^b = c$ $\quad p^2 = \log_c b$

$S_{\triangle ATX} = 9 \cdot \left(\frac{4}{3}\right)^2 = 16.$

$b = \log_{\sqrt{a}} c = 2 \cdot \log_a c$

$\log_a b = 4 \cdot \log_c a$

$\frac{b}{2} = \log_a c$

$b = 2 \cdot \log_a c$

log

$\cancel{\log_a b = 2}$

$a^b = c^2$ $\quad a^{\frac{b}{2}} = c$

$b = \log_a c^2$

$b = 2 \cdot \log_a c$ $\quad \frac{b}{2} = \log_a c$

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Чистовик. Вариант 21. <u>№4</u>

$(a;b;c)=35=5^1\cdot7^1;\qquad a,b,c\in\mathbb{N}$

$[a;b;c]=5^{18}\cdot7^{16};$

$$\left.\begin{matrix}5^{18}\cdot7^{16}\ \vdots\ a,\ b,\ c\\ a,b,c\ \vdots\ (5^1\cdot7^1)\end{matrix}\right\}\Rightarrow\begin{matrix}a=5^{\alpha}\cdot7^{\beta};\\ b=5^{n}\cdot7^{m};\\ c=5^{x}\cdot7^{y};\end{matrix}\qquad \alpha,\beta,n,m,x,y\in\mathbb{N}$$

$$\begin{cases}\min(\alpha;n;x)=1\\ \min(\beta;m;y)=1\\ \max(\alpha;n;x)=18\\ \max(\beta;m;y)=16\end{cases}$$

$$\underset{(18-2)}{\overset{16}{\shortparallel}}\cdot\frac{3!}{\cancel{1!1!1!}}+\cancel{1}\cdot\frac{3!}{\cancel{2!1!}}+\cancel{1}\cdot\frac{3!}{\cancel{2!1!}}+(16-2)\cdot\frac{3!}{\cancel{1!1!1!}}+\cancel{1}\cdot\frac{3!}{\cancel{2!1!}}+\cancel{1}\cdot\frac{3!}{\cancel{2!1!}}=$$

$$=16\cdot3!+3+3+14\cdot3!+3+3=30\cdot3!+4\cdot3=30\cdot6+12=192$$

Ответ: 192.

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Чистовик. Вариант 21. **№5**

$$\log_{(\sqrt{2x-3})}(x+1);\quad \log_{(2x^2-3x+5)}(2x-3)^2;\quad \log_{(x+1)}(2x^2-3x+5)$$

О.О.У.:

$$\left.\begin{cases} 2x-3 \geq 0; & 2x^2-3x+5>0; & x+1>0; \\ \sqrt{2x-3}>0; & 2x^2-3x+5 \neq 1; & x+1 \neq 1; \\ \sqrt{2x-3} \neq 1; & 2x-3>0; & 2x^2-3x+5>0. \\ x+1>0; & & \end{cases}\right\} \Rightarrow$$

$$\Rightarrow \begin{cases} 2x-3>0 \Rightarrow x>\frac{3}{2} \\ 2x-3 \neq 1 \Rightarrow x \neq 2 \\ x+1>0 \Rightarrow x>-1 \\ x+1 \neq 1 \Rightarrow x \neq 0 \\ 2x^2-3x+5>0 \text{ — } x \in \mathbb{R} \\ 2x^2-3x+5 \neq 1 \text{ — } x \in \mathbb{R} \end{cases} \Rightarrow x \in (1{,}5;2) \cup (2;+\infty).$$

$a = \sqrt{2x-3}$ 1) $\log_{\sqrt{a}} b = 2 \cdot \log_a b$

$b = x+1$ 2) $\log_c a^2 = 2 \cdot \log_c a$

$c = 2x^2-3x+5$ 3) $\log_b c$

$$①=②: \begin{cases} 2\cdot\log_a b = 2\cdot\log_c a \Rightarrow \overset{t+1}{\log_a b} = \overset{t+1}{\log_c a} \Rightarrow \dfrac{\log_c b}{\log_c a} \overset{=(t+1)^2}{} = \log_a b \\ 2\cdot\log_a b = \log_b c + 1 = \log_b c + \log_b b = \log_b bc \\ \frac{1}{2}\log_c a = \underset{t}{\log_b c} + 1 = \log_b bc \end{cases}$$

$$\begin{cases} \log_c b = (t+1)^2 \\ \log_b c = t \end{cases} \qquad \log_c b = (\log_b(bc))^2$$