Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21102927**

ID профиля: **169305**

Вариант 21

Чистовик. Вариант 21. Математика 11 кл.

№1. $S = a_1 + .. + a_7 = 7 \cdot a_1 + 21d$

$$\begin{cases} a_8 \cdot a_{17} = (a_1+7d)(a_1+16d) = a_1^2 + 23 a_1 \cdot d + 112 d^2 > 7a_1 + 21d + 27 \\ a_{11} \cdot a_{14} = (a_1 + 10d)(a_1 + 13d) = a_1^2 + 23a_1 d + 130 d^2 < 7a_1 + 21d + 60 \end{cases}$$

$$\begin{cases} a_1^2 + 23a_1 d + 112 d^2 > 7a_1 + 21d + 27 \iff -a_1^2 - 23a_1 d - 112 d^2 < -7a_1 - 21d - 27 \\ a_1^2 + 23 a_1 d + 130 d^2 < 7a_1 + 21d + 60 \end{cases}$$

$(1)+(2)$: $18d^2 < 33$

$d^2 < \frac{11}{6}$, $\Rightarrow$ $d \in \left(-\sqrt{\frac{11}{6}}; \sqrt{\frac{11}{6}}\right)$

Т.к. $a_1, a_2, \dots,$ — целые числа, и $a_1 < a_2 < a_3 < \dots$, $\Rightarrow$ $d = 1$.

Значит, $S = 7a_1 + 21$.

$$\begin{cases} a_1^2 + 23a_1 + 112 > 7a_1 + 21 + 27 \\ a_1^2 + 23a_1 + 130 < 7a_1 + 21 + 60 \end{cases} \quad \begin{cases} a_1^2 + 16a_1 + 64 > 0 \\ a_1^2 + 16a_1 + 49 < 0 \end{cases} \quad (a_1+8)^2 > 0, \Rightarrow a_1 \neq -8$$

$D = 256 - 196 = 60$

$a_1 = \frac{-16 \pm 2\sqrt{15}}{2} = -8 \pm \sqrt{15}$, $\Rightarrow$ $a_1 \in (-8-\sqrt{15}; -8+\sqrt{15})$

Т.к. a_1 — целое, $\Rightarrow$ $a_1 \in [-11; -5]$.

Значит, $a_1 \in [-11; -8) \cup (-8; -5]$, или $a_1 = \{-11; -10; -9; -7; -6; -5\}$

Ответ: $a_1 = \{-11; -10; -9; -7; -6; -5\}$

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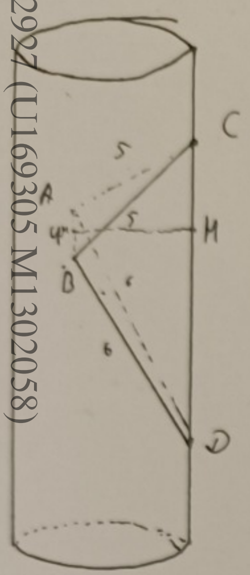
Чистовик. Вариант 21 Математика 11 кл.

№2.

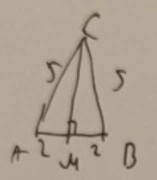


Так как $CD \parallel$ оси цилиндра, $AC = CB$, $AD = BD$, $\Rightarrow AB \perp CD$.

Тогда $AB \parallel$ осн. цилиндра, откуда $2r \geq AB$.

Значит, минимальное значение $r = 2$. В этом случае AB - диаметр.

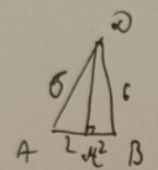
Пусть M - середина AB. Опустим из т. M высоту на CD в точке H.



$CM = \sqrt{21}$

$HM = 2$, $MC = \sqrt{21}$, $\Rightarrow CH = \sqrt{21-4} = \sqrt{17}$

$MD = 4\sqrt{2}$, $\Rightarrow DH = \sqrt{32-4} = 2\sqrt{7}$.



$DM = 4\sqrt{2}$.

Значит, $CD = 2\sqrt{7} + \sqrt{17}$

Ответ: $CD = 2\sqrt{7} + \sqrt{17}$.

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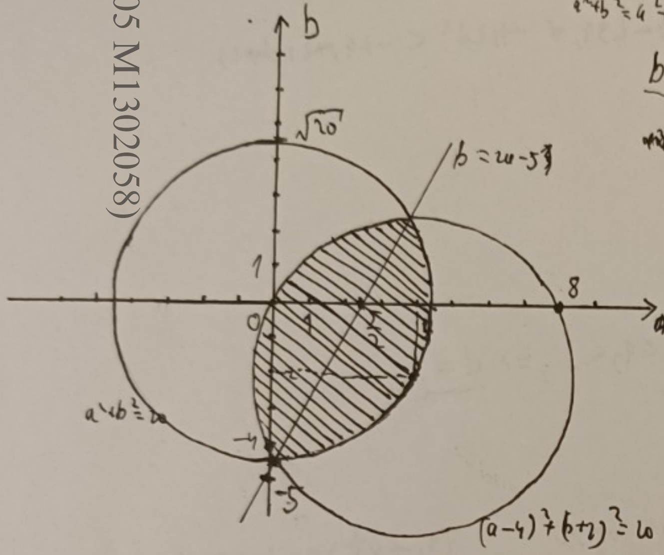
Чистовик. Вариант 21. Математика 11 кл.

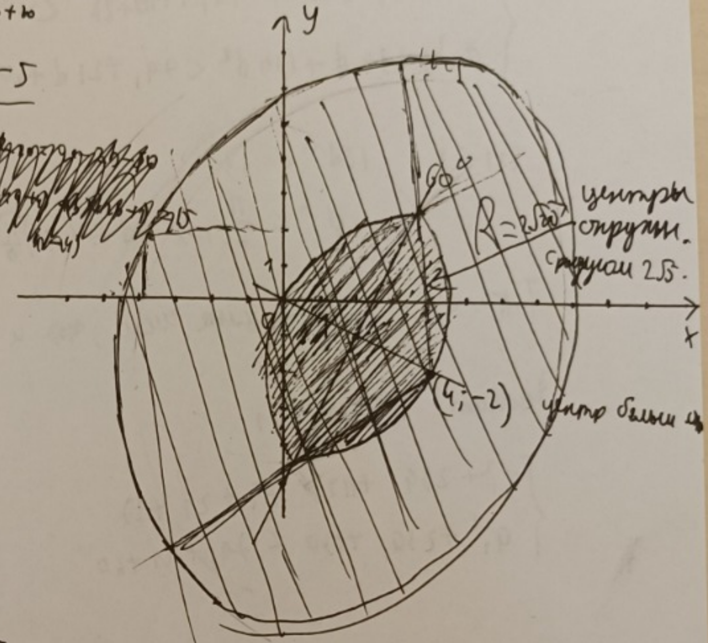
№3.

$$\begin{cases}(x-a)^2+(y-b)^2\le 20\\ a^2+b^2\le \min(8a-4b;20)\end{cases} \iff \begin{cases}(x-a)^2+(y-b)^2\le 20\\ a^2+b^2\le 20\\ a^2-8a+16+b^2+4b+4\le 20\end{cases} \quad \begin{cases}(x-a)^2+(y-b)^2\le 20\\ (a-4)^2+(b+2)^2\le 20\\ a^2+b^2\le 20\end{cases}$$

$a^2+b^2=a^2+b^2-8a+4b+20$

$b=2a-5$





$$\begin{cases}b=\sqrt{20-a^2}\\ b=2a-5\end{cases}$$

$\Rightarrow \sqrt{20-a^2}=2a-5 \quad / a\in[-\sqrt{20};\sqrt{20}]$

$20-a^2=4a^2-20a+25$

$5a^2-20a+5=0$

$a^2-4a+1=0$

$D=16-4=12$

$a=\frac{4\pm2\sqrt3}{2}=2\pm\sqrt3 \Rightarrow$ $(2-\sqrt3;\ -2\sqrt3-1)$, $(2+\sqrt3;\ 2\sqrt3-1)$ — координаты точек перес

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$l=\sqrt{(2\sqrt3-1+2\sqrt3+1)^2+(2+\sqrt3-2+\sqrt3)^2}=\sqrt{(4\sqrt3)^2+(2\sqrt3)^2}=$

$=\sqrt{16\cdot3+4\cdot3}=2\sqrt{15}$

$R=2\sqrt{20}=\sqrt{80}$

$60=20+20-2\cdot20\cdot20\cos\alpha$

$20=-2\cdot20\cdot20\cos\alpha$

$-\frac12=\cos\alpha \Rightarrow \boxed{\alpha=120°}$, $\Rightarrow$ ~~$S=\frac{\pi r^2}{3}-\frac12 r^2\sin\alpha=\frac{20\pi}{3}-5\sqrt3$~~

$S=x+2\cdot\left(\frac{\pi R^2}{3}-\frac12R^2\sin\alpha\right)=\frac{2\pi R^2}{3}-R^2\sin\alpha+x=\frac{160\pi}{3}-40\sqrt3+x=60\pi-40\sqrt3.$

$x=2\cdot\frac{\pi r^2}{6}=\frac{\pi r^2}{3}=\frac{20\pi}{3}$

Ответ: $S=60\pi-40\sqrt3$

$S = a_1 + a_2 + \dots + a_7 = 7 \cdot a_2 + 21d$

$a_8 \cdot a_{12} = (a_1 + 7d)(a_1 + 16d) > 7a_1 + 21d + 27$

$a_{11} \cdot a_{14} = (a_1 + 11d)(a_1 + 13d) < 7a_1 + 21d + 60$

$a_1^2 + a_1(23d - 7) + 130d^2 - 21d - 60 < 0$

$D = 529d^2 - 322d + 49 - 520d^2 + 84d + 240 = 9d^2 - 238d + 289 = (3d-17)^2 - 136d$

$\begin{cases} a_1^2 + 23a_1 d + 112d^2 > 7a_1 + 21d + 27 \\ a_1^2 + 23a_1 d + 130d^2 < 7a_1 + 21d + 60 \end{cases} \Leftrightarrow \begin{cases} -a_1^2 + 23a_1 d - 112d^2 < -7a_1 - 21d - 27 \\ a_1^2 + 23a_1 d + 130d^2 < 7a_1 + 21d + 60 \end{cases}$

$(1)+(2): 18d^2 < 33$

$6d^2 < 11$

$d^2 < \frac{11}{6}$

$d \in \left[0; \sqrt{\frac{11}{6}}\right)$

$d = 0: S = 7a_1$

$a_1^2 + a_1(23d - 7) + 112d^2 - 21d - 27 > 0$

$a_8 \cdot a_{12} = a_1^2 > 7a_1 + 27$

$a_1^2 < 7a_1 + 60$

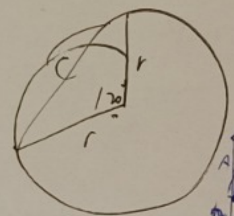
$D = 529d^2 - 322d + 49 - 448d^2 + 84d + 108 =$

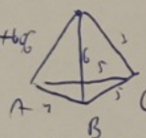
$= 81d^2 - 238d + 157 =$

$= (9d - 13)^2 - 4d - 12$

$AB = 4$

$AC = CB = 5, \quad AD = DB = 6$





Черновик

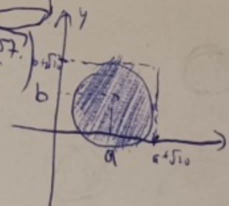
$\begin{cases} a_1^2 - 7a_1 - 27 > 0 \\ a_1^2 - 7a_1 - 60 < 0 \end{cases}$ $D = 49 + 108 = 157$ $a_1 \in \left(\frac{7 - \sqrt{157}}{2}; \frac{7 + \sqrt{157}}{2}\right)$

$D = 49 + 240 = 289$

$a_1 = \frac{7 \pm 17}{2}; \quad a_1 \in (-5; 12)$

$a_1 \in \left(\frac{7 - \sqrt{157}}{2}; \frac{7 + \sqrt{157}}{2}\right)$

$\boxed{a_1 \in [-2; 9]}$ $d = 0$



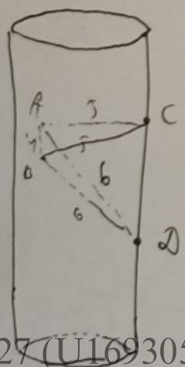
$-d^2 + 4b + 20 = 0$

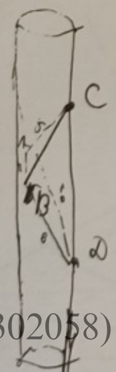
$b = 2 \pm \sqrt{20 - a^2}$

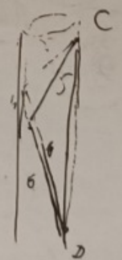
$\begin{cases} (\dots)^2 + (y - b)^2 \le 20 \\ a^2 + b^2 \le 84 - 4b \\ a^2 + b^2 \le 20 \end{cases}$ $\begin{cases} (x - a)^2 + (y - b)^2 \le 20 \\ (a - 4)^2 + (b + 2)^2 \le 20 \\ a^2 + b^2 \le 20 \end{cases}$

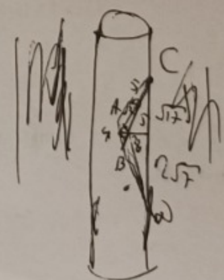
$d = 1:$

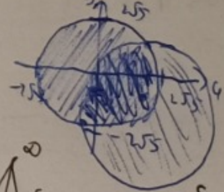
min $r = 2$.

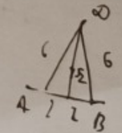


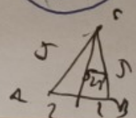












21102927 (U169305 M1302058)

Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21102927**

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Вариант 21

Чистовик. Вариант 21. Математика 11 кл.

№4.

$$\begin{cases} НОД(a;b;c) = 35 \\ НОК(a;b;c) = 5^{18}\cdot 7^{16} \end{cases}$$

$\Rightarrow a = 35\cdot a_1,\ b = 35\cdot b_1,\ c = 35\cdot c_1.$

$НОК(a;b;c) = 35\cdot a_1\cdot b_1\cdot c_1 = 5^{18}\cdot 7^{16} \Rightarrow a_1\cdot b_1\cdot c_1 = 5^{17}\cdot 7^{15}$

Кол-во 5:

a_1	b_1	c_1	
17	0	0	1
16	0	1	2
16	1	0	
15	0	2	3
15	1	1	
15	2	0	
...	...	...	
0	0	17	18
0	1	16	
0	2	15	
...	...		
0	16	1	
0	17	0	

$1+2+\ldots+18 = \frac{18\cdot 19}{2} = 171.$

Кол-во 7:

a_1	b_1	c_1	
15	0	0	1
14	0	1	2
14	1	0	
...	...	...	
0	0	14	15
0	1	13	
...	...		
0	14	0	

$1+2+\ldots+15 = \frac{15\cdot 16}{2} = 120$

Значит, всего вариантов троек:

$171\cdot 120 = 20\,520$

Ответ: 20520.

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21102927 (U169305 M1302059)

Чистовик. Вариант 21 Математика 11кл

№5.

$\log_{\sqrt{2x-3}}(x+1)$, $\log_{2x^2-3x+5}(2x-3)^2$, $\log_{(x+1)}(2x^2-3x+5)$, $x>\frac{3}{2}$, $x\neq 2$.

$\parallel$

$\log_{2x-3}(x+1)^2$

1) $\log_{2x-3}(x+1)^2 = \log_{2x^2-3x+5}(2x-3)^2$, $\log_{(2x-3)}(x+1)^2 = \log_{(x+1)}(2x^2-3x+5)+1$

$\log_{(2x-3)}(x+1) = \log_{(2x^2-3x+5)}(2x-3)$: $x+1=2x-3$, $\Rightarrow x=4$.

$2x^2-3x+5=2x-3$, $2x^2-5x+8=0$

$D=25-4\cdot 2\cdot 8<0$.

$x=4$: $\log_{2x-3}(x+1)^2=2$

$\log_{2x^2-3x+5}(2x-3)^2=1$

$\log_{x+1}(2x^2-3x+5)=2$

1) $\Rightarrow$ при $\begin{cases}\log_{2x-3}(x+1)^2=\log_{(x+1)}(2x^2-3x+5)\\ \log_{2x^2-3x+5}(2x-3)^2+1=\log_{(x+1)}(2x^2-3x+5)\end{cases}$ $x=4$.

~~$\log_{(2x-3)}(x+1)=\log_{(2x^2-3x+5)}(2x-3)$~~

~~$\log^2_{(2x-3)}(x+1)=\log_{(2x^2-3x+5)}(2x-3)\cdot\log_{(2x-3)}(x+1)=\log_{(2x^2-3x+5)}(2x-3)^{\log_{(2x-3)}(x+1)}=\log_{(2x^2-3x+5)}(x+1)$~~

~~$\log_{(x+1)}(2x^2-3x+5)=\log_{(2x-3)}\frac{(x+1)^2}{2x-3}$~~

~~$\log_{(2x^2-3x+5)}(x+1)=\log_{\frac{(x+1)^2}{2x-3}}2x-3=$~~

②

Ответ: $x=4$.

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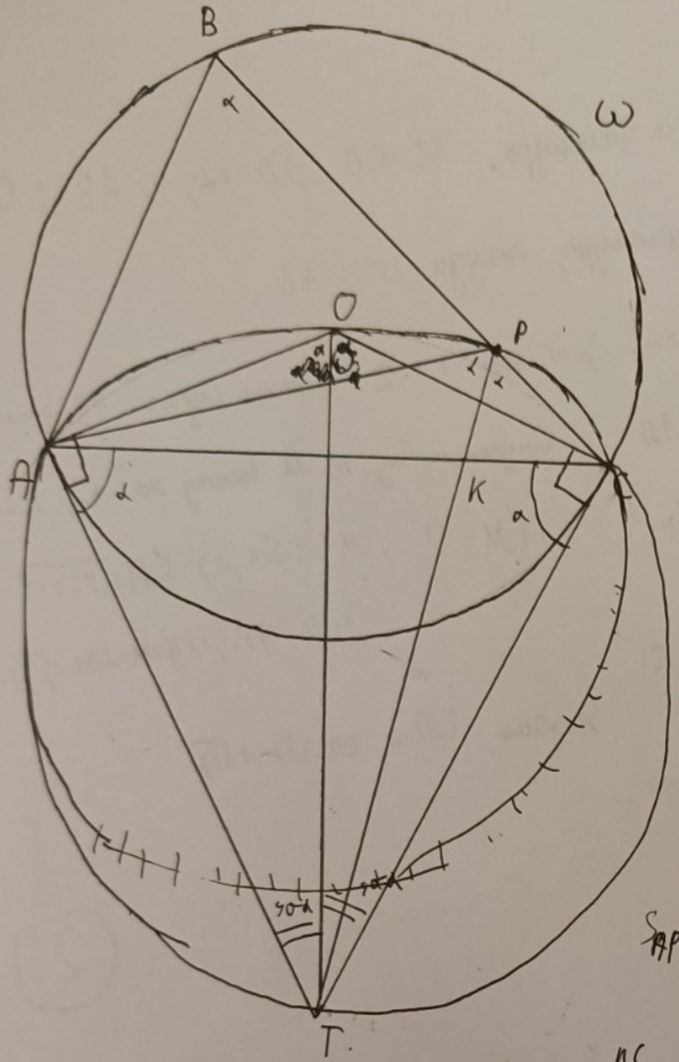
Чистовик. Вариант 21 Математика 11кл

№8.



а) Пусть $\angle ABC = \alpha$. Тогда $\angle AOC = 2\alpha$.

$\angle ATC = 360 - (90 + 90 + 2\alpha) = 180 - 2\alpha$, =)

=) точка $T \in$ окружн. AOC.

$\angle APC = 2\alpha$ (т.к. опирается на дугу AC)

$AO = OC$, $\angle OAT = \angle OCT = 90°$, OT - общ., =)

=) $\triangle OAT = \triangle OCT$, =) $AT = TC$, =) $\angle AOT = \angle COT = \alpha$.

$\angle TPC = \angle TOC = \alpha$, =) PT - биссектриса $\angle APC$.

$S_{APK} = 12$, $S_{CPK} = 9$ (усл.)

$S_{APK} = \frac{1}{2} AP \cdot PK \cdot \sin\alpha$, $S_{CPK} = \frac{1}{2} PC \cdot PK \cdot \sin\alpha$, =) $\frac{AP}{PC} = \frac{4}{3}$

$\frac{AP}{PC} = \frac{AK}{KC} = \frac{4}{3}$, $AP = \frac{4}{3} PC$

$S_{APC} = 21$. Т.к. $\angle KPC = \angle ABC$, =) $\triangle ABC \sim \triangle KPC$.

$\angle BCA = \angle PCK$, =)

$\frac{KC}{AC} = \frac{3}{7}$, =) $\frac{S_{KPC}}{S_{ABC}} = \frac{9}{49}$, =) $\underline{S_{ABC} = 49}$.

б) $\angle ABC = \operatorname{arctg}\frac{3}{7}$, =) $\alpha = \operatorname{arctg}\frac{3}{7} = \arcsin\frac{3}{\sqrt{58}} = \arccos\frac{7}{\sqrt{58}}$

$S_{APC} = \frac{1}{2} AP \cdot PC \cdot \sin 2\alpha = \frac{1}{2} PC^2 \cdot \frac{4}{3} \cdot 2 \cdot \frac{3}{\sqrt{58}} \cdot \frac{7}{\sqrt{58}} = PC^2 \cdot \frac{28}{58} = PC^2 \cdot \frac{14}{29} = 21$, =) $PC^2 = \frac{29 \cdot 3}{2} = \frac{87}{2}$

$PC = \sqrt{\frac{87}{2}}$, $AP = \sqrt{\frac{8 \cdot 87}{2 \cdot 3}} = \sqrt{\frac{232}{3}}$

$9 = \frac{1}{2} \cdot \sqrt{\frac{87}{2}} \cdot PK \cdot \frac{3}{\sqrt{58}} = \frac{PK \cdot 3\sqrt{3}}{4}$, =) $PK = 4\sqrt{3}$, =) $AB = \frac{28}{\sqrt{3}}$

~~для~~

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Ответ: а) $S_{ABC} = 49$.

21102927 (U169305 M1302059)

Черновик

$$\begin{cases} НОД(a;b;c) = 5 \cdot 7 \\ НОК(a;b;c) = 5^{18} \cdot 7^{16} \end{cases}$$

$a = 35 \cdot a_1$, $b = 35 \cdot b_1$, $c = 35 \cdot c_1$, $НОД = 35$

$НОК = a_1 b_1 c_1 \cdot 35 = 5^{17} \cdot 7^{15} \cdot 35$

$a_1 b_1 c_1 = 5^{17} \cdot 7^{15}$

$5^{a_1 \ldots} \cdot 7^{a_1 \ldots}$

$2\log_{(2x-3)}(x+1) = 2\log_{(2x^2-3x+5)}(2x-3)$

$2\log_{(2x-3)}(x+1) = \log_{(x+1)}\left((x+1)(2x^2-3x+5)\right)$

17	0	0
16	1	0
16	0	1
15	2	0
15	0	2
15	1	1
14	3	0
14	0	3
14	2	1
14	1	2
13	4	0
13	0	4
13	1	3
13	3	1
13	2	2

$1+2+3+\ldots+18 = 9 \cdot 19 = 171$

0	17	0
0	16	1
0	15	2
0	14	3
...		
0	1	16
0	0	17

сл. 1: $1) = 2)$, $1) + 1 = 3)$

сл. 2: $2) = 3)$, $2) + 1 = 1)$

сл. 3: $3) = 1)$, $3) + 1 = 2)$ $\boxed{x = 4}$

$\boxed{x=4:}$ $2 \cdot \log_5 5 = 2$, $2 \cdot \log_{25} 5 = 1$, $\log_{(x+1)}(2x^2-3x+5)$

$\log_5 25 = 2$

$x = 4: \log_5 5 = \log_{32-12+5} 5$

$\log_5 25 = 2$

$x > \frac{3}{2}$

$2x^2 - 3x + 5 = 4x^2 - 12x + 9$

$2x^2 - \ldots + 5 = 0$

$2x^2 - 9x + 4 = 0$

$D = 81 - 32 = 49$

$x = \frac{9 \pm 7}{4} = 4$

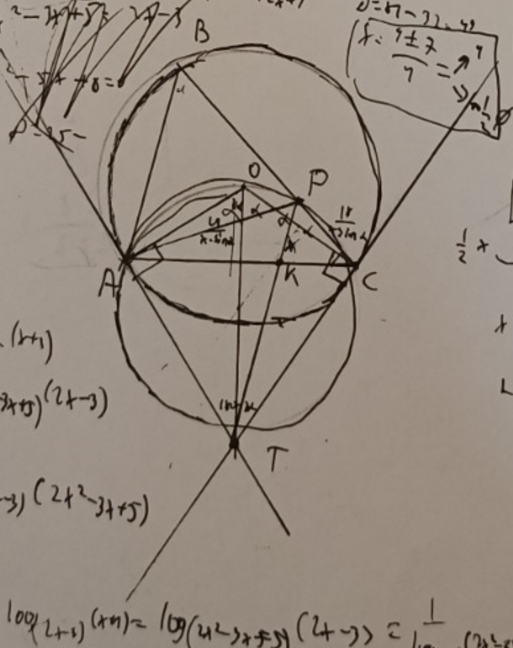
$D = 25$

1) $2\log_{(2x-3)}(x+1)$

2) $2\log_{(2x^2-3x+5)}(2x-3)$

3) $\log_{(x+1)}(2x^2-3x+5)$

$$\begin{array}{r} 171 \\ \times\ 120 \\ \hline 342 \\ +\ 171 \\ \hline 20520 \end{array}$$



$\frac{1}{2} x \ldots \sin\alpha = 9$

$x \cdot \ldots = \frac{18}{\sin\alpha}$

$\ldots = \frac{18}{x \cdot \sin\alpha}$

$\log_5 25 \cdot \log_5 5^5 = 1$

$\log_5 125 \cdot \log_{25} \sqrt[3]{5} = 1$

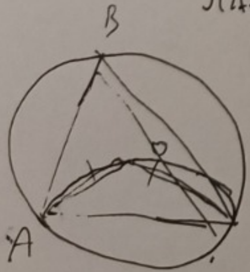
$\frac{AP}{PC} = \frac{4}{7} = \frac{AK}{KC}$

$\log^2_{(2x-3)}(x+1) = \log_{2x^2-3x+5}(x+1)$

$\log_{2x^2-3x+5}(2x-3) \cdot \log_{(2x-3)}(x+1) = \log_{(2x^2-3x+5)}(2x-3)$

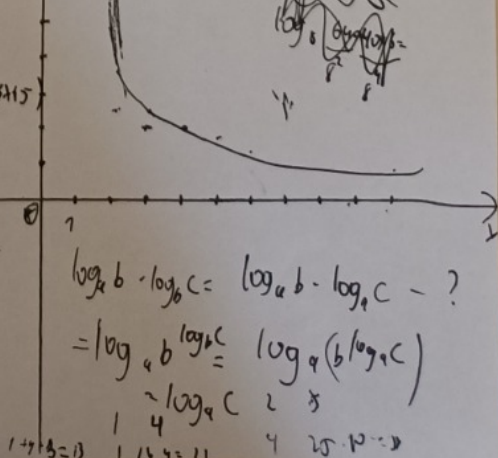
$\log_{(x+1)}(2x-3)$

$\log_{(2x-3)}(x+1) = \frac{1}{\log_{(2x-3)}(2x^2-3x+5)}$



сл. 1: $\log_{(2x-3)}(x+1) = \log_{(2x^2-3x+5)}(2x-3) = \frac{1}{\log_{(x+1)}(2x^2-3x+5)}$

$\log_{(2x-3)}(x+1) \cdot \log_{(x+1)}(2x^2-3x+5) = 1$



$4x^2 - 12x + 9 = 2x^2 - 3x + 5$

$2x^2 - 9x + 4 = 0$

$2x^2 - 3x + 5 = x + 1$

$2x^2 - 4x + 1 = 0$

$x^2 + 2x + 1 = 2x - 3$

$x^2 + 4 = 0$ $\varnothing$

$x = 4$

$\boxed{\Pi}$

$(x+1) = \sqrt[n]{y}$

$1 = (x+1)^{n^2} = 2x^2 - 3x + 5$

$2x^2 - 3x + 5 = y^n$

$\log_a b \cdot \log_b c = \log_a b \cdot \log_a c$ – ?

$= \log_a b^{\log_a c} = \log_a\left(b^{\log_a c}\right)$

$= \log_a c$

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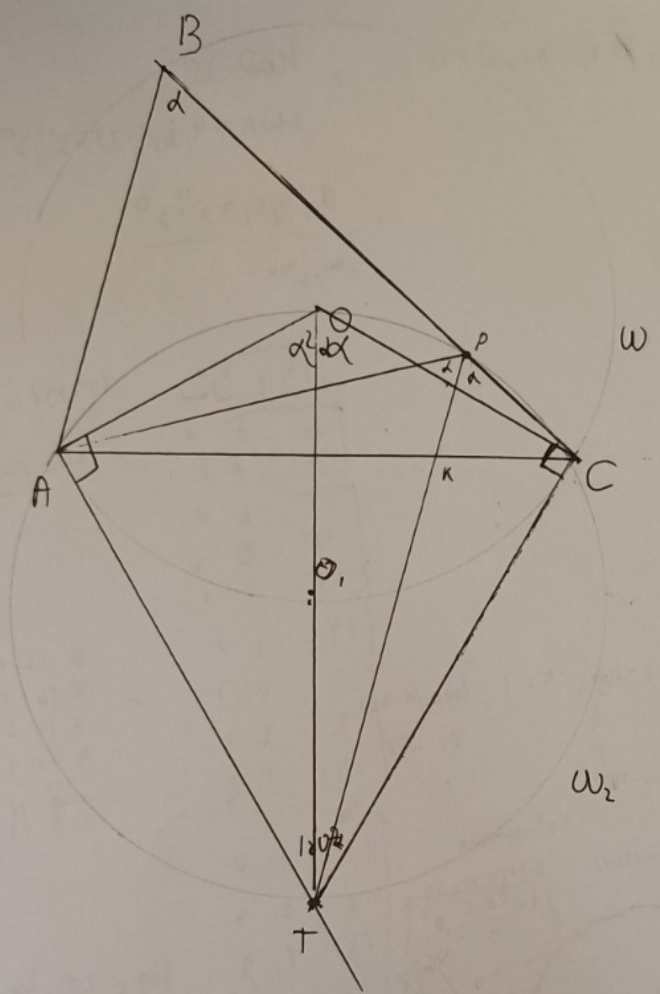
Черновик.



$$2R_{\omega} = \frac{AC}{\sin\alpha}$$

$$2R_{\omega_1} = \frac{AC}{\sin 2\alpha}$$

$$\frac{R_{\omega}}{R_{\omega_1}} = \frac{\frac{AC}{\sin\alpha}}{\frac{AC}{\sin 2\alpha}} = \frac{\sin 2\alpha}{\sin\alpha} = 2\cos\alpha$$

$$\begin{array}{r} 29 \\ \times\ 8 \\ \hline 232 \end{array}$$

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