Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21102901**

ID профиля: **152495**

Вариант 21

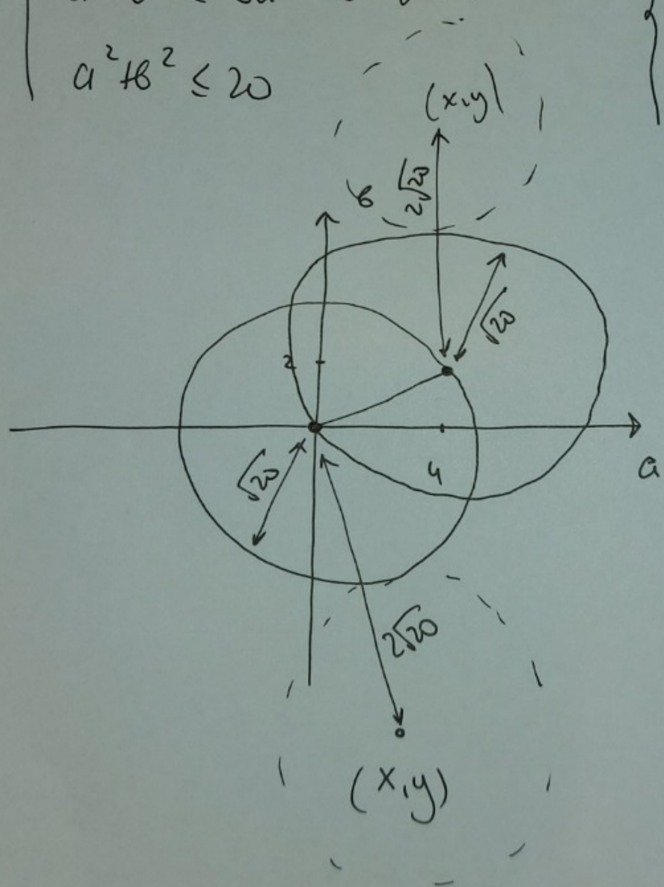
Чистовик

№3.

$$\begin{cases} (x-a)^2+(y-b)^2 \le 20 \\ a^2+b^2 \le \min(8a-4b, 20) \end{cases}$$

$$\Updownarrow$$

$$\begin{cases} (x-a)^2+(y-b)^2 \le 20 \\ a^2+b^2 \le 8a-4b \\ a^2+b^2 \le 20 \end{cases} \Leftrightarrow \begin{cases} (a-x)^2+(b-y)^2 \le 20 \\ (a-4)^2+(b-2)^2 \le 20 \\ a^2+b^2 \le 20 \end{cases}$$



• нужны такие x, y, чтобы система имела хотя бы одно решение

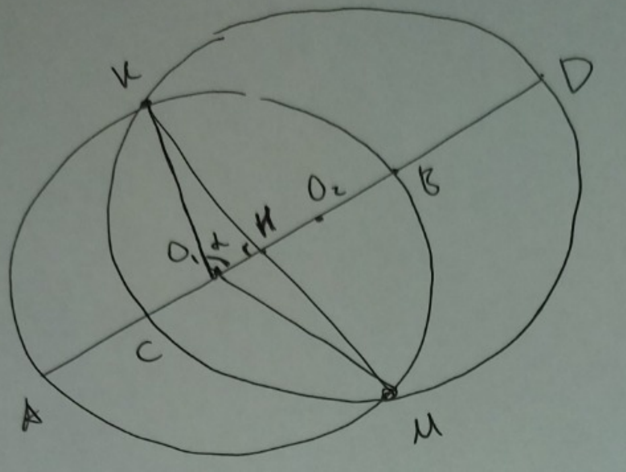
• Тогда такие x, y это 2 круга

① центр в $(0;0)$ $R = 2\sqrt{20}$

② центр в $(4;2)$ $R = 2\sqrt{20}$

Чистовик

2



① $O_1H = O_2H = \frac{O_1O_2}{2} = \sqrt{5}$

② $KH = \sqrt{16\cdot 5 - 5} = 5\sqrt{3}$

③ $\sin\angle KO_1H = \sin\angle\alpha = \frac{5\sqrt{3}}{4\sqrt{5}} = \frac{\sqrt{15}}{4}$

④ $S_1 = \pi R^2 = \pi\cdot(2\sqrt{20})^2 = 80\pi \ (= S_0)$

⑤ $S_2 = S_{\triangle KO_1M} = 2\cdot\left(\frac{1}{2}KH\cdot O_1H\right) =$

$= 5\sqrt{15}$

⑥ $S_3 = S_{\text{сект}} = \frac{R^2\cdot 2\alpha}{2} = 80\arcsin\left(\frac{\sqrt{15}}{4}\right)$

⑦ $S = 2(S_1 - (S_3 - S_2)) = 160\pi + 10\sqrt{15} - 160\arcsin\left(\frac{\sqrt{15}}{4}\right)$

Ответ: $S = 160\pi + 10\sqrt{15} - 160\arcsin\left(\frac{\sqrt{15}}{4}\right)$

Чистовик

№1.

$(a_1 = a,\ \text{разность} = в)$

$S = S_7 = \dfrac{a+a+6в}{2} \cdot 7 = 7(a+3в)$

$a, в \in \mathbb{Z}$, по условию

3

$a_8 \cdot a_{17} = (a+7в)(a+16в) > S+27$

$a_{11} \cdot a_{14} = (a+10в)(a+13в) < S+60$

$\begin{cases} a^2+2в+112в^2 > 7a+27 \quad (1) \\ a^2+2в+130в^2 < 7a+60 \quad (2) \end{cases}$

$\begin{cases} a^2+2в+112в^2-7a > 27 \\ a^2+2в+112в^2-7a < 60-18в^2 \end{cases}$

$\Downarrow$

$27 < 60-18в^2$

$в^2 < \frac{33}{18} = \frac{11}{6} < 2$, {т.к. $в \in \mathbb{Z}$, $в > 0$ по условию}

$\boxed{в = 1}$

$a^2+114 > 7a+27$

$a^2-7a+87 > 0$

$D < 0$

верно всегда

$a^2+132 < 7a+60$

$a^2-7a+72 < 0$

$D < 0$

не верно никогда

$\varnothing$

Ответ: ~~подходящих~~ a нет.

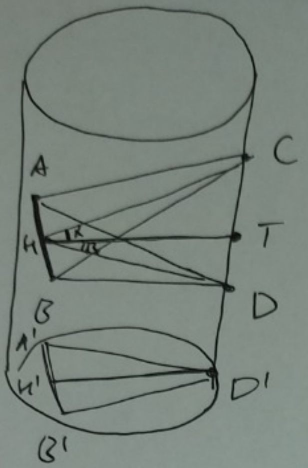
подходящих

Чистовик

№2.

2 картинки

I



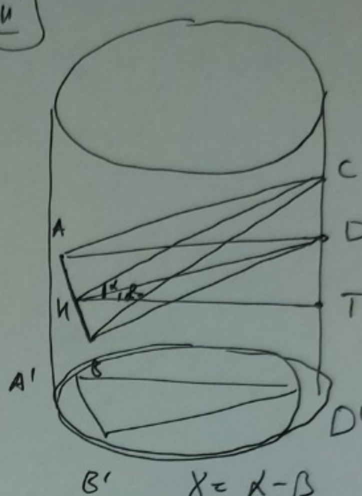
$AB = 4$

$AC = CB = 5$

$AD = DB = 6$

$\gamma = \alpha + \beta$

II



$\gamma = \alpha - \beta$

① проведём высоты DH и CH в $\triangle ABD$ и $\triangle$ ~~ABD~~ ABC

② проведём HT: $HT \parallel$ основанию цилиндра

③ построим проекцию $\triangle ACB$ и $\triangle ADB$ на основание цилиндр. $(A'B'D')$

④ $\angle CHT = \alpha$, $\angle$ ~~CH~~ $DHT = \beta$, ~~в~~ $\angle CHD = \gamma$

⑤ ~~HT = HT~~ из р/б $\triangle ABC$ и $\triangle ABD$

$CH = 4\sqrt{2}$, $DH = \sqrt{21}$

⑥ $HT = HT$

$CH \cos\alpha = DH \cos\beta$

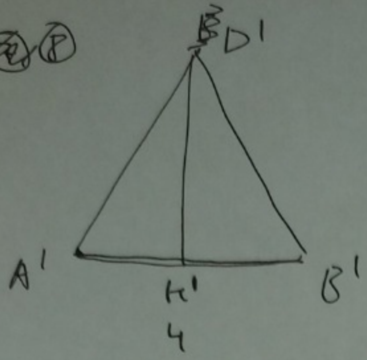
$\cos\beta = \dfrac{4\sqrt{2}}{\sqrt{21}} \cos\alpha$

~~⑦~~ ⑦ тк $\triangle ABC$ и $\triangle ABD$ р/б, $CD \perp$ основанию, $AB \parallel$ основанию

Чистовик

5

(2)(8)



$A'B' = AB = 4$ (т.к. $AB \parallel A'B'$)

$D'H' = 4\sqrt{2}\cos\alpha$

$A'D' = B'D' = \sqrt{4+32\cos^2\alpha}$

$S_{\triangle A'B'D'} = \frac{1}{2}\cdot A'B'\cdot D'H' = 8\sqrt{2}\cos\alpha$

$$S_{\triangle A'B'D'} = \frac{A'D'\cdot A'B'\cdot B'D'}{4R} =$$

$$= \frac{4(4+32\cos^2\alpha)}{4R}$$

$S = S$

$$8\sqrt{2}\cos\alpha = \frac{4+32\cos^2\alpha}{R}$$

$$R = \frac{1+8\cos^2\alpha}{2\sqrt{2}\cos\alpha}\ ;\ \cos\alpha = a$$

$$R = \frac{1+8a^2}{2\sqrt{2}a}$$

исследуем $R(a)$ на $0 \le a \le 1$

$$R'(a) = \frac{16a\cdot 2\sqrt{2}a - 2\sqrt{2} - 8\sqrt{2}\cdot 16\sqrt{2}a^2}{8a^2}\ ;\ R'(a) = 0$$

a	$\left(0;\frac{1}{2\sqrt{2}}\right)$	$\frac{1}{2\sqrt{2}}$	$\left(\frac{1}{2\sqrt{2}};1\right]$
$R'(a)$	$-$	0	$+$
$R(a)$	$\searrow$	R_{min}	$\nearrow$

$16\sqrt{2}a^2 = 2\sqrt{2}$

$a^2 = \frac{1}{8}$

$a = \frac{1}{2\sqrt{2}}$

$$R_{min} = R\left(\frac{1}{2\sqrt{2}}\right) = \frac{1+1}{1} = 2$$

т.к. на исследуемом промежутке только одна стационарная точка – точка минимума, в ней функция наименьшая

(9) ~~I: CD =~~

$\cos\alpha = \frac{1}{2\sqrt{2}}$ $\quad$ $\cos\beta = \frac{2}{\sqrt{21}}$

$\sin\alpha = \frac{\sqrt{7}}{2\sqrt{2}}$ $\quad$ $\sin\beta = \frac{\sqrt{17}}{\sqrt{21}}$

(10) I: $CD_{I} = CH \cdot \sin\alpha + DH \sin\beta$

$CD_{I} = 2\sqrt{7} + \sqrt{17}$

II: $CD_{II} = CH \sin\alpha - DH \sin\beta = 2\sqrt{7} - \sqrt{17}$

Ответ: $R_{min} = 2$; $CD = 2\sqrt{7} \pm \sqrt{17}$.

Черновик

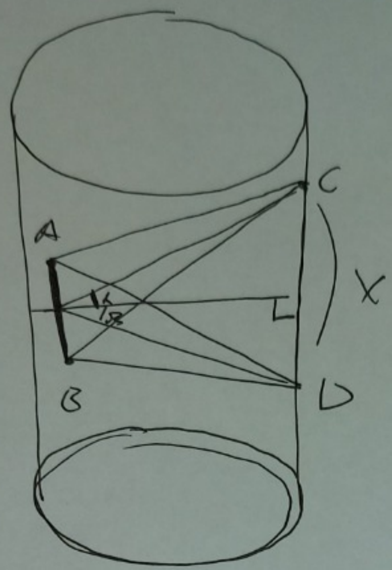
$\cos\beta \leq 1$

$\cos\alpha \leq \frac{\sqrt{21}}{4\sqrt{2}}$

$AB = 4$

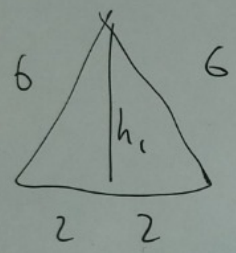
$AC = CK = 5$

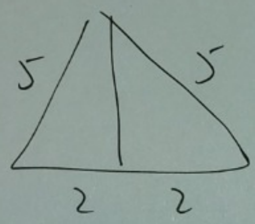
$AD = DK = 6$



$h_1 = 4\sqrt{2}$

$h_2 = \sqrt{21}$





$4\sqrt{2}\cos\alpha = \sqrt{21}\cos\beta$

$\cos\beta = \frac{4\sqrt{2}}{\sqrt{21}}\cos\alpha$

$\cos^2\beta = \frac{32}{21}\cos^2\alpha$

$R = \frac{c}{2}$

$x^2 = 32 + 21 - 8\sqrt{42}\cos\gamma$ $\quad \sin^2\beta = \frac{21 - 32\cos^2\alpha}{21}$

$\frac{abc}{4 \cdot \frac{c}{2}} =$

$\cos\gamma = \frac{53 - x^2}{8\sqrt{42}}$

$= \frac{ab}{2}$

$\cos\gamma = \cos(\alpha \pm \beta)$

$$\frac{53 - x^2}{8\sqrt{42}} = \frac{4\sqrt{2}}{\sqrt{21}}\cos^2\alpha \mp \sqrt{1-\cos^2\alpha} \cdot \sqrt{1 - \frac{32}{21}\cos^2\alpha}$$

$$\frac{(53-x^2)^2}{64 \cdot 42} + \frac{32}{21}\cos^4\alpha - \frac{53-x^2}{21} \cdot \cos^2\alpha = (1-\cos^2\alpha)(1 - \frac{32}{21}\cos^2\alpha)$$

21102901 (U152495 M1298837)

Чернови

$\sin\beta = \sqrt{1 - \frac{32}{21}\cos^2\alpha}$

$\sin\beta \cancel{<} \cancel{3} > 0$

$\sqrt{1-\frac{32}{21}\cos^2\alpha} < \cancel{3} > 0$

$\cancel{32}$ $1 - \frac{32}{21}\cos^2\alpha > 0$

$\cos^2\alpha < \frac{21}{32}$

$CD = \sqrt{53 - 8\sqrt{42}\cos\gamma}$

CD

$\cancel{4\sqrt{2}}$ $16\sqrt{2}\cos\alpha$

$4\sqrt{21}\cos\beta$

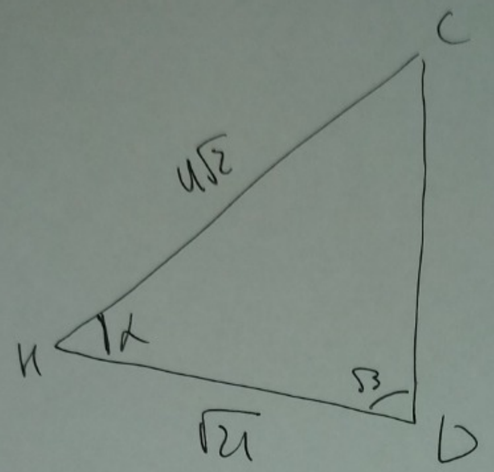
$\sqrt{1-\frac{1}{8}} = \frac{\sqrt{7}}{2\sqrt{2}}$

$21-4=17$

28

$\cancel{17}$

Черновик



$$CD = \sqrt{32+21-\cancel{2}8\sqrt{42}\cos\alpha}$$

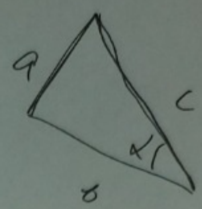
$$\sqrt{\cancel{32+21}}$$

$$\frac{\sqrt{\cancel{4+3}\,53-8\sqrt{42}\cos\alpha}}{\sin\alpha} = \frac{4\sqrt{2}}{\sin\beta}$$

$$\sin\beta = \frac{4\sqrt{2}\sin\alpha}{\sqrt{53-8\sqrt{42}\cos\alpha}}$$

$$R = \frac{9}{2\sqrt{2}}\sin\beta = \frac{18\sin\alpha}{\sqrt{53-8\sqrt{42}\cos\alpha}}$$

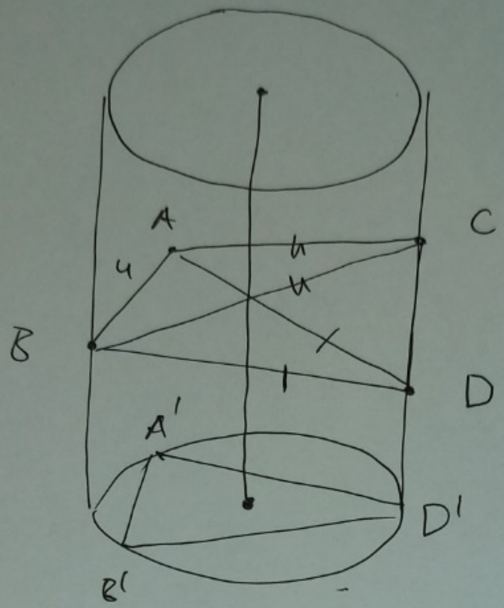
Черновик



$\sin\alpha = \frac{a}{c}$

$R = \frac{c}{2}$

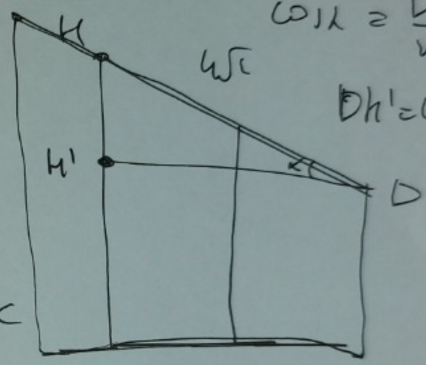
$\sin\alpha = \frac{a}{2R}$; $\frac{a}{\sin\alpha} = 2R$



$AC = CB = 5$

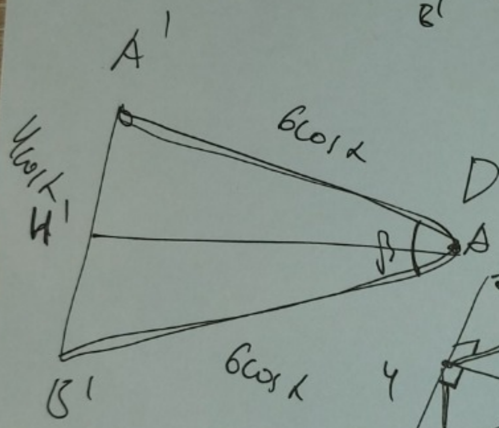
$AD = DB = 6$

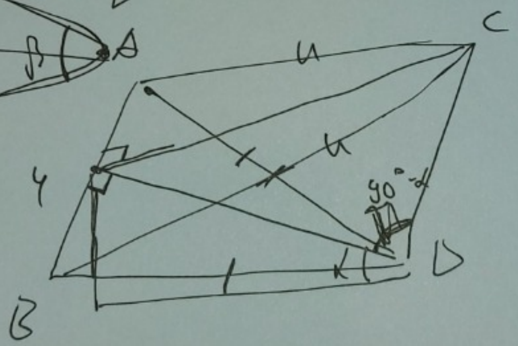
$AB = 4$



$\cos\alpha = \frac{DH'}{4\sqrt{2}}$

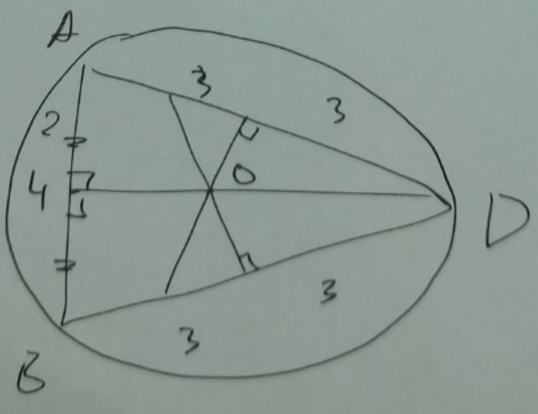
$DH' = 4\sqrt{2}\cos\alpha$





$R = 2\sqrt{2}\cos\alpha$

$AH' = 4\sqrt{2}\cos\alpha$



$DH^2 + 4 = 36$

$DH = \sqrt{32} = 4\sqrt{2}$

Четвёрки

$a^2 - 7a + 87 > 0$

$\cancel{\mathcal{D}}^2\ 49 - 4 \cdot 87$

$a^2 + 132 - 7a - 60 < 0$

$a^2 - 7a + 72 < 0$

$49 - 4 \cdot 72$

нет таких a.

№3.

$$\begin{cases} (x-a)^2 + (y-b)^2 \le 20 \\ a^2 + b^2 \le \min(8a - 4b, 20) \end{cases}$$

$(x-a)^2 + (y-b)^2 \le 20$

$a^2 + b^2 \le 20$

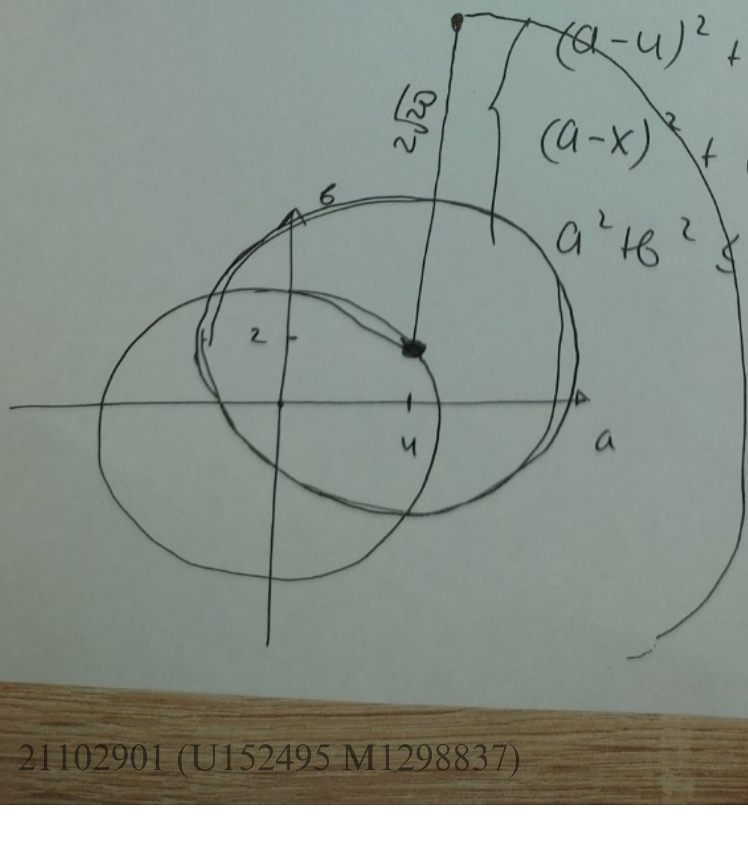
$a^2 + b^2 \le 8a - 4b$

$a^2 - 8a + 16 + b^2 - 4b + 4 \le 20$

$(a-4)^2 + (b-2)^2 \le 20$

$(a-x)^2 + (b-y)^2 \le 20$

$a^2 + b^2 \le 20$



$2\sqrt{5}$

$\cancel{20 = 16 + 4}$

Чернова

$S_7 = S$

$S = \frac{a+a+6b}{2} \cdot 7 = 7(a+3b)$

$(a+7b)(a+16b) > S+27$

$a^2 + 23b + 112b^2 > 7a + 21b + 27$

$\boxed{a^2 + 2b + 112b^2 > 7a + 27}$

$a, b \in \mathbb{Z}$

$(a+10b)(a+13b) < 7(a+3b) + 60$

$a^2 + 23b + 130b^2 < 7a + 21b + 60$

$\boxed{a^2 + 2b + 130b^2 < 7a + 60}$

$18b^2 < 33$

$b^2 < \frac{11}{6} = 1\frac{5}{6}$

$b > 0$ - бгр

$b = 1$

$a^2 + 114 > 7a + 27$

$a^2 - 7a + 87 > 0$

$114 - 27 = 87$

$a^2 + 2b + 112b^2 - 7a > 27$

$a^2 + 2b + 112b^2 - 7a < 60 - 18b^2$

$60 - 18b^2 > 27$

$18b^2 < 33$ $\boxed{b = 1}$

$8 \cdot 7 = 56$

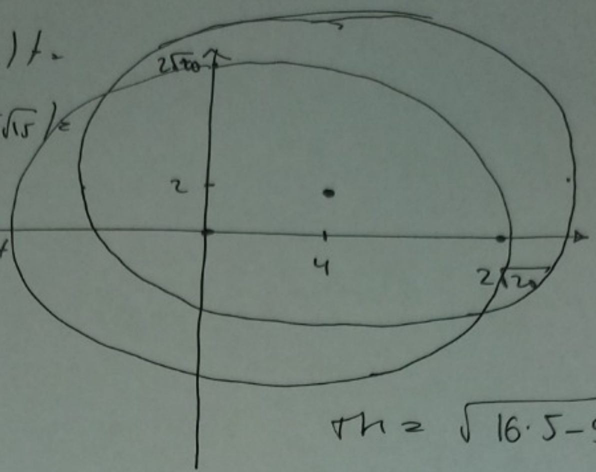
Черновик

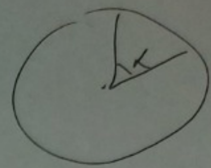
$S = 2(S_1 - (S_3 - S_2)) + =$

$= 2(80\pi - 80\arcsin + 5\sqrt{15}) =$

$= 160\pi - 160\arcsin(\ ) +$

$+ 10\sqrt{15}$





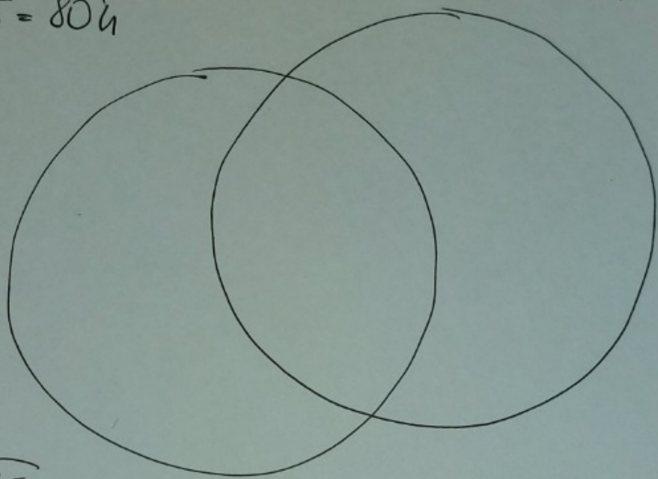
$\frac{\alpha}{2\pi} \cdot \pi r^2 =$

$=$

$TH = \sqrt{16 \cdot 5 - 5} = \sqrt{15 \cdot 5} = 5\sqrt{3}$

$S_1 = \pi \cdot 16 \cdot 5 = 80\pi$

$\sin\alpha = \frac{5\sqrt{3}}{4\sqrt{5}} = \frac{\sqrt{15}}{4}$

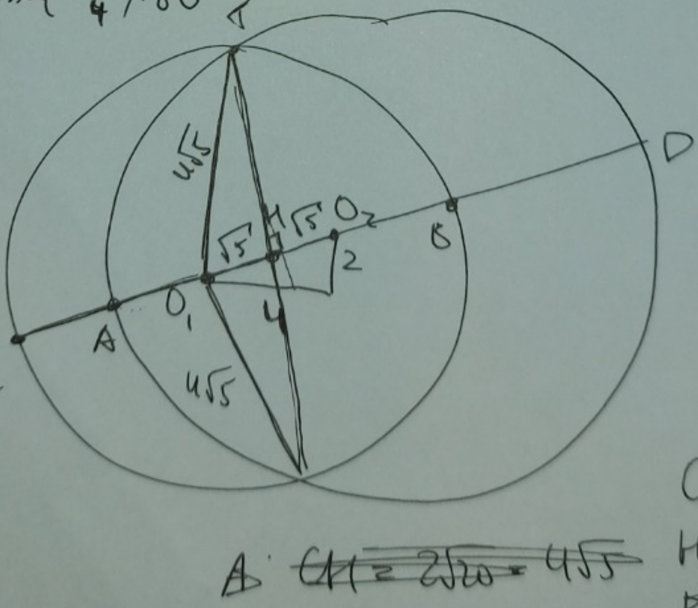


$S_2 = \sqrt{5} \cdot 5\sqrt{3} =$

$= 5\sqrt{15}$

$\sqrt{20} = 2\sqrt{5}$

$S_3 = \cancel{2 \cdot \alpha}\ \arcsin\left(\frac{\sqrt{15}}{4}\right) \cdot 80$



$O_1C = 4\sqrt{5}$

$HC = 5\sqrt{5} = HD$

$BH = 3\sqrt{5} = AH$

A: $\cancel{CH = 2\sqrt{20} = 4\sqrt{5}}$

40

Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21102901**

ID профиля: **152495**

Вариант 21

Чистовик

(1)

№1.

пусть:

$\ln(2x-3)=k$

$\ln(x+1)=m$

$\ln(2x^2-3x+5)=n$

~~$(k,m,n\neq 0)$~~

Тогда:

$A=\log_{\sqrt{2x-3}}(x+1)=\dfrac{2m}{k}$

$B=\log_{2x^2-3x+5}(2x-3)^2=\dfrac{2k}{n}$

$C=\log_{(x+1)}(2x^2-3x+5)=\dfrac{n}{m}$

$$*:\ \begin{cases} x>\frac{3}{2} \\ x>-1 \\ x\neq 2 \\ x\neq 0 \end{cases} \Leftrightarrow \begin{cases} x>\frac{3}{2} \\ x\neq 2 \end{cases}$$

$k,m,n\neq 0$

(1) $A=B=\cancel{B}\cdot C+1$

$$\begin{cases} nk+mk=2m^2 \\ mn=k^2 \end{cases} \Rightarrow n^3+2mn^2+m^2n-4m^3=0$$

	1	$2m$	m^2	$-4m^3$	
m	1	$3m$	$4m^2$	0	$\boxed{n=m}$

$n^2+3mn+4m^2=0$

$D=9m^2-16m^2<0$

$n=m\ (1) \Rightarrow k=m\ (2)$

$x+1=2x^2-3x+5$

$2x^2-4x+4=0$

$x^2-2x+2=0$

$D=4-4\cdot 2<0$

.должны выполняться оба условия (1) и (2)

(2) $A=C=B+1$

$$\begin{cases} 2m^2=kn \\ 2k^2+kn=2mn \end{cases} \Rightarrow n^3-mn^2-4m^3=0$$

	1	$-m$	0	-4	
$2m$	1	m	$2m^2$	0	$\boxed{n=2m}$

Чистовик ②

$n^2+mn+2m^2=0$

$D=m^2-4\cdot 2m^2<0$

(1) $n=2m$ ~~tx~~ $\Rightarrow$ $k=3m$ (2)

$2x^2-3x+5=x^2+2x+1$ $\quad$ $2x-3=x+1$

$x^2-5x+4=0$ $\quad$ $X=4$

подх. в *

$D=25-16=9$

$x=\frac{5\pm 3}{2}$ $\quad$ $X=4$ $\quad$ или $\quad$ $X=1$

подх. в * $\quad$ не подх. в *

• должны выполняться и (1), и (2), след. $\boxed{X=4}$

③ $B=C=A+1$

$$\begin{cases} 2km=n^2 \\ 2mn+kn=2k^2 \end{cases} \Rightarrow n^3-mn^2-4m^3=0$$

~~аналогично ②~~

~~$X=4$~~

аналогично ②

$n=2m$

~~Ответ: $X=4$.~~

(1) $n=2m$ $\Rightarrow$ $k=2m$ (2)

$X=4$ $\quad$ $2x-3=x^2+2x+1$

подх. в *

$x^2=-4$

$\varnothing$

· должны выполняться оба условия (1) и (2)

след. ~~[illegible]~~ в ③ решений нет

Ответ: $X=4$.

21102901 (U152495 M1298838)

Чистовик

3

№1.

$$\begin{cases} НОД = 35 = 5\cdot 7 \\ НОК = 5^{18}\cdot 7^{16} \end{cases}$$

т.к. $НОК = 5^{18}\cdot 7^{16}$, $НОД = 35$

$a = 5^{\alpha_1+1}\cdot 7^{\alpha_2+1}$

$b = 5^{\beta_1+1}\cdot 7^{\beta_2+1}$

$c = 5^{\gamma_1+1}\cdot 7^{\gamma_2+1}$

должны выполняться следующие условия

① хотя бы одно из $\alpha_1, \beta_1, \gamma_1 = 0$

② хотя бы одно из $\alpha_1, \beta_1, \gamma_1 = 17$

③ $0 \le \alpha_1, \beta_1, \gamma_1 \le 17$

④ хотя бы одно из $\alpha_2, \beta_2, \gamma_2 = 0$

⑤ хотя бы одно из $\alpha_2, \beta_2, \gamma_2 = 15$

⑥ $0 \le \alpha_2, \beta_2, \gamma_2 \le 15$

~~I · не нарушая общности (затем умножим кол-во на 3) $\alpha_1 = 0$, $\beta_1 = 17$~~

I) $\alpha_1, \beta_1, \gamma_1$ – различны

$\alpha_2, \beta_2, \gamma_2$ различны

$S = \underbrace{3\cdot 2\cdot 16}\cdot\underbrace{3\cdot 2\cdot 14} = 36\cdot 14\cdot 16$

II $\alpha_1, \beta_1, \gamma_1$ – различны

$\alpha_2, \beta_2, \gamma_2$ – не все различны

$S = \underbrace{3\cdot 2\cdot 16}\cdot 2\cdot 3 = 36\cdot 16$

III $\alpha_1, \beta_1, \gamma_1$ – ~~различн~~ не все различны

$\alpha_2, \beta_2, \gamma_2$ – различны

$S = 2\cdot 3\cdot 3\cdot 2\cdot 14 =$
$= 36\cdot 14$

Чистовик

4

IV $\alpha_1, \beta_1, \gamma_1$ - не все различны

$\alpha_2, \beta_2, \gamma_2$ - не все различны

$S = \cancel{4 \cdot 2 \cdot 3 \cdot 6 \cdot 4}\ 4 \cdot 9 = 36$

$S = S_{I} + S_{II} + S_{III} + S_{IV} = 36(14 \cdot 16 + 16 + 14 + 1) =$

$= 36 \cdot (14+1)(16+1) = 36 \cdot 15 \cdot 17 = 9186$

Ответ: 9186.

Чистовик

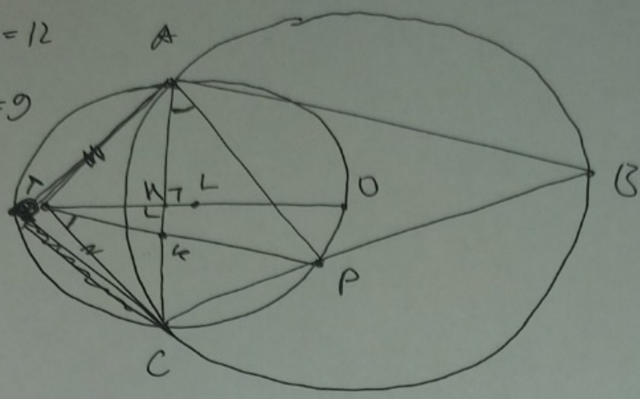
№6.

5

Дано:

$S_{\triangle APK} = 12$

$S_{\triangle CPK} = 9$



① ~~Е~~ пусть вторая окружность – φ

② тк $\varphi \cap BC = P$

$R_\varphi < R_\omega$

③ $AO = OC = R_\omega$

$AT = CT$ – ~~кас из~~ касательные из одной точки

след. центр φ: ~~__~~ точка L

~~$L \in TO$~~

$L \in TO$

④ $TO \cap AC = H$

⑤ $\frac{AK}{KC} = \frac{\sqrt{S_{\triangle APK}}}{\sqrt{S_{\triangle CPK}}} = \frac{2}{\sqrt{3}}$

⑥ $OA \perp TA$, тк AT – касательная

$\angle OTAO = 90° \Rightarrow T \in \varphi$ (T лежит на окружности OT – диаметр)

⑦ $S_{\triangle APC} = S_{\triangle APK} + S_{\triangle CPK} = 21$

⑧ ~~$S_{\triangle ABC}$~~ $\frac{S_{\triangle APC}}{S_{\triangle ABC}} = \frac{PC}{BC}$; $S_{\triangle ABC} = \frac{BC}{PC} \cdot 21$

Чернова

НОД = 35 = 5 · 7

НОК = $5^{18} \cdot 7^{16}$

~~$a = 5^{\alpha} \cdot 7^{\beta}$~~

~~$b = 5^{\gamma} \cdot 7^{4}$~~

~~$c =$~~

$a = 35 \cdot 5^{\alpha_1} \cdot 7^{\alpha_2}$

$b = 35 \cdot 5^{\beta_1} \cdot 7^{\beta_2}$

$c = 35 \cdot 5^{\gamma_1} \cdot 7^{\gamma_2}$

$\alpha_1 / \beta_1 / \gamma_1 = 0$ 3 вар.

$\alpha_2 / \beta_2 / \gamma_2 = 0$ 3 вар.

$\alpha_1 / \beta_1 / \gamma_1 = 17$ 2 вар.

$\alpha_2 / \beta_2 / \gamma_2 = 15$ 2 вар.

от. $\alpha_1 / \beta_1 / \gamma_1 \in [0; 17] \rightarrow$ 18 вар.

от. $\alpha_2 / \beta_2 / \gamma_2 \in [0; 15] \rightarrow$ 16 вар.

$2^3 \cdot 2^4 = 2^7$

$3^2 \cdot 2^2 \cdot 18 \cdot 16 = 9 \cdot 4 \cdot 18 \cdot 16 =$

$= 8 \cdot 81 \cdot 16 =$

$= 128 \cdot 81$

$$
\begin{array}{r}
128 \\
\times\ 81 \\
\hline
128 \\
+\ 924 \\
\hline
\boxed{9768}
\end{array}
$$

21102901 (U152495 M1298838)

Чернови

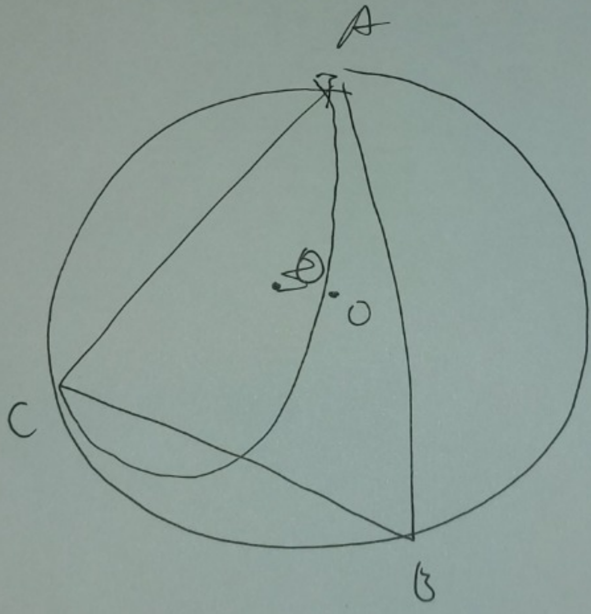
$(2x-3)(x+1) = 2x^2 - \cancel{3}x - 3 =$

$= 2x^2 - 3x + 5 + 2x - 8$

$\cancel{4x^2 + 9 - 6}\,12x$

$$\begin{array}{l|l} 2x^2-3x+5 & 2x-3 \\ \hline -2x^2-3x & x \\ \hline \quad 5 & \end{array}$$

$\frac{2x^2-3x+5}{2x-3} = x + \frac{5}{2x-3}$



21102901 (U152495 M1298838)

Чернови

$A = \log_{\sqrt{2x-3}}(x+1) \quad B = \log_{2x^2-3x+5}(2x-3)^2$

$C = \log_{(x+1)}(2x^2-3x+5)$

$A = \dfrac{2\ln(x+1)}{\ln(2x-3)} \quad B = \dfrac{2\ln(2x-3)}{\ln(x(2x-3)+5)} \quad C = \dfrac{\ln(x(2x-3)+5)}{\ln(x+1)}$

$2x^2 - 3x + 5 = 0$

$D = 9 - 4\cdot 5 \cdot 2 < 0$

$\ln(2x-3) = k$

$\ln(x+1) = m$

$\ln(2x^2-3x+5) = n$

$A = \dfrac{2m}{k} \quad B = \dfrac{2k}{n} \quad C = \dfrac{n}{m}$

① $A = B: \quad \dfrac{m}{k} = \dfrac{k}{n}; \quad mn = k^2$

$mn = k^2$

$\dfrac{n}{m} + 1 = \dfrac{2m}{k}$

$\begin{cases} nk + km = 2m^2; \; k = \dfrac{2m^2}{m+n} \\ mn = k^2 \end{cases}$

$mn = \dfrac{4m^4}{m^2+2mn+n^2}$

$nm^2 + 2mn^2 + n^3 = 4m^3$

$n^3 + 2mn^2 + m^2n - 4m^3 = 0$

$\dfrac{n+m}{m} = \dfrac{2m}{k}$

$kn + mk = 2m^2$

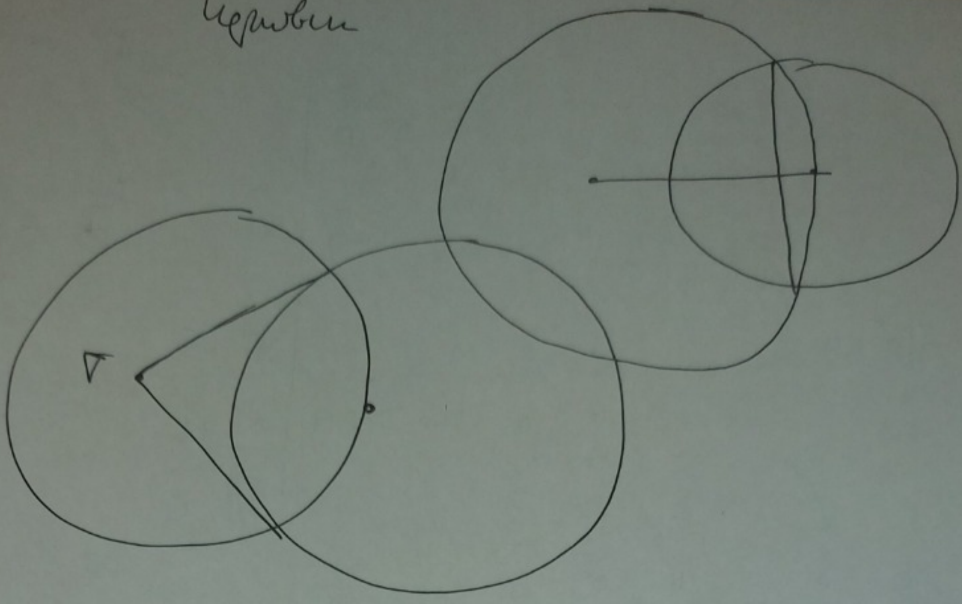
$\dfrac{n}{k} + \dfrac{m}{k} = \dfrac{2m^2}{k^2}$

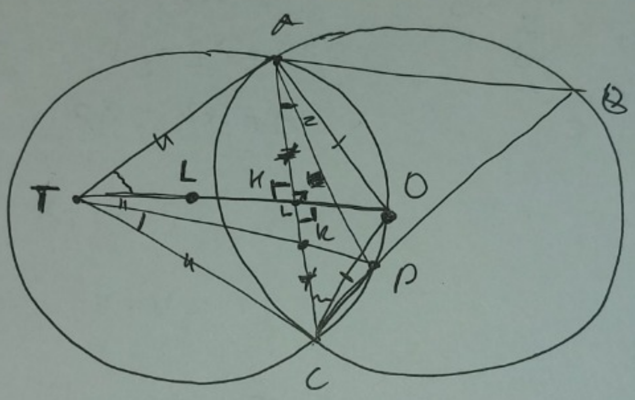
$\dfrac{m}{k} + \dfrac{k}{m} = \dfrac{2m^2}{k^2}$

$\dfrac{m^2+k^2}{m} = \dfrac{2m^2}{k}$

$m^2k + k^3 = 2m^3$

Чернови

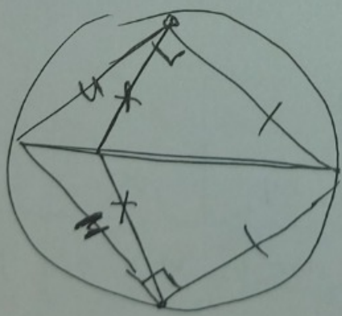




$AH = CH$

$\frac{AK}{KC} = \frac{\sqrt{12}}{\sqrt{9}} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$

$\triangle ACP = 21$



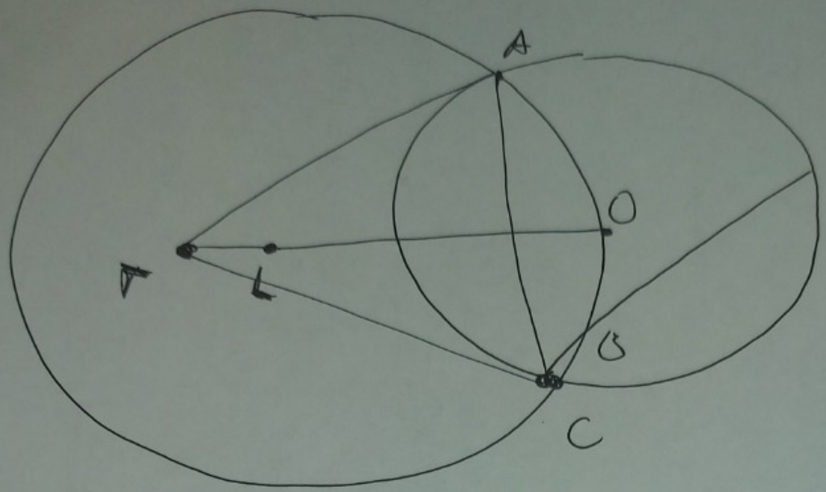
$AC = x$

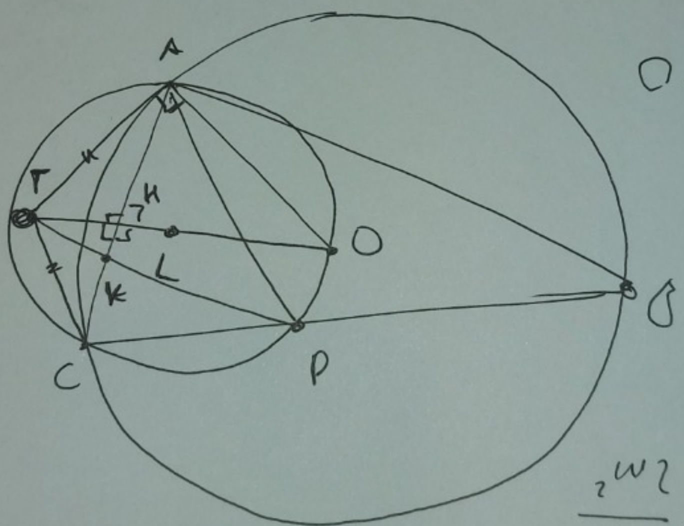
$AH = CH = \frac{x}{2}$

$\cancel{AH} = AK = \cancel{\frac{2+\sqrt{3}}{\sqrt{3}}} \frac{2}{2+\sqrt{3}} x$

$CK = \frac{2\sqrt{3}}{2+\sqrt{3}} x$

Чернови





$$\frac{2k}{n} = \frac{n}{m}$$

$$2km = n^2$$

$$\cancel{2k}\ k = \frac{n^2}{2m} \qquad \frac{2m+k}{k} = \frac{2k}{n}$$

$$2mn + kn = 2k^2$$

$$2mn + \frac{n^3}{2m} = \frac{n^4}{2m^2}$$

$$4m^3n + mn^3 = n^4$$

$$4m^3 + mn^2 = n^3$$

$$n^3 - mn^2 - 4m^3 = 0$$

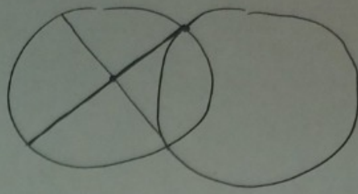
$$n = km$$

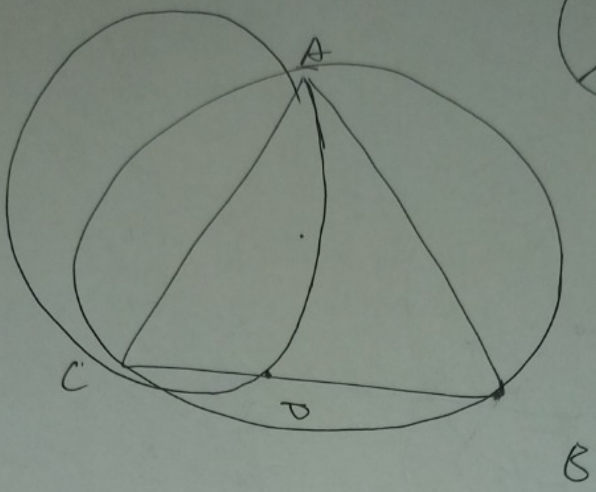
$$k^3 - k^2 - 4 = 0$$

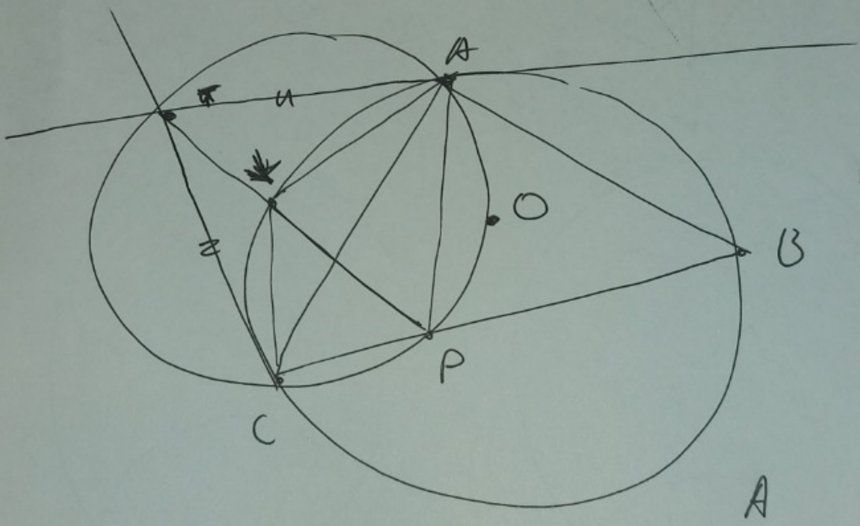
$$k^3 = k^2 + 4 = 0$$

21102901 (U152495 M1298838)

Чернова

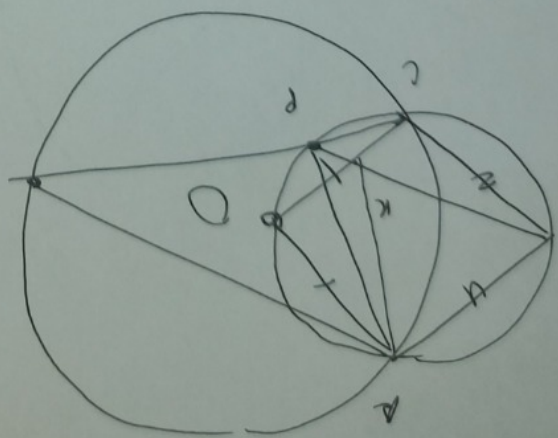






$S_{\Delta APK} = 12$

$S_{\Delta CPK} = 9$



$2x^2 - 3x + 4$

$9 - 4 \cdot 4$