

Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

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ID профиля: **74985**

Вариант 21

Begrüßung 21. Exam 1. Nummer

Begrüßung 1

$$\begin{aligned} a_8 &= a_1 + 7d \\ a_{12} &= a_1 + 16d \\ a_{11} &= a_1 + 10d \\ a_{14} &= a_1 + 13d \end{aligned}$$

$$\begin{aligned} a_8 \cdot a_{12} &> S + 27 \Leftrightarrow (a_1 + 7d)(a_1 + 16d) > S + 27 \\ a_{11} \cdot a_{14} &< S + 60 \Leftrightarrow (a_1 + 10d)(a_1 + 13d) < S + 60 \end{aligned}$$

⇓

$$(a_1 + 7d)(a_1 + 16d) + S + 60 > S + 27 + (a_1 + 10d)(a_1 + 13d) \Leftrightarrow$$

$$a_1^2 + 23da_1 + 112d^2 + 8 + 60 > 8 + 27 + a_1^2 + 23da_1 + 130d^2 \Leftrightarrow$$

$$18d^2 < 33 \Leftrightarrow \text{m.k. } d > 0 \text{ und } d \in \mathbb{Z} \text{ (no numbers)} \Rightarrow d < \sqrt{\frac{33}{18}} \Rightarrow$$

$$\boxed{d=1} \Rightarrow (a_1 + 7d)(a_1 + 16d) > S + 27, \quad S = a_1 + a_2 + \dots + a_9 = 7a_1 + 21d \Rightarrow$$

$$(a_1 + 7)(a_1 + 16) > 7a_1 + 48 \Leftrightarrow a_1^2 + 16a_1 + 64 > 0 \Leftrightarrow (a_1 + 8)^2 > 0 \Leftrightarrow \boxed{a_1 \neq -8}$$

$$\Rightarrow (a_1 + 10d)(a_1 + 13d) < 7a_1 + 21d + 60 \Leftrightarrow a_1^2 + 16a_1 + 49 < 0 \cdot \quad \Delta = 256 - 196 = 60$$

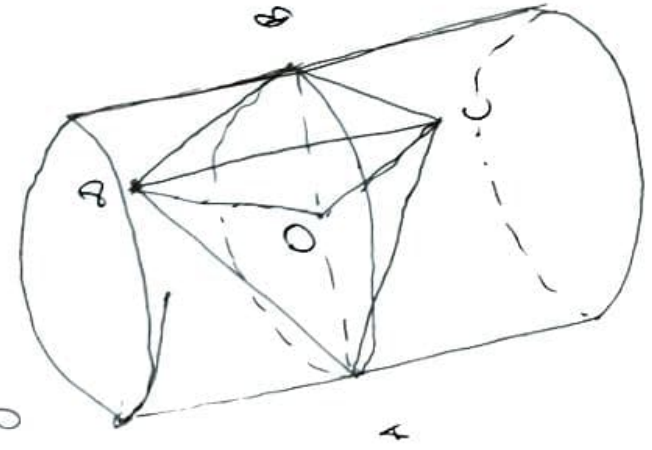
$$a_1 = \frac{-16 \pm \sqrt{60}}{2} = -8 \pm \sqrt{15} \Rightarrow \frac{-8 - \sqrt{15}}{-8 + \sqrt{15}} \Rightarrow a_1 \in \mathbb{Z} \text{ (no numbers)}$$

$$a_1 \Rightarrow \{ -11, -10, -9, -7, -6, -5 \}$$

Ordnung:

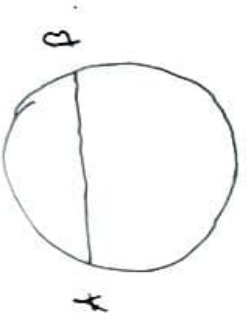
$$\begin{array}{ccc} a_1 = -11 & a_1 = -9 & a_1 = -6 \\ a_1 = -10 & a_1 = -7 & a_1 = -5 \end{array}$$

Загара № 2



Заманаси, зно AB ~~сыйра~~ сыйра
 ремангун бугуну огурунуну.

AB = 4.



$AC = CB = 5$; $AD = DB = 6$; CD напар.

керне си ~~укумугу~~ ~~укумугу~~ \Rightarrow AB ремон на огурунуну напареле.
 нои ордонмо укумугу. Торга пагузе укумугу дугеи навалентуу
 еси AB - гуурун. $\Rightarrow r = \frac{AB}{2} = 2$. ~~Аде~~ Талуурун $\triangle ADC$.

$AD + AC > CD \Leftrightarrow CD < 11$; ~~AC~~ $AC + CD > AD \Leftrightarrow CD > 1$.

Талуурун $\triangle ODC$, где O - саргуну AB. $\Rightarrow OC \perp AB$. м.к. $\triangle AOC = \triangle OBC$
 ($AC = CB$, OC - огузу сугуна, $AO = OB$ (пагурун)). $\Rightarrow OC = \sqrt{AC^2 - AO^2} = \sqrt{25 - 4} = \sqrt{21}$.

$\triangle AOD = \triangle BOD$ (OD - огузу сугуна, $AO = OB$, $AD = DB$) $\Rightarrow OD \perp AB \Rightarrow$ ~~оо~~ \Rightarrow

$OD = \sqrt{AD^2 - AO^2} = \sqrt{32}$ $\Rightarrow CD + \sqrt{21} > \sqrt{32}$, $CD < \sqrt{32} + \sqrt{21}$.

Сугурун $\sqrt{32} + \sqrt{21} \neq 11 \Rightarrow 32 + 2 \cdot \sqrt{32 \cdot 21} + 21 > 121 \Leftrightarrow \sqrt{34} > \sqrt{32 \cdot 21}$.

$\Rightarrow CD < \sqrt{32} + \sqrt{21}$. Сугурун $1 \vee \sqrt{32} - \sqrt{21} \Leftrightarrow 26 > \sqrt{32 \cdot 21}$ ~~оо~~

$26 > \sqrt{32 \cdot 21} \Rightarrow CD > 1 \Rightarrow CD \in (1; \sqrt{32} + \sqrt{21})$

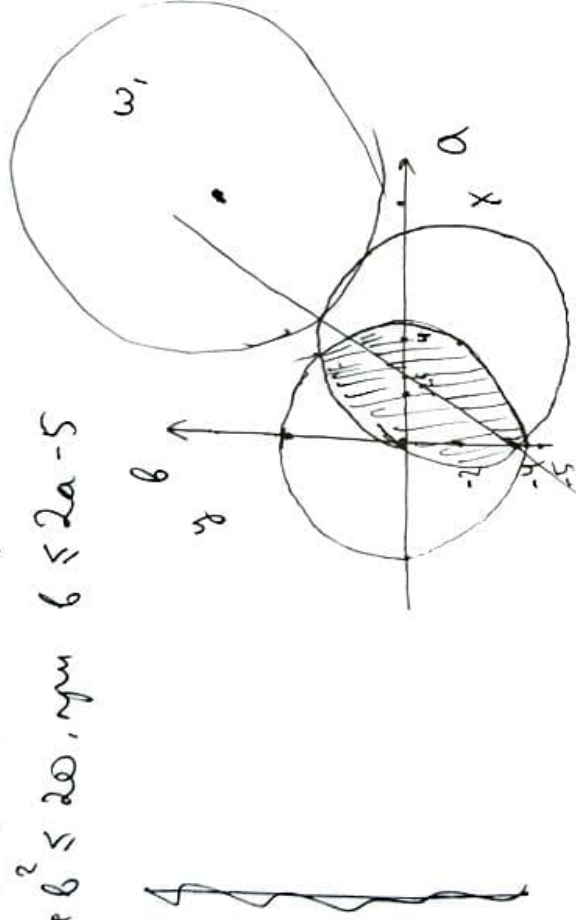
Оубен: $CD \in (1; \sqrt{32} + \sqrt{21})$

Задача №3

Учебник. Бакин 21. Задача 1.

$$\begin{cases} (x-a)^2 + (y-b)^2 \leq 20 & (\Rightarrow) \\ a^2 + b^2 \leq \min(8a-4b, 20) \\ a^2 + b^2 \leq 20, \text{ при } 8a-4b \leq 20 & (\Rightarrow) \\ a^2 + b^2 \leq 20, \text{ при } 8a-4b \geq 20 \end{cases}$$

$$\Leftrightarrow \begin{cases} (a-x)^2 + (b-y)^2 \leq 20 \\ (a-4)^2 + (b+2)^2 \leq 20, \text{ при } b > 2a-5 \\ a^2 + b^2 \leq 20, \text{ при } b \leq 2a-5 \end{cases}$$

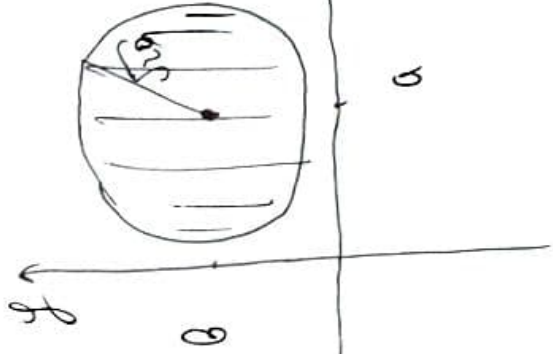


а, b - диаметр окружности, если ω₁ и ω₂ имеют общую точку
 площадь заштрихованной части: $(x-4)^2 + (y+2)^2 \leq (\sqrt{20})^2$
 - ~~площадь~~

Площадь фигуры M - диаметр общей хорды окружностей $\sqrt{20}$.

$$S = \pi R^2 = 20\pi$$

$$\text{Ответ: } S = 20\pi.$$

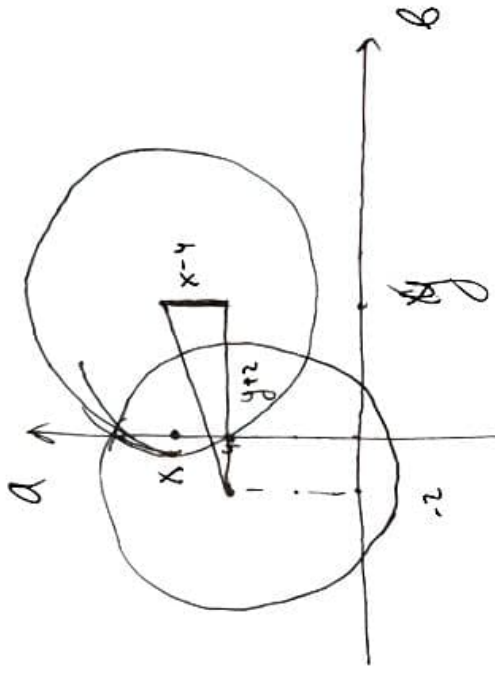
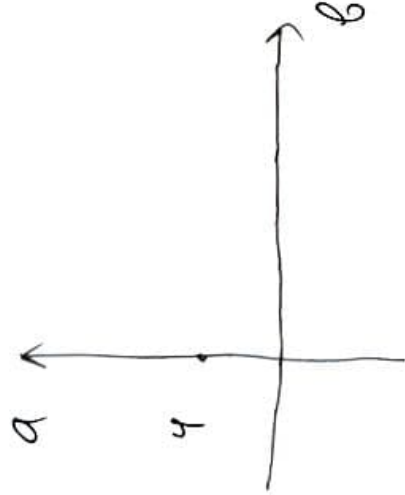


$$] \quad 8a - 4b < 20$$

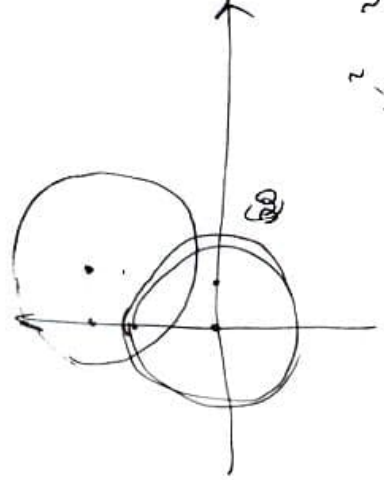
$$a^2 + b^2 < 8a - 4b$$

$$a - 8a + 16 + b^2 + 4b + 4 < 20$$

$$a - 8a + 16 + (a-4)^2 + (b+2)^2 < 20$$



$$(2\sqrt{20})^2 < (x-4)^2 + (y-2)^2$$

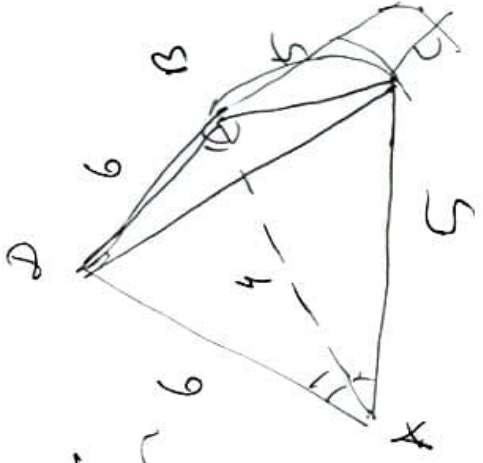


$$x^2 + y^2 < (2\sqrt{20})^2$$

48.

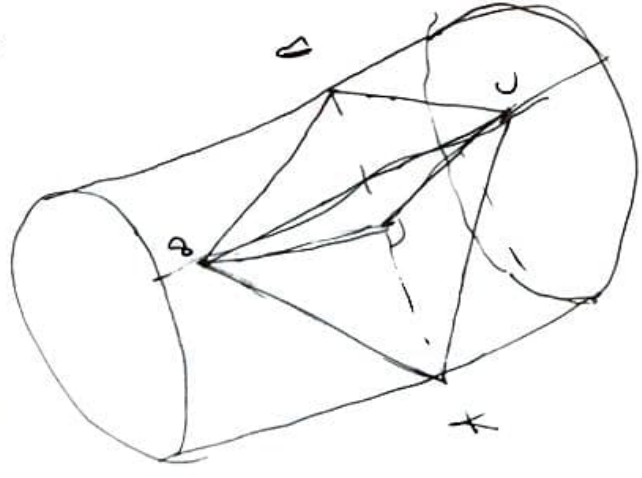
$$\begin{array}{r} 26 \\ \times 26 \\ \hline 156 \\ 520 \\ \hline 676 \end{array}$$

$$\begin{array}{r} 321 \\ \times 22 \\ \hline 642 \\ 642 \\ \hline 7056 \end{array}$$



$$\begin{array}{r} 57 \\ \times 57 \\ \hline 399 \\ 3249 \\ \hline 3249 \end{array}$$

$CD < 11$



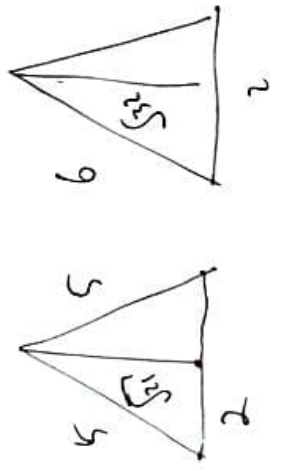
$\sqrt{32}$

1V

$$53 - 2\sqrt{32} \cdot 21$$

$$26 \cdot \sqrt{32} \cdot 21$$

$CD < \sqrt{32} + \sqrt{21}$



$5 + CD > 6 \implies CD + \sqrt{21} > \sqrt{32}$

$CD > 1 \implies CD > \sqrt{32} - \sqrt{21}$

$\sqrt{32} + \sqrt{21} \approx 11$

$53 + 2\sqrt{32 \cdot 21} \approx 121$

$34 \approx \sqrt{32} \cdot 21$

$$a_1 + a_2 + \dots + a_7 = S$$

$$a_1 + (a_1 + d) + \dots + a_1 + 6d = \frac{(1+n-1) \cdot n-1}{2} \cdot a_1 + \frac{n(n-1)d}{2}$$

$$4 \cdot 3 + 5 + 7 = 1 \cdot 4 + \frac{4(3) \cdot 2}{2} = 4 + 12 = 16$$

$$\begin{aligned} a_9 \cdot a_{17} &> S + 27 \\ a_{11} \cdot a_{19} &< S + 60 \end{aligned}$$

$$(a_1 + 7d)(a_1 + 16d) > S + 27$$

$$(a_1 + 10d)(a_1 + 13d) < S + 60$$

$$a_1^2 + 23ad + 112d^2 > 7a_1 + \cancel{76d} = 7a_1 + 21d + 27$$

$$a_1^2 + 23ad + 130d^2 < 7a_1 + 21d + 60$$

$$a_1^2 + 23ad + 112d^2 + 7a_1 + 21d + 60 > 7a_1 + 21d + 27 + \cancel{a_1^2 + 23ad + 130d^2} + 18d^2$$

$$60 > 27 + 18d^2 \Rightarrow 33 > 18d^2$$

$$d > 0 \Rightarrow d < \sqrt{\frac{33}{18}} \Rightarrow \boxed{d = 1}$$

$$a_1^2 + 23a_1 + 112 > 27a_1 + 21 + 27$$

$$a_1^2 - 4a_1 + 64 > 0$$

$$a_1^2 + 23a_1 + 130 < 7a_1 + 21 + 60$$

$$a_1^2 + 16a_1 + 49 < 0 \quad a_{12} = \frac{16 \pm \sqrt{60}}{2} = 8 \pm \sqrt{15}$$

$$D = 256 - 196 = 60$$

$$\begin{array}{r} \times 16 \\ \hline 16 \\ \hline 96 \\ \hline 256 \end{array}$$

$$456 - 111$$

$$(x-a)^2 + (y-b)^2 \leq 20$$

$$a^2 + b^2 \leq \min(8a-4b, 20)$$

$$(x-a)^2 + (y-b)^2 \leq 20$$

$$a^2 + b^2 \leq 8a-4b, \text{ if } 8a-4b < 20$$

$$a^2 + b^2 \leq 20, \text{ if } 8a-4b \geq 20$$

$$a^2 + b^2 \leq 8a-4b$$

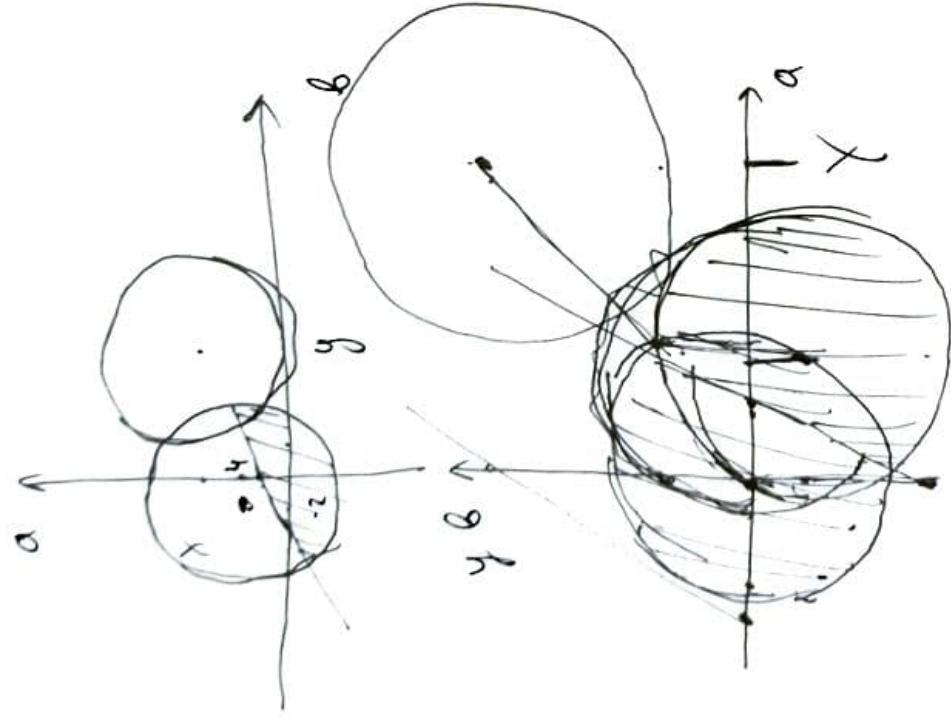
$$\textcircled{1} (a-4)^2 + (b+2)^2 \leq 20$$

$$a < \frac{b}{2} + \frac{5}{2}$$

$$4b > 8$$

$$b > 2a - 5$$

$$b \leq 2a - 5$$



$$(y+2)^2 + (x-4)^2 \leq (2\sqrt{5})^2$$

$$x^2 + y^2 \leq 2\sqrt{5}0$$

$$(a-x)^2 + (b-y)^2$$

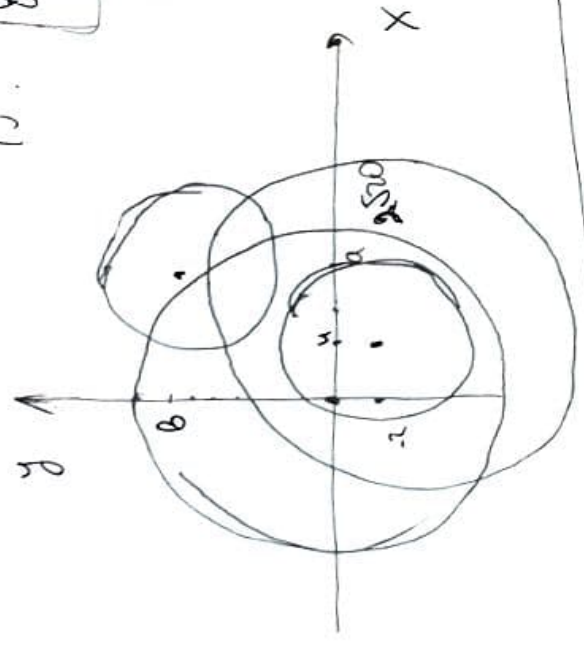
$$(x-4)^2 + (y+2)^2 \leq (\sqrt{20})^2$$

if: $8a - 4b < 20$.

$2a - b < 5$

elseif $8a - 4b > 20$.

$2a - b > 20$



$\frac{4 \cdot 16}{112}$

$S = a_1 + a_2 + \dots + a_7 = 7a_1 + \frac{7 \cdot 6}{2} = 7a_1 + 21d$.

$a_8 = a_1 + 7d \Rightarrow (a_1 + 7d)(a_1 + 6d) > S \cdot 27$.

$a_{17} = a_1 + 16d$.

$(a_1 + 10d)(a_1 + 13d) < S \cdot 60$.

$a_{11} = a_1 + 10d$

$a_{14} = a_1 + 13d$,

$a_1 \leq 7$
 $a_1 \in (-11; -5) \cup (1; 8)$

$a_1^2 + 16da_1 + 7da_1 + 112d^2 > S \cdot 27 = 7a_1 + 21d + 27$

$a_1^2 + 23a_1d + 130d^2 < S \cdot 60$

$a_1^2 + 23a_1d + 112d^2 + 8 + 60 > a_1^2 + 23a_1d + 130d^2 + 8 + 27$

$d^2 < \frac{33}{18} \Rightarrow d < 2 \Rightarrow d = 1$

$a_1^2 + 23a_1 + 112 > 7a_1 + 21 + 27$.

$a_1^2 + 16a_1 + 64 \geq 0$.

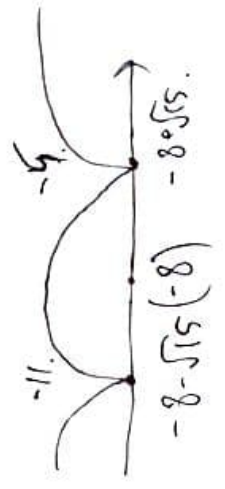
$(a_1 + 8)^2 > 0 \Rightarrow a_1 \neq -8$.

$a_1^2 + 23a_1 + 130 < 7a_1 + 21 + 60$.

$a_1^2 + 16a_1 + 49 < 0$.

$\Delta = 256 - 196 = 60$.

$a_{12} = \frac{-16 \pm \sqrt{60}}{2} = -8 \pm \sqrt{15}$.



Barisan 21. Zamb 1. Zumbuk

Zagara no 1

$$a_8 = a_1 + 7d$$

$$a_{12} = a_1 + 16d$$

$$a_{11} = a_1 + 10d$$

$$a_{14} = a_1 + 13d$$

$$a_8 \cdot a_{12} > S + 27 \Leftrightarrow (a_1 + 7d)(a_1 + 16d) > S + 27$$

$$a_{11} \cdot a_{14} < S + 60 \Leftrightarrow (a_1 + 10d)(a_1 + 13d) < S + 60$$

⇐

$$(a_1 + 7d)(a_1 + 16d) + S + 60 > S + 27 + (a_1 + 10d)(a_1 + 13d) \Leftrightarrow$$

$$a_1^2 + 23da_1 + 112d^2 + S + 60 > S + 27 + a_1^2 + 23da_1 + 130d^2 \Leftrightarrow$$

$$18d^2 < 33 \Leftrightarrow \text{m.k. } d > 0 \text{ u } d \in \mathbb{Z} \text{ (no yumburo)} \Rightarrow d < \sqrt{\frac{33}{18}} \Rightarrow$$

$$\boxed{d=1} \Rightarrow (a_1 + 7d)(a_1 + 16d) > S + 27; S = a_1 + a_2 + \dots + a_7 = 7a_1 + 21d \Rightarrow$$

$$(a_1 + 7)(a_1 + 16) > 7a_1 + 48 \Leftrightarrow a_1^2 + 16a_1 + 64 > 0 \Leftrightarrow (a_1 + 8)^2 > 0 \Leftrightarrow \boxed{a_1 \neq -8}$$

$$\Rightarrow (a_1 + 10d)(a_1 + 13d) < 7a_1 + 21d + 60 \Leftrightarrow a_1^2 + 16a_1 + 49 < 0. \Delta = 256 - 196 = 60$$

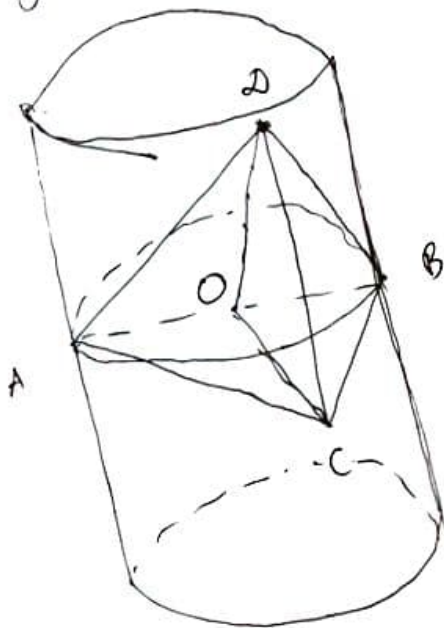
$$a_1 = \frac{-16 \pm \sqrt{60}}{2} = -8 \pm \sqrt{15} \Rightarrow \begin{array}{c} \text{p} \quad \text{p} \\ \text{---} \quad \text{---} \\ -8 - \sqrt{15} \quad -8 + \sqrt{15} \end{array} \Rightarrow a_1 \notin \mathbb{Z} \text{ (no yumburo)}$$

$$\Rightarrow \cancel{a_1 \in \mathbb{Z}} \quad a_1 = \{-11, -10, -9, -7, -6, -5\}$$

Ombem:

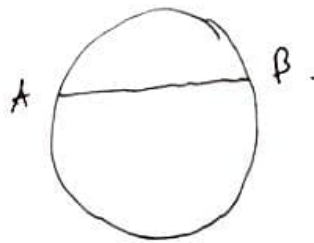
$a_1 = -11$	$a_1 = -9$	$a_1 = -6$
$a_1 = -10$	$a_1 = -7$	$a_1 = -5$

Задача № 2



Задача, что AB — хорда окружности
лежащий выше от окружности.

$$AB = 4.$$



$$AC = CB = 5; AD = DB = 6; CD \text{ парал-}$$

лельна оси цилиндра цилиндра. \Rightarrow AB лежит на окружности параллель-
ной основанию цилиндра. Тогда радиус цилиндра будет наименьшим
если AB — диаметр. $\Rightarrow r = \frac{AB}{2} = 2$. ~~АДС~~ Рассмотрим $\triangle ADC$;

$$AD + AC > CD \Leftrightarrow CD < 11; \quad AC + CD > AD \Leftrightarrow CD > 1.$$

Рассмотрим $\triangle ODC$, где O — середина AB. $\Rightarrow OC \perp AB$. т.к. $\triangle AOC = \triangle OBC$

$$(AC = CB, OC \text{ — общая сторона, } AO = OB \text{ (радиусы)}). \Rightarrow OC = \sqrt{AC^2 - AO^2} = \sqrt{25 - 4} = \sqrt{21}.$$

$$\triangle AOD = \triangle BOD. (OD \text{ — общая сторона, } AO = OB, AD = DB) \Rightarrow OD \perp AB. \Rightarrow \dots \Rightarrow$$

$$OD = \sqrt{AD^2 - AO^2} = \sqrt{32} \Rightarrow CD + \sqrt{21} > \sqrt{32}, \quad CD < \sqrt{32} + \sqrt{21}.$$

$$\text{Сравним } \sqrt{32} + \sqrt{21} \text{ и } 11 \Rightarrow 32 + 2\sqrt{32 \cdot 21} + 21 > 121 \Leftrightarrow \boxed{34 > \sqrt{32 \cdot 21}}$$

$$\Rightarrow CD < \sqrt{32} + \sqrt{21}. \text{ Сравним } 1 \text{ и } \sqrt{32} - \sqrt{21} \Leftrightarrow 2 < 6\sqrt{32 \cdot 21} \Leftrightarrow$$

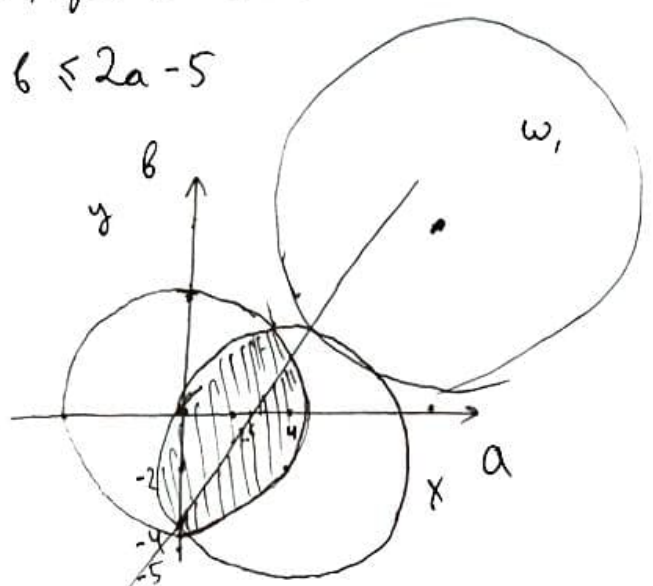
$$26 > \sqrt{32 \cdot 21} \Rightarrow CD > 1. \Rightarrow CD \in (1; \sqrt{32} + \sqrt{21})$$

$$\text{Ответ: } CD \in (1; \sqrt{32} + \sqrt{21})$$

Задача №3

$$\begin{cases} (x-a)^2 + (y-b)^2 \leq 20 \\ a^2 + b^2 \leq \min(8a-4b, 20) \end{cases} \Leftrightarrow \begin{cases} (a-x)^2 + (b-y)^2 \leq 20 \\ a^2 + b^2 \leq 8a - 4b, \text{ при } 8a - 4b < 20 \Leftrightarrow \\ a^2 + b^2 \leq 20, \text{ при } 8a - 4b \geq 20 \end{cases}$$

$$\Leftrightarrow \begin{cases} (a-x)^2 + (b-y)^2 \leq 20 \\ (a-4)^2 + (b+2)^2 \leq 20, \text{ при } b > 2a-5 \\ a^2 + b^2 \leq 20, \text{ при } b \leq 2a-5 \end{cases}$$

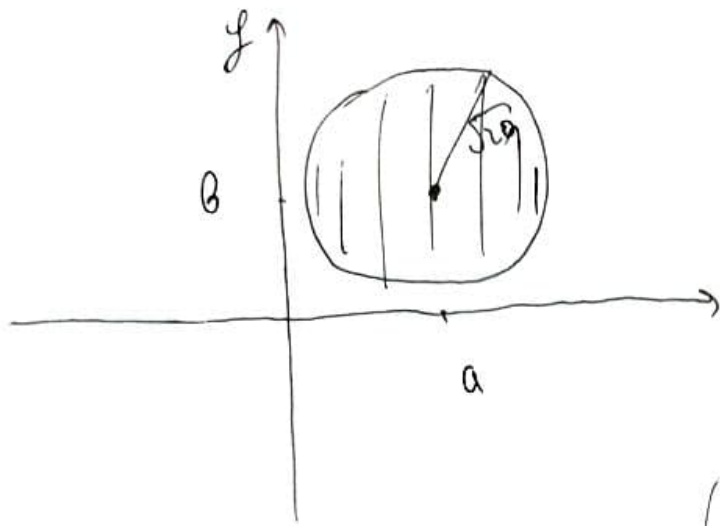


a, b - будуть циркульовати, если ω_1 будет касаться оси b ^{касается} или не-касается ^{или не-}
 переать ограниченную часть: $(x-4)^2 + (y+2)^2 \leq (\sqrt{20})^2$ - ~~определ~~

Площадь фигуры M - будет площадью окружности радиусом $\sqrt{20}$.

$$S = \pi R^2 = 20\pi$$

Ответ: $S = 20\pi$.

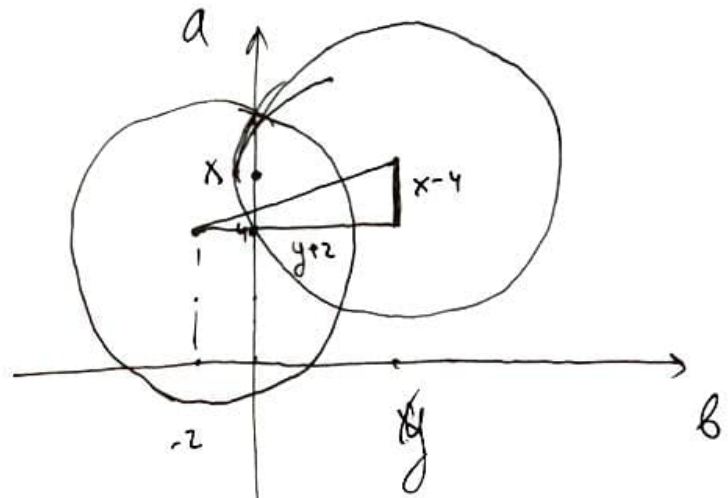
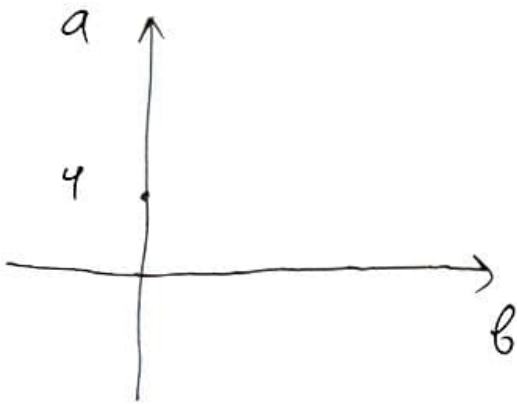


$$] \quad 8a - 4b < 20$$

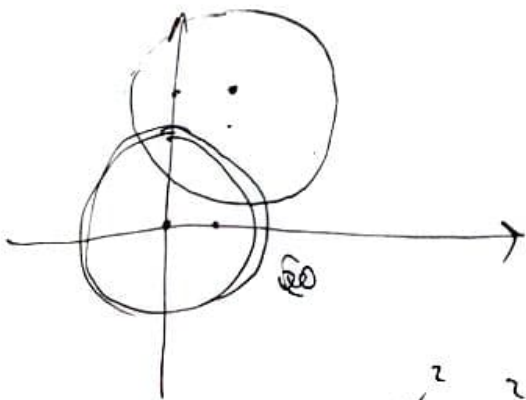
$$a^2 + b^2 < 8a - 4b$$

$$x \cdot a - 8a + 16 + b^2 + 4b + 4 < 20$$

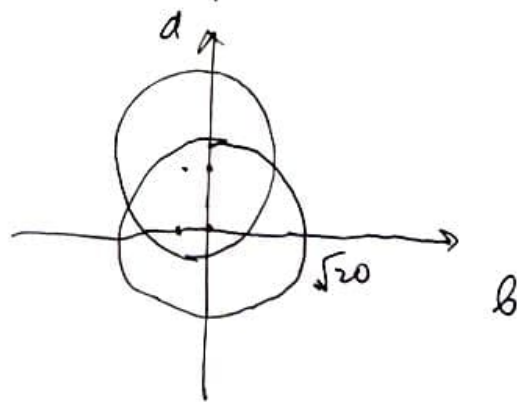
$$a - 8a + 16 + (a-4)^2 + (b+2)^2 < 20$$



$$(2\sqrt{20})^2 \leq (x-4)^2 + (y+2)^2$$



$$x^2 + y^2 < (2\sqrt{20})^2$$



$$a_1 + a_2 + \dots + a_7 = S$$

$$a_1 + (a_2 + d) + \dots + 7a_1 + \frac{(1+n-1) \cdot n-1}{2} \cdot d = \frac{n(n-1)d}{2} + n \cdot a_1$$

$$1+3+5+7 = 1 \cdot 4 + \frac{4(3) \cdot 2}{2} = 4 + 12 = 16$$

$$a_8 \cdot a_{17} > S + 27$$

$$a_{11} \cdot a_{14} < S + 60$$

$$(a_1 + 7d)(a_1 + 16d) > S + 27$$

$$(a_1 + 10d)(a_1 + 13d) < S + 60$$

$$a_1^2 + 23ad + 112d^2 > 7a_1 + \frac{7 \cdot 6 \cdot d}{2} = 7a_1 + 21d + 27$$

$$a_1^2 + 23ad + 130d^2 < 7a_1 + 21d + 60$$

$$a_1^2 + 23ad + 112d^2 + 7a_1 + 21d + 60 > 7a_1 + 21d + 27 + a_1^2 + 23ad + 130d^2$$

$$60 > 27 + 18d^2 \quad 33 > 18d^2$$

$$d > 0 \Rightarrow d < \sqrt{\frac{33}{18}} \Rightarrow \boxed{d=1}$$

$$\begin{array}{r} \times 16 \\ 16 \\ \hline 96 \\ 16 \\ \hline 256 \end{array} \quad 456 - 11$$

$$a_1^2 + 23a_1 + 112 > 27a_1 + 21 + 27$$

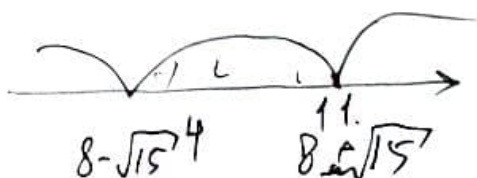
$$a_1^2 - 4a_1 + 64 > 0$$

$$a_1^2 + 23a_1 + 130 < 7a_1 + 21 + 60$$

$$a_1^2 + 16a_1 + 49 < 0$$

$$a_{12} = \frac{16 \pm \sqrt{60}}{2} = 8 \pm \sqrt{15}$$

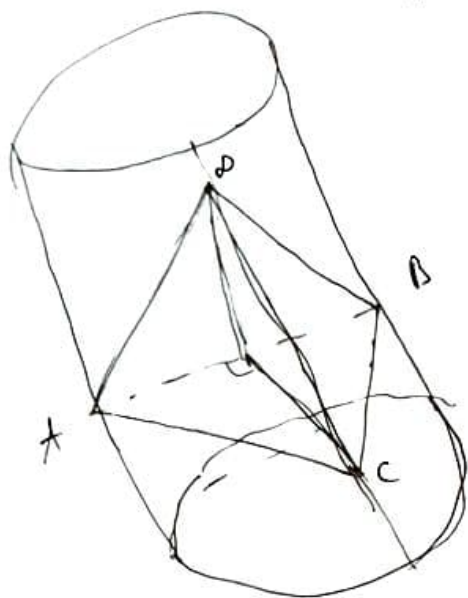
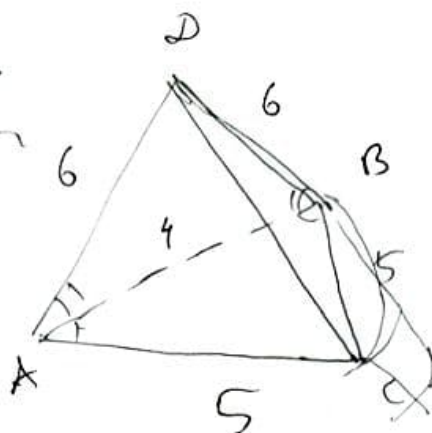
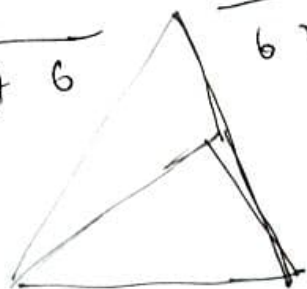
$$D = 256 - 196 = 60$$



$$\begin{array}{r} \times 26 \\ 26 \\ \hline 156 \\ 52 \\ \hline 676 \end{array}$$

$$\begin{array}{r} \times 32 \\ 21 \\ \hline 32 \\ 64 \\ \hline 672 \end{array}$$

4.8

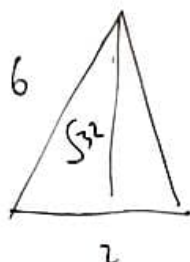
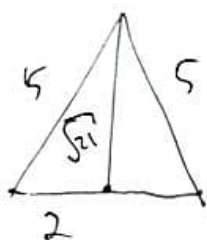


$$CD < 11$$

$$\begin{array}{r} \times 57 \\ 57 \\ \hline 399 \\ 285 \\ \hline 3249 \end{array}$$

$$\sqrt{32}$$

$$1 \vee 53 - 2\sqrt{32 \cdot 21}$$



$$CD < \sqrt{32} + \sqrt{21}$$

$$26 \vee \sqrt{32 \cdot 21}$$

$$5 + CD > 6$$

$$CD + \sqrt{21} > \sqrt{32}$$

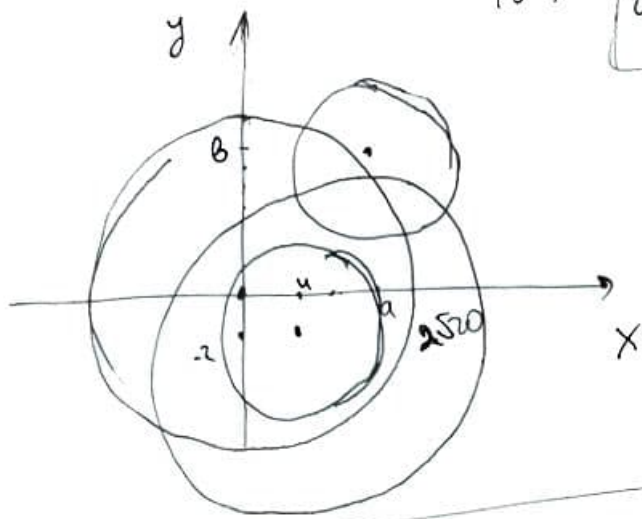
$$CD > 1$$

$$CD > \sqrt{32} - \sqrt{21}$$

$$\sqrt{32} + \sqrt{21} \vee 11$$

$$3 + 2\sqrt{32 \cdot 21} \vee 121$$

$$34 \vee \sqrt{32 \cdot 21}$$



if: $8a - 4b < 20$

$2a - b < 5$

elseif $8a - 4b > 20$

$2a - b > 5$

$\frac{+16}{7}$
 $\frac{112}{112}$

$S = a_1 + a_2 + \dots + a_7 = 7a_1 + \frac{7d \cdot 6}{2} = 7a_1 + 21d$

$a_8 = a_1 + 7d \Rightarrow (a_1 + 7d)(a_1 + 16d) > S + 27$
 $a_{17} = a_1 + 16d \Rightarrow (a_1 + 10d)(a_1 + 13d) < S + 60$
 $a_{11} = a_1 + 10d$
 $a_{14} = a_1 + 13d$

$a_1 \in \mathbb{Z}$
 $a_1 \in (-11; -5) \setminus \{-8\}$

$a_1^2 + 16da_1 + 7da_1 + 112d^2 > S + 27 = 7a_1 + 21d + 27$

$a_1^2 + 23ad + 130d^2 < S + 60$

$a_1^2 + 23ad + 112d^2 + 8 + 60 > a_1^2 + 23ad + 130d^2 + 8 + 27$

$d^2 < \frac{33}{18} \Rightarrow d^2 < 2 \Rightarrow d = 1$

$a_1^2 + 23a_1 + 112 > 7a_1 + 21 + 27$

$a_1^2 + 16a_1 + 64 \geq 0$

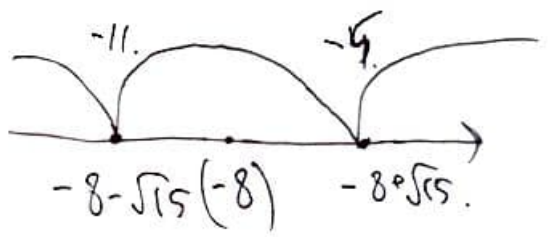
$(a_1 + 8)^2 > 0 \Rightarrow a_1 \neq -8$

$a_1^2 + 23a_1 + 130 < 7a_1 + 21 + 60$

$a_1^2 + 16a_1 + 49 < 0$

$D = 256 - 196 = 60$

$a_{12} = \frac{-16 \pm \sqrt{60}}{2} = -8 \pm \sqrt{15}$



$$\begin{cases} (x-a)^2 + (y-b)^2 \leq 20 \\ a^2 + b^2 \leq \min(8a-4b, 20) \end{cases}$$

$$\begin{cases} (x-a)^2 + (y-b)^2 \leq 20 \\ a^2 + b^2 \leq 8a-4b, \text{ if } 8a-4b < 20 \\ a^2 + b^2 \leq 20, \text{ if } 8a-4b \geq 20 \end{cases}$$

$$a^2 + b^2 \leq 8a-4b$$

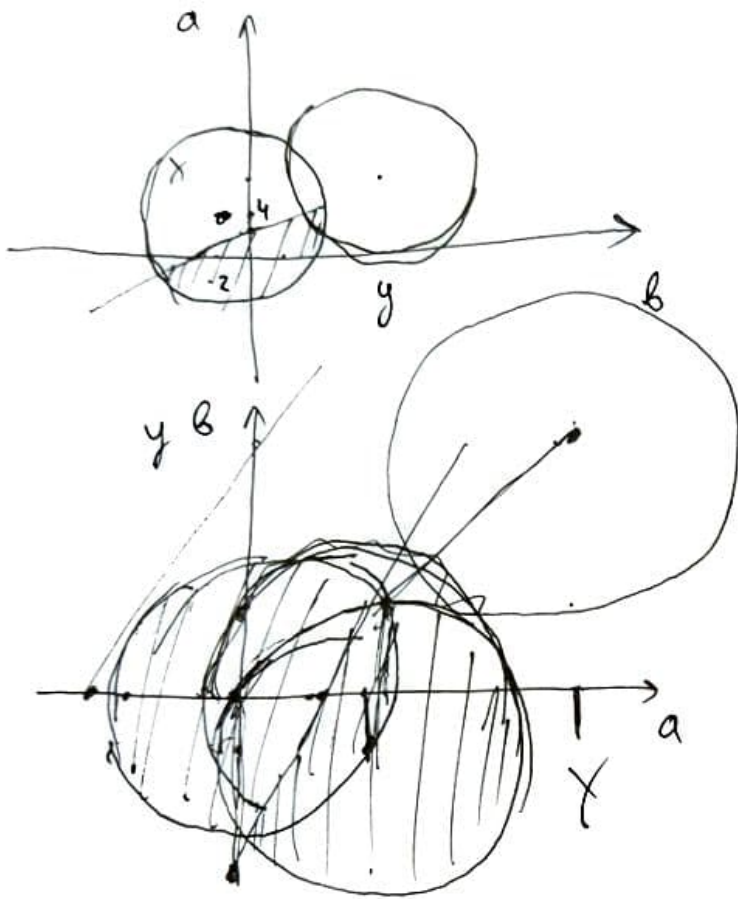
$$(a-4)^2 + (b+2)^2 \leq 20$$

$$a < \frac{b}{2} + \frac{5}{2}$$

$$4b > 8$$

$$b > 2a - 5$$

$$b \leq 2a - 5$$



$$\begin{aligned} & \cancel{(a-x)^2 + (b-y)^2} \\ & (x-4)^2 + (y+2)^2 \leq (\sqrt{20})^2 \end{aligned}$$

$$\begin{aligned} & \cancel{(y+2)^2 + (x-4)^2 \leq (2\sqrt{20})^2} \\ & \cancel{x^2 + y^2 \leq 2\sqrt{20}} \end{aligned}$$

Barisan 21. Zamb 1. Zumbuk

Zagara no 1

$$a_8 = a_1 + 7d$$

$$a_{12} = a_1 + 16d$$

$$a_{11} = a_1 + 10d$$

$$a_{14} = a_1 + 13d$$

$$a_8 \cdot a_{12} > S + 27 \Leftrightarrow (a_1 + 7d)(a_1 + 16d) > S + 27$$

$$a_{11} \cdot a_{14} < S + 60 \Leftrightarrow (a_1 + 10d)(a_1 + 13d) < S + 60$$

⇐

$$(a_1 + 7d)(a_1 + 16d) + S + 60 > S + 27 + (a_1 + 10d)(a_1 + 13d) \Leftrightarrow$$

$$a_1^2 + 23da_1 + 112d^2 + S + 60 > S + 27 + a_1^2 + 23da_1 + 130d^2 \Leftrightarrow$$

$$18d^2 < 33 \Leftrightarrow \text{m.k. } d > 0 \text{ u } d \in \mathbb{Z} \text{ (no yumburo)} \Rightarrow d < \sqrt{\frac{33}{18}} \Rightarrow$$

$$\boxed{d=1} \Rightarrow (a_1 + 7d)(a_1 + 16d) > S + 27; S = a_1 + a_2 + \dots + a_7 = 7a_1 + 21d \Rightarrow$$

$$(a_1 + 7)(a_1 + 16) > 7a_1 + 48 \Leftrightarrow a_1^2 + 16a_1 + 64 > 7a_1 + 48 \Leftrightarrow (a_1 + 8)^2 > 0 \Leftrightarrow \boxed{a_1 \neq -8}$$

$$\Rightarrow (a_1 + 10d)(a_1 + 13d) < 7a_1 + 21d + 60 \Leftrightarrow a_1^2 + 16a_1 + 49 < 0. \Delta = 256 - 196 = 60$$

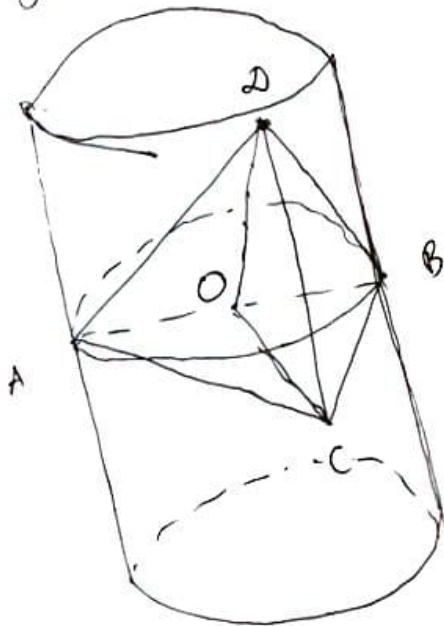
$$a_1 = \frac{-16 \pm \sqrt{60}}{2} = -8 \pm \sqrt{15} \Rightarrow \begin{array}{c} \text{p} \quad \text{p} \\ \text{---} \quad \text{---} \\ -8 - \sqrt{15} \quad -8 + \sqrt{15} \end{array} \Rightarrow a_1 \notin \mathbb{Z} \text{ (no yumburo)}$$

$$\Rightarrow \cancel{a_1 = -11, -10, -9, -7, -6, -5} \quad a_1 = \{-11, -10, -9, -7, -6, -5\}$$

Ombem:

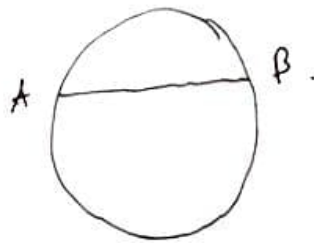
$a_1 = -11$	$a_1 = -9$	$a_1 = -6$
$a_1 = -10$	$a_1 = -7$	$a_1 = -5$

Задача № 2



Задача, что AB — хорда окружности
лежащий выше от окружности.

$AB = 4$.



$AC = CB = 5$; $AD = DB = 6$; CD парал-

лельна оси цилиндра. \Rightarrow AB лежит на окружности параллель-
ной основанию цилиндра. Тогда радиус цилиндра будет наименьшим
если AB — диаметр. $\Rightarrow r = \frac{AB}{2} = 2$. Также применим $\triangle ADC$;

$AD + AC > CD \Leftrightarrow CD < 11$; $AC + CD > AD \Leftrightarrow CD > 1$.

Применим $\triangle ODC$, где O — середина AB. $\Rightarrow OC \perp AB$. т.к. $\triangle AOC = \triangle OBC$

($AC = CB$, OC — общая сторона, $AO = OB$ (радиусы)). $\Rightarrow OC = \sqrt{AC^2 - AO^2} = \sqrt{25 - 4} = \sqrt{21}$.

$\triangle AOD = \triangle BOD$. (OD — общая сторона, $AO = OB$, $AD = DB$) $\Rightarrow OD \perp AB$. \Rightarrow

$OD = \sqrt{AD^2 - AO^2} = \sqrt{32} \Rightarrow CD + \sqrt{21} > \sqrt{32}$, $CD < \sqrt{32} + \sqrt{21}$.

Сравним $\sqrt{32} + \sqrt{21}$ с 11 $\Rightarrow 32 + 2\sqrt{32 \cdot 21} + 21 > 121 \Leftrightarrow 34 > \sqrt{32 \cdot 21}$

$\Rightarrow CD < \sqrt{32} + \sqrt{21}$. Сравним $1 > \sqrt{32} - \sqrt{21} \Leftrightarrow 2 > \sqrt{32 \cdot 21}$

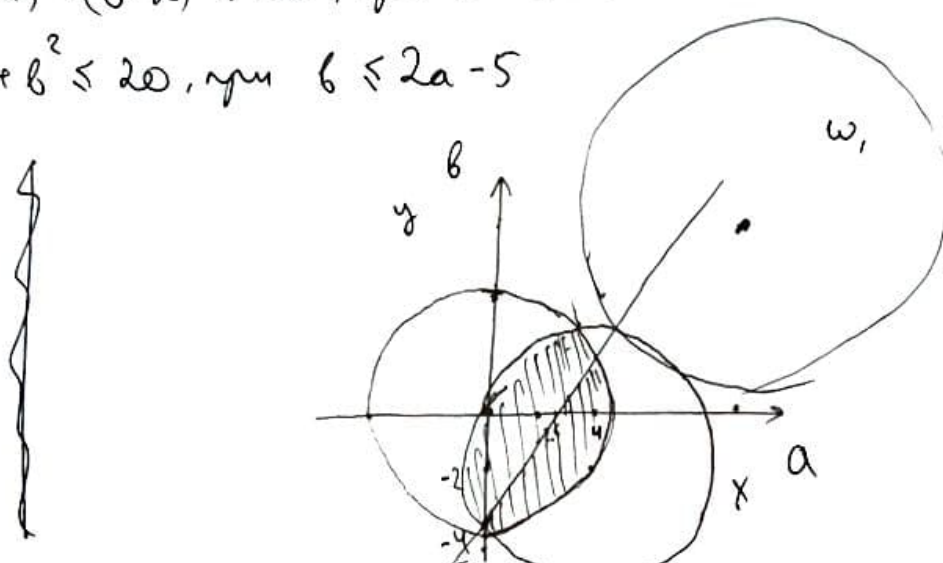
$2 > \sqrt{32 \cdot 21} \Rightarrow CD > 1 \Rightarrow CD \in (1; \sqrt{32} + \sqrt{21})$

Ответ: $CD \in (1; \sqrt{32} + \sqrt{21})$

Задача №3

$$\begin{cases} (x-a)^2 + (y-b)^2 \leq 20 \\ a^2 + b^2 \leq \min(8a-4b, 20) \end{cases} \Leftrightarrow \begin{cases} (a-x)^2 + (b-y)^2 \leq 20 \\ a^2 + b^2 \leq 8a - 4b, \text{ при } 8a - 4b \leq 20 \Leftrightarrow \\ a^2 + b^2 \leq 20, \text{ при } 8a - 4b \geq 20 \end{cases}$$

$$\Leftrightarrow \begin{cases} (a-x)^2 + (b-y)^2 \leq 20 \\ (a-4)^2 + (b+2)^2 \leq 20, \text{ при } b > 2a-5 \\ a^2 + b^2 \leq 20, \text{ при } b \leq 2a-5 \end{cases}$$

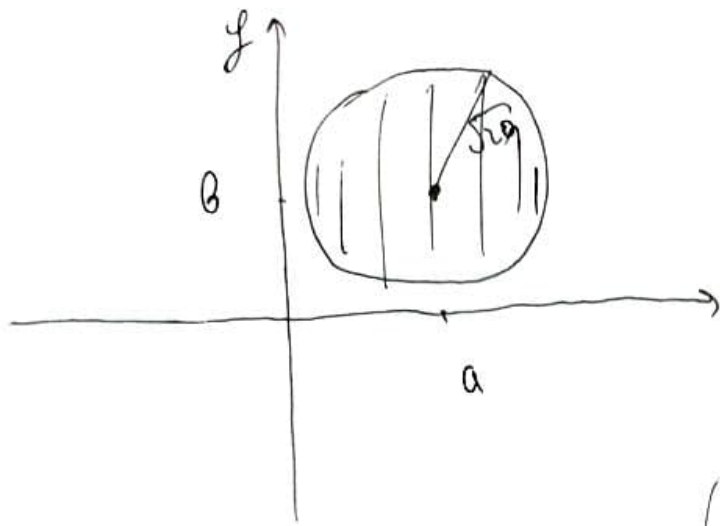


a, b - будуть суцільнобачити, если ω_1 будет хотя бы касаться или не-пересекать ограниченную часть: $(x-4)^2 + (y+2)^2 \leq (\sqrt{20})^2$ - ~~определен~~

Площадь фигуры M - будет площадью окружности радиусом $\sqrt{20}$.

$$S = \pi R^2 = 20\pi$$

Ответ: $S = 20\pi$.

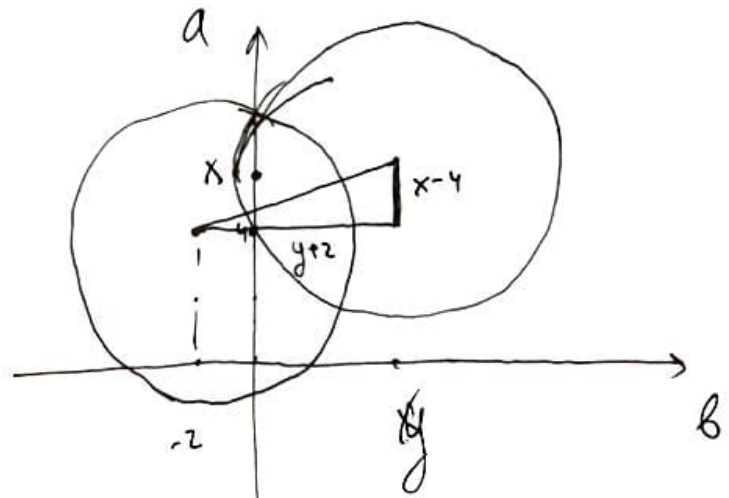
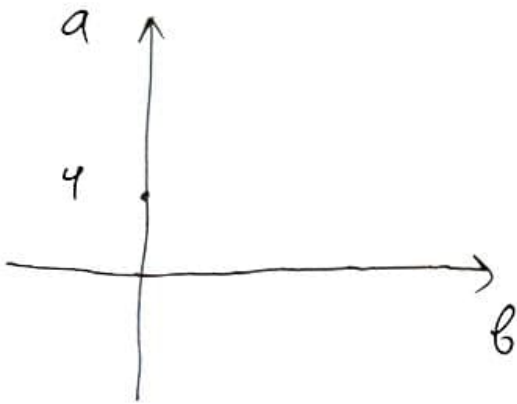


$$] \quad 8a - 4b < 20$$

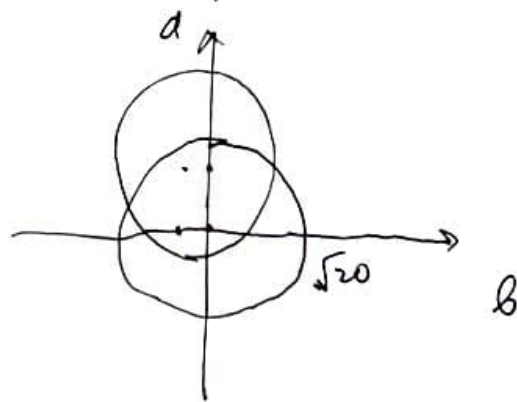
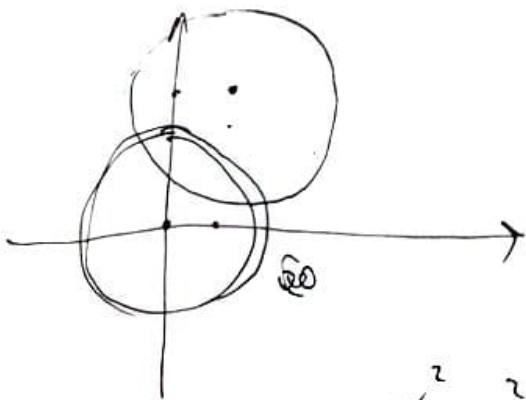
$$a^2 + b^2 < 8a - 4b$$

$$x \cdot a - 8a + 16 + b^2 + 4b + 4 < 20$$

$$a - 8a + 16 + (a-4)^2 + (b+2)^2 < 20$$



$$(2\sqrt{20})^2 \leq (x-4)^2 + (y+2)^2$$



$$x^2 + y^2 < (2\sqrt{20})^2$$

$$a_1 + a_2 + \dots + a_7 = S$$

$$a_1 + (a_2 + d) + \dots + 7a_1 + \frac{(1+n-1) \cdot n-1}{2} \cdot d = \frac{n(n-1)d}{2} + n \cdot a_1$$

$$1+3+5+7 = 1 \cdot 4 + \frac{4(3) \cdot 2}{2} = 4 + 12 = 16$$

$$a_9 \cdot a_{17} > S + 27$$

$$a_{11} \cdot a_{19} < S + 60$$

$$(a_1 + 7d)(a_1 + 16d) > S + 27$$

$$(a_1 + 10d)(a_1 + 13d) < S + 60$$

$$a_1^2 + 23ad + 112d^2 > 7a_1 + \frac{7 \cdot 6 \cdot d}{2} = 7a_1 + 21d + 27$$

$$a_1^2 + 23ad + 130d^2 < 7a_1 + 21d + 60$$

$$a_1^2 + 23ad + 112d^2 + 7a_1 + 21d + 60 > 7a_1 + 21d + 27 + a_1^2 + 23ad + 130d^2$$

$$60 > 27 + 18d^2 \quad 33 > 18d^2$$

$$d > 0 \Rightarrow d < \sqrt{\frac{33}{18}} \Rightarrow \boxed{d=1}$$

$$\begin{array}{r} \times 16 \\ 16 \\ \hline 96 \\ 16 \\ \hline 256 \end{array} \quad 456 - 11$$

$$a_1^2 + 23a_1 + 112 > 27a_1 + 21 + 27$$

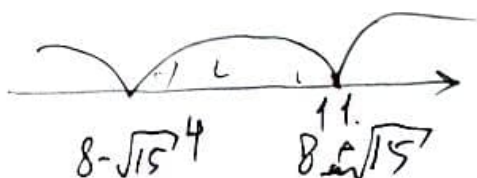
$$a_1^2 - 4a_1 + 64 > 0$$

$$a_1^2 + 23a_1 + 130 < 7a_1 + 21 + 60$$

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$$a_{12} = \frac{16 \pm \sqrt{60}}{2} = 8 \pm \sqrt{15}$$

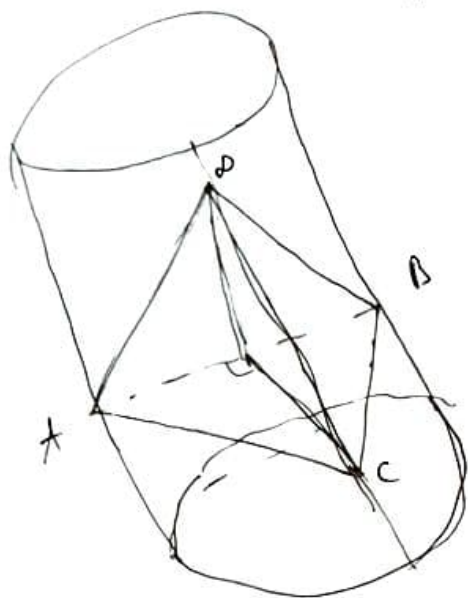
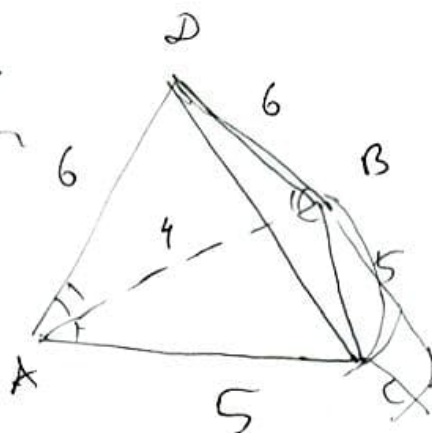
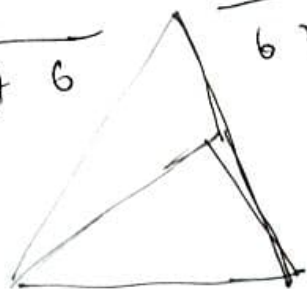
$$D = 256 - 196 = 60$$



$$\begin{array}{r} \times 26 \\ 26 \\ \hline 156 \\ 52 \\ \hline 676 \end{array}$$

$$\begin{array}{r} \times 32 \\ 21 \\ \hline 32 \\ 64 \\ \hline 672 \end{array}$$

4.8

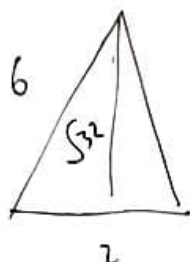
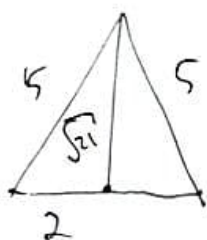


$$CD < 11$$

$$\begin{array}{r} \times 57 \\ 57 \\ \hline 399 \\ 285 \\ \hline 3249 \end{array}$$

$$\sqrt{32}$$

$$1 \vee 53 - 2\sqrt{32 \cdot 21}$$



$$CD < \sqrt{32} + \sqrt{21}$$

$$26 \vee \sqrt{32 \cdot 21}$$

$$5 + CD > 6$$

$$CD + \sqrt{21} > \sqrt{32}$$

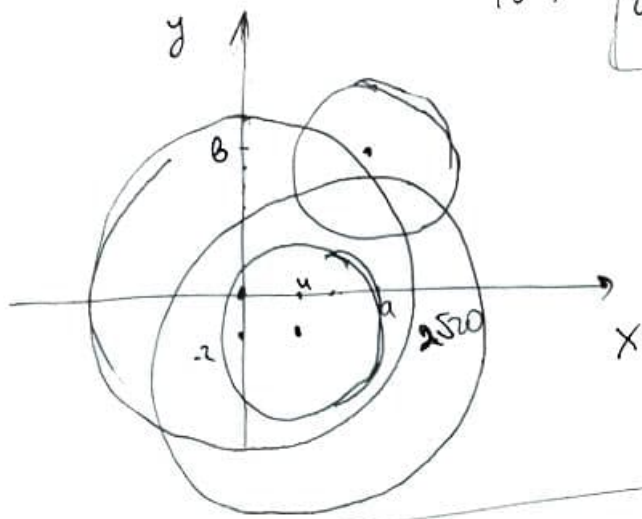
$$CD > 1$$

$$CD > \sqrt{32} - \sqrt{21}$$

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$$34 \vee \sqrt{32 \cdot 21}$$



if: $8a - 4b < 20$

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$\frac{+16}{7}$
 $\frac{112}{112}$

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$a_1 \in \mathbb{Z}$
 $a_1 \in (-11; -5) \cup (-8)$

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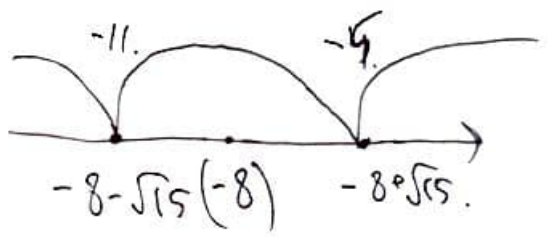
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$$\begin{cases} (x-a)^2 + (y-b)^2 \leq 20 \\ a^2 + b^2 \leq 8a-4b, \text{ if } 8a-4b < 20 \\ a^2 + b^2 \leq 20, \text{ if } 8a-4b \geq 20 \end{cases}$$

$$a^2 + b^2 \leq 8a-4b$$

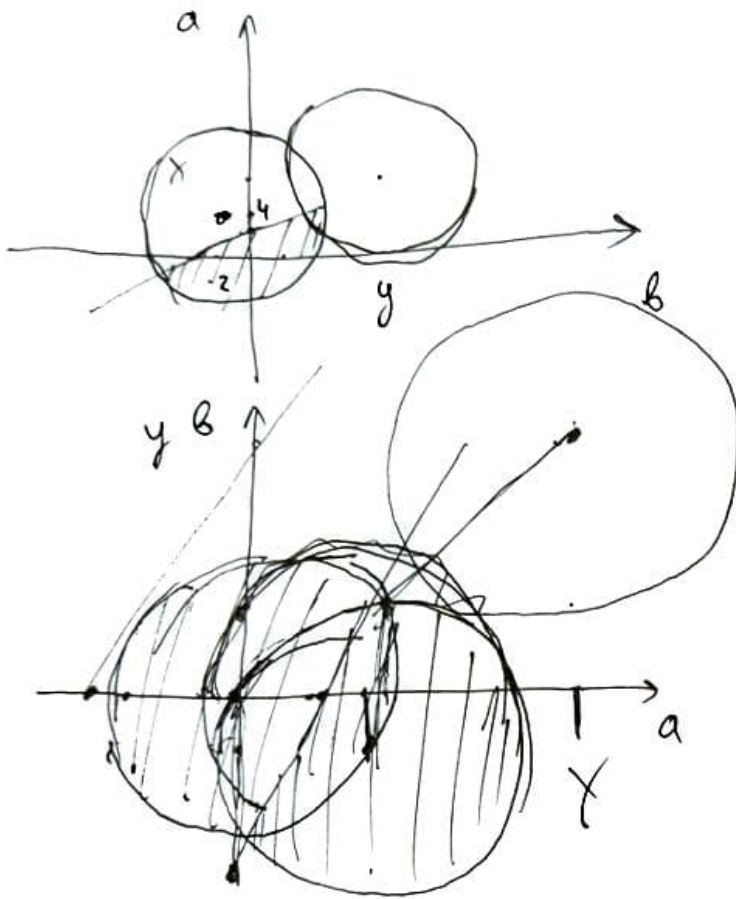
$$(a-4)^2 + (b+2)^2 \leq 20$$

$$a < \frac{b}{2} + \frac{5}{2}$$

$$4b > 8$$

$$b > 2a - 5$$

$$b \leq 2a - 5$$



$$\begin{aligned} & \cancel{(a-x)^2 + (b-y)^2} \\ & (x-4)^2 + (y+2)^2 \leq (\sqrt{20})^2 \end{aligned}$$

$$\begin{aligned} & \cancel{(y+2)^2 + (x-4)^2 \leq (2\sqrt{20})^2} \\ & \cancel{x^2 + y^2 \leq 2\sqrt{20}} \end{aligned}$$

Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21101289**

ID профиля: **74985**

Вариант 21

Баърам 21. Заам 2. Теъмобух

Загара №4

$$\begin{cases} \text{НОД}(a, b, c) = 35 \\ \text{НОК}(a, b, c) = 5^{18} \cdot 7^{36} \end{cases} \rightarrow \begin{cases} a = 35 \cdot p \\ b = 35 \cdot q \\ c = 35 \cdot k \end{cases} \text{, шундан } \text{НОД}(p, q, k) = 1$$

$$\Rightarrow \text{НОК}(a, b, c) = 5^{18} \cdot 7^{36} = 35 \cdot p \cdot q \cdot k \Leftrightarrow p \cdot q \cdot k = 5^{17} \cdot 7^{35}$$

$$\begin{array}{ccc} \underline{5^{17}} & \underline{7^{35}} & \underline{7^{35-i}} \\ & \rightarrow & \rightarrow 36 \text{ баърамвор} \end{array}$$

$$\begin{array}{ccc} \underline{5^{16}} & \underline{7^i \cdot 5^1} & \underline{7^{35-i}} \\ & \rightarrow & \rightarrow 36 \cdot 2 \text{ баър.} \end{array}$$

$$\begin{array}{ccc} \underline{5^{15}} & \underline{7^i \cdot 5^2} & \underline{7^{35-i}} \\ & \rightarrow & \rightarrow 36 \cdot 3 \text{ баър.} \end{array}$$

⋮

$$\begin{array}{ccc} \underline{5} \cdot \underline{7^i \cdot 5^{16}} & \underline{7^{35-i}} & \rightarrow 36 \cdot 17 \text{ баър.} \end{array}$$

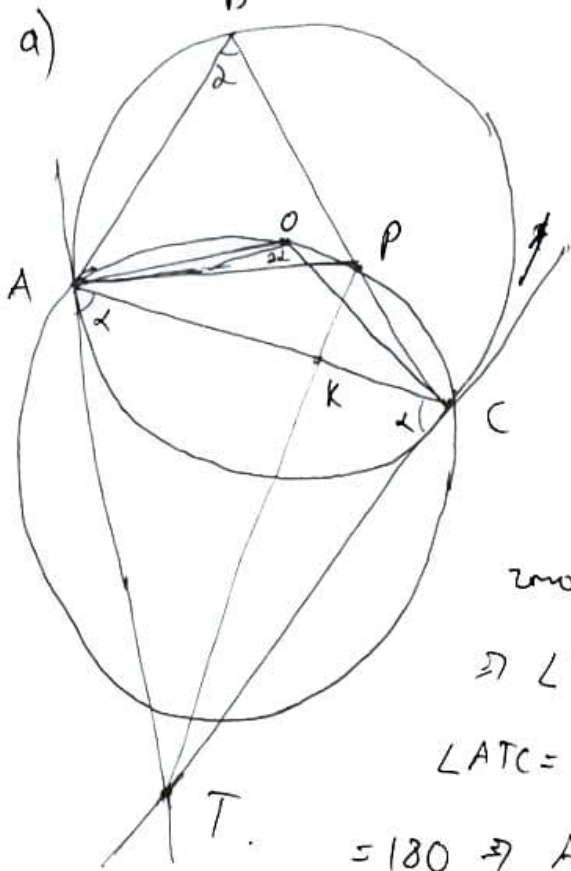
⌈

Бешта баърамвор: $36 \cdot (1+2+\dots+17) \cdot 6$ (шундан $каВ, мк.$
 $(a, b, c) (b, a, c) (b, c, a) (a, c, b) (c, a, b) (c, b, a)$ — рағзона)

$$\text{Жавоб: } 36 \cdot (1+2+\dots+17) \cdot 6 = 33048$$

Вариант 21. Часть 2. Задача 6

Задача №6



$$S_{APK} = 12$$

$$S_{CPK} = 9$$

Докажем, что T - центр на окружности симметричной относительно биссектрисы $\angle AOC$. $\angle ABC = \delta \Rightarrow$

$$\angle AOC = 2\delta \text{ (центральный угол)}$$

$\triangle AOC$ - равнобедренный ($AO = OC$) (радиусы). \Rightarrow

$$\angle OCA = \angle OAC = \frac{180 - 2\delta}{2} = 90 - \delta. \text{ Заметим,}$$

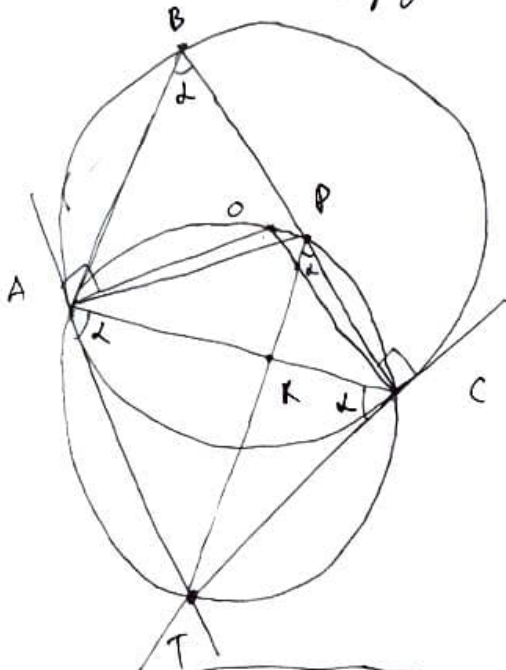
что $OC \perp CT$ и $AO \perp AT$ (радиусы к касательным).

$$\Rightarrow \angle TAC = \angle TCA = 90 - \angle OCA = 90 - \angle OAC = \delta. \Rightarrow$$

$$\angle ATC = 180 - \delta - \delta = 180 - 2\delta. \Rightarrow \angle AOC + \angle ATC = 2\delta + 180 - 2\delta =$$

$$= 180 \Rightarrow AOC - \text{вписанный четырехугольник.} \Rightarrow$$

T - центр на этой окружности, симметричной:



$$\angle TPC = \angle TAC \text{ (вписанные углы)} \Rightarrow$$

рассмотрим $\triangle ABC$ и $\triangle KPC$:

$$\angle KCP - \text{общий} \quad \angle KPC = \delta = \angle ABC \Rightarrow$$

$$\triangle ABC \sim \triangle KPC \Rightarrow S_{ABC} = S_{KPC} \cdot K^2$$

рассмотрим $\triangle APC$:

$$S_{KPC} = h \cdot \frac{KC}{2} = 9$$

$$S_{APK} = h \cdot \frac{AK}{2} = 12 \Rightarrow$$

$$\frac{KC}{AK} = \frac{9}{12} \Leftrightarrow \boxed{KC = AK \cdot \frac{9}{12}} \Rightarrow AC = AK + KC = \frac{7}{4} AK \Rightarrow K = \frac{AC}{KC} = \frac{7}{3} \Rightarrow$$

$$\boxed{S_{ABC} = S_{KPC} \cdot K^2 = 9 \cdot \frac{49}{9} = 49}$$

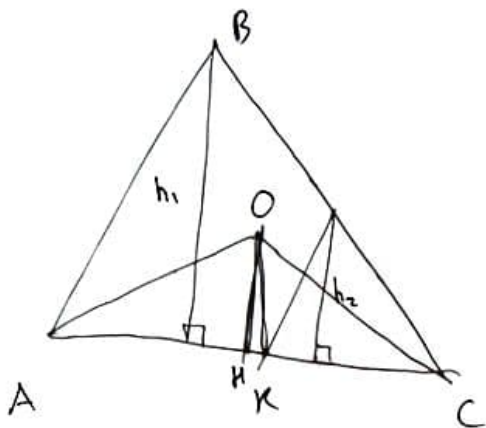
Вариант 21. Задача. Умножить

Задача № 6 (продолжение)

д) $\angle ABC = \alpha = \arctg \frac{3}{7}$ $S_{ABC} = S_{ABK} + S_{CBK} = 21$

$$h_1 = \frac{AC}{2} = 49 \Rightarrow \frac{h_1}{h_2} = \frac{7}{3}$$

$$h_2 = \frac{KC}{2} = 9$$



$OK = \frac{1}{2} BC$ (свойство) $\Rightarrow \angle KOC = \alpha$ (свойство).

$$\text{tg } \alpha = \frac{AC}{2h_2} \Rightarrow \frac{AC}{2 \cdot \frac{18}{\frac{3}{7}AC}} \Rightarrow AC = \sqrt{\frac{7}{3} \text{tg } \alpha \cdot 2 \cdot 18} = 6$$

Ответ: а) $S_{ABC} = 49$

б) $AC = 6$

Бауырым 21. Тапсырма 2. Түрлендіру

Зағара №5

$$\log_{\sqrt{2x-3}}(x+1), \log_{2x^2-3x+5}(2x-3), \log_{x+1}(2x^2-3x+5)$$

$$\begin{aligned} a &= 2x-3 \\ b &= x+1 \\ c &= 2x^2-3x+5 \end{aligned} \Rightarrow \begin{aligned} &2\log_a b, 2\log_c a, \log_b c \end{aligned}$$

ОДЗ:

$$\begin{aligned} x &> \frac{3}{2} \\ 2x^2-3x+5 &> 0 \\ 2x^2-3x+5 &\neq 1 \\ x &\neq 0 \\ x &> -1 \\ x &\neq 2 \end{aligned}$$

$$\begin{cases} 2\log_a b = 2\log_c a \\ \log_b c + 1 = 2\log_c a \end{cases} \Leftrightarrow \begin{cases} \frac{\ln b}{\ln a} = \frac{\ln a}{\ln c} \\ \frac{\ln c + \ln b}{\ln b} = 2\frac{\ln a}{\ln c} \end{cases} \Rightarrow$$

$$(\ln b + \ln c) \cdot \ln c = 2 \ln a \cdot \ln b \Leftrightarrow \ln^2 a \cdot \ln c + \ln^3 c = 2 \ln^3 a$$

$$\begin{aligned} \ln a &\equiv t \\ \ln c &\equiv m \end{aligned} \Rightarrow t^2 m + m^3 = 2t^3. \quad \boxed{t=m} \text{ - абстрация көрсетеді } \Rightarrow$$

$$2t^3 - t^2 m - m^3 = (t-m)(2t^2 + mt + m^2) = 0. \Leftrightarrow \begin{cases} t=m \\ 2t^2 + mt + m^2 = 0 \end{cases} \Rightarrow$$

$$D = m^2 - 8m^2 = -7m^2 < 0. \Rightarrow \boxed{t=m} \Leftrightarrow \ln a = \ln c \Leftrightarrow a=c. \Leftrightarrow$$

$$2x-3 = 2x^2-3x+5 \Leftrightarrow 2x^2-5x+8=0 \quad D=25-64 < 0. \text{ көрсеткенінше}$$

$$\begin{cases} 2\log_a b = \log_b c \\ \log_b c = 2\log_c a + 1 \end{cases} \Leftrightarrow \begin{cases} \frac{2\ln b}{\ln a} = \frac{\ln c}{\ln b} \\ \frac{\ln c}{\ln b} = \frac{2\ln a + \ln c}{\ln c} \end{cases} \Rightarrow$$

$$\ln^2 c = 2\ln a \cdot \ln b + \ln c \cdot \ln b \Leftrightarrow \ln^3 c = 4\ln^3 b + \ln^2 c \ln b \quad | \quad t = \ln c, \quad m = \ln b.$$

$$t = 2m \text{ - көрсеткенінше } \Rightarrow t^3 - mt^2 - 4m^3 = (t-2m)(t^2 + mt + 2m^2) = 0. \Leftrightarrow$$

$$\begin{cases} t=2m \\ t^2 + mt + 2m^2 \end{cases} \Rightarrow D = m^2 - 8m^2 < 0 \Rightarrow t=2m \Leftrightarrow \ln c = \ln b^2 \Leftrightarrow 2x^2-3x+5 = x^2+2x+1 \Leftrightarrow$$

$$x^2 - 5x + 4 = 0 \Rightarrow D = 25 - 16 = 9 \quad x_{1,2} = \frac{5 \pm 3}{2} = \begin{cases} 4 \\ 1 \end{cases} \Rightarrow \boxed{x=4}$$

сұрақ. 4

Бапуанон 21. Таамб 2. Теоморук.

Загара №5 (нгуогаменне)
И уонб

$$\begin{cases} 2 \log_c a = \log_b c \\ \log_b c = 2 \log_a b + 1 \end{cases} \Leftrightarrow \begin{cases} \frac{2 \ln a}{\ln c} = \frac{\ln c}{\ln b} \\ \frac{\ln c}{\ln b} = \frac{2 \ln b + \ln a}{\ln a} \end{cases} \Rightarrow$$

$$\ln c \cdot \ln a = (2 \ln b + \ln a) \ln b = 2 \ln^2 b + \frac{\ln^2 c}{2} = \ln c \cdot \frac{\ln^2 c}{2 \ln b} \Leftrightarrow$$

$$4 \ln^3 b + \ln^2 c \ln b = \ln^3 c \quad \left| \begin{array}{l} t = \ln c \\ m = \ln b \end{array} \Rightarrow \right.$$

$$t^3 - mt^2 - 4m^3 = 0 \quad t = 2m - \text{корень} \Rightarrow t^3 - mt^2 - 4m^3 = (t - 2m)(t^2 + mt + 2m^2)$$

$$= 0 \Leftrightarrow \begin{cases} t = 2m \\ t^2 + mt + 2m^2 = 0 \end{cases} \Rightarrow D = m^2 - 8m^2 < 0 \Rightarrow t = 2m \quad \ln c = 2 \ln b \Leftrightarrow \boxed{x = 4}$$

(См. нгуогууун оубер).

Оубер: $\boxed{x = 4}$

$$\frac{2\ln a}{\ln c} = \frac{\ln c}{\ln b}$$

$$\frac{\ln c}{\ln b} = \frac{2\ln b + \ln a}{\ln a}$$

$$\ln c \cdot \ln a = 2\ln^2 b + \ln a \cdot \ln b = 2\ln^2 b + \frac{\ln^2 c}{2} = \ln c \cdot \frac{\ln^2 c}{2\ln b}$$

$$2\ln a \cdot \ln b = \ln^2 c$$

$$4\ln^2 b + \ln^2 c = \frac{\ln^3 c}{\ln b}$$

$$4\ln^2 b \cdot \ln b + \ln^2 c \cdot \ln b = \ln^3 c$$

$$h_1 \cdot \frac{Ac}{2} = 49$$

$$t^3 - mt^2 - 4m^3 = 0 \quad (t = 2m)$$

$$h_2 \cdot \frac{Kc}{2} = 9 \quad \frac{h_1}{h_2} = \frac{7}{3}, \frac{49}{9}$$

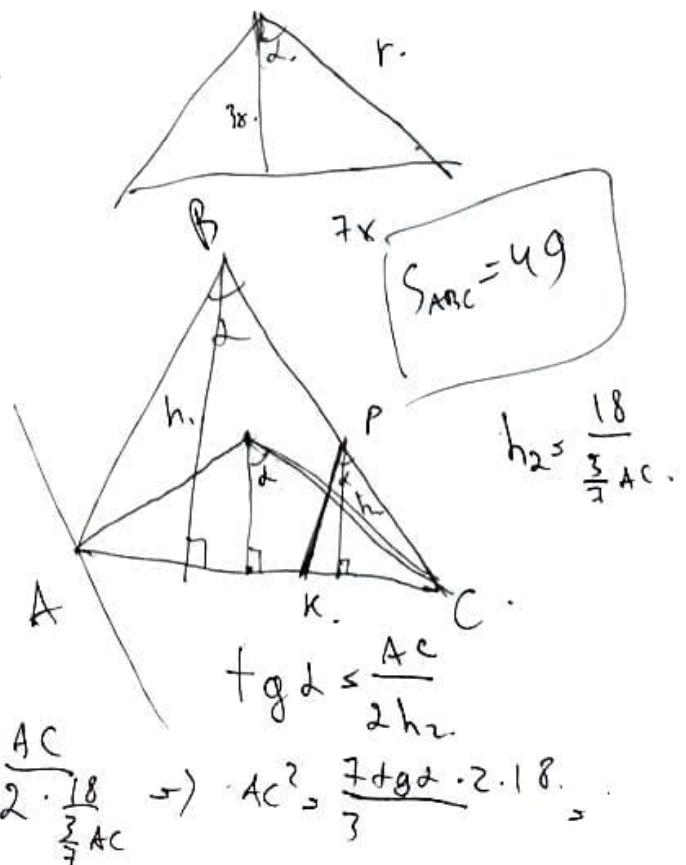
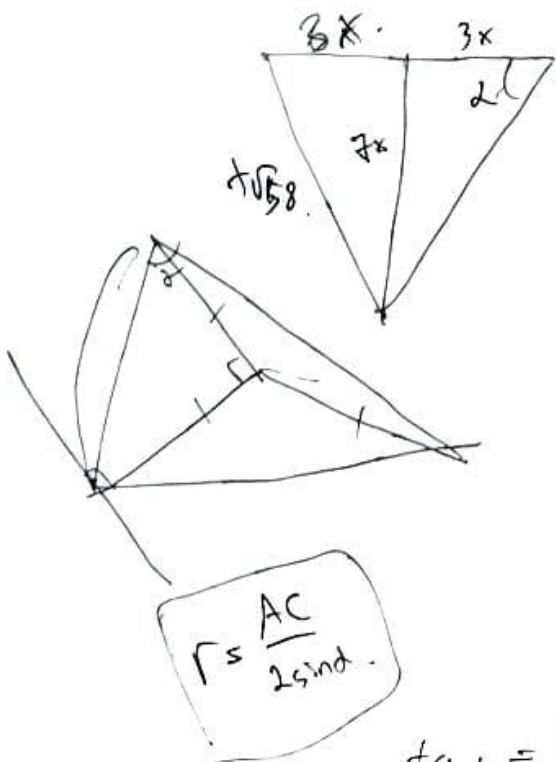
$$\frac{t^3 - mt^2 - 4m^3}{t^3 - 2mt^2} \left| \begin{array}{l} t-2m \\ (t^2 + mt + 2m^2)(t-2m) = 0 \end{array} \right.$$

$$\frac{h_1}{h_2} = \frac{7}{3} \quad AC = \frac{7}{3} Kc$$

$$\frac{2mt^2 - 4m^3}{mt^2 - 2m^2t} = \frac{2m^2t - 4m^3}{2m^2t - 4m^3}$$

$$\ln c = 2\ln b$$

$$r = \sqrt{58}$$



$$2 \log_a b = \log_a^2 C = \frac{2 \ln b}{\ln a} = \frac{\ln C}{\ln b} \quad \ln a = \frac{2 \ln^2 b}{\ln C}$$

$$\log_b C = 2 \log_c a + 1 = \frac{\ln C}{\ln b} = \frac{2 \ln a + \ln C}{\ln C}$$

$$\ln^2 C = 2 \ln a \cdot \ln b + \ln C \cdot \ln b$$

$$\ln^2 C = 2 \cdot \frac{2 \ln^3 b}{\ln C} + \ln^2 C \cdot \ln b \quad \ln^3 C = 4 \ln^3 b + \ln^2 C \ln b$$

$$\ln C = 2 \ln b$$

$$\ln^3 C - 4 \ln^3 b - \ln^2 C \ln b$$

$$8 - 4 - 4$$

$$\textcircled{A} \quad t^3 - 4m^3 - t^2 m \quad | \quad t - 2m$$

$$\cdot \quad \frac{t^3 - 2mt^2}{(t^2 + mt + 2m^2)(t - 2m)} = 0$$

$$\frac{\cancel{t^3} \quad mt^2 - 4m^3}{mt^2 - 2m^2 t}$$

$$\cancel{t^3}$$

$$t = 2m$$

$$2m^2 - 4m^3$$

$$2m^2 - 4m^3$$

$$\ln C = \ln 9 b^2$$

$$C = b^2$$

$$2x^2 - 3x + 5 = x^2 + 2x + 1$$

$$x^2 - 5x + 4 = 0$$

$$D = 25 - 16 = 9$$

$$x_{1,2} = \frac{5 \pm 3}{2} = \textcircled{4}$$

$$\log_{\sqrt{2x-3}}(x+1)$$

$$\log_{2x^2-3x+5} (2x-3)^2 \log_{x+1} (2x^2-3x+5)$$

$$x > -1 \quad x \neq 0$$

$$x \neq \frac{3}{2}$$

$$2x^2 - 3x + 5 > 0$$

$$D = 9 - 40$$

$$\sqrt{2x-3} = a$$

$$x+1 = b$$

$$2x^2 - 3x + 5 = c$$

$$\ln a \neq 0$$

$$\ln c \neq 0$$

$$\begin{cases} 2 \log_a b = \log_c a^2 \\ \log_b c + 1 = 2 \log_c a \end{cases} \Rightarrow \log_a b = \log_c a = \frac{\ln b}{\ln a} = \frac{\ln a}{\ln c} \Rightarrow$$

$$\ln^2 a = \ln b \cdot \ln c \quad \frac{\ln c}{\ln b} + \frac{\ln b}{\ln c} = 2 \frac{\ln a}{\ln c} = \frac{\ln(bc)}{\ln b} = \frac{2 \ln a}{\ln c}$$

$$(\ln b + \ln c) \ln c = 2 \ln a \cdot \ln b$$

$$\ln^2 a + \ln^2 c = 2 \ln a \cdot \frac{\ln^2 a}{\ln c}$$

$$\ln^2 a \ln c + \ln^3 c = 2 \ln^3 a$$

$$\begin{aligned} t &= \ln a \\ -m &= \ln c \end{aligned}$$

$$t^2 \cdot m + m^3 = 2t^3$$

$$(2t^2 + mt + m^2)(t - m) = 0$$

$$\frac{2t^3 - t^2 m = m^3}{2t^3 - 2t^2 m} = \frac{m^3}{2t^2 + mt + m^2}$$

$$D = m^2 - 8m^2 < 0$$

$$t = m$$

$$\frac{t^2 m - m^3}{t^2 m - m^2 t}$$

$$\ln a = \ln c$$

$$m^2 t - m^3$$

$$a = c$$

$$2x-3 = 2x^2-3x+5$$

$$2x^2 - 5x + 8 = 0$$

$$D = 25 -$$

~~a.b.c~~

$$a = 35p, \quad 35p \cdot kq = 5^{18} \cdot 7^{36}$$

$$b = 35k$$

$$c = 35q, \quad p \cdot k \cdot q = 5^{17} \cdot 7^{35}$$

18.

$$\text{HOD}(p, k, q) = 1$$

$$\boxed{18 \cdot 36 \cdot 3} = 6^3 \cdot 3^2$$

1 + 17 =

$$\frac{(1+17) \cdot 17}{2} = 9 \cdot 17$$

$$5^{17} \cdot 7^3$$

$$5^{16} \cdot 7^k \cdot 5$$

$$5^{15} \cdot 7^k \cdot 5^2$$

$$5^{14} \cdot 7^k \cdot 5^3$$

$$5^k \cdot 7 \cdot 5^{16} \cdot 7^{35-k}$$

$$7^k \cdot 5^{17}$$

36

$$36 \cdot 2$$

$$36 \cdot 3$$

$$36 \cdot 4$$

$$36 \cdot 17$$

$$36 \cdot 18$$

$$\boxed{36 \cdot \sum_{i=1}^{18} i \cdot 3}$$

$$\begin{array}{r} \times 36 \\ 216 \end{array}$$

$$\begin{array}{r} \times 153 \\ \hline 918 \\ 153 \\ 306 \\ \hline 33048 \end{array}$$

$$\alpha = \arctg \frac{3}{7}$$

$$2\alpha = 2 \arctg \frac{3}{7}$$

$$S_{AOC} = 21$$

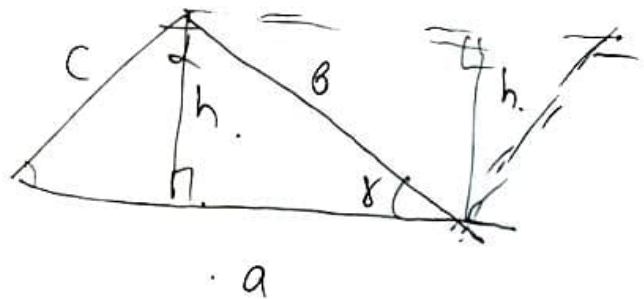
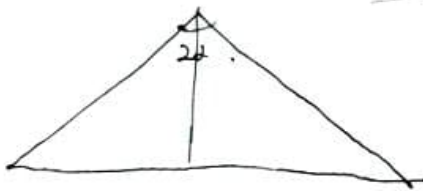
$$r = \frac{AC}{2 \sin \alpha}$$

$$S_{AOC} = \frac{AC \cdot h}{2} = \frac{AC \cdot r \cos \alpha}{2} = 21 = \frac{AC \cdot \frac{AC}{2 \sin \alpha} \cdot \cos \alpha}{2} = 21$$

$$AC^2 \cdot \frac{\operatorname{ctg} \alpha}{4} = 21$$

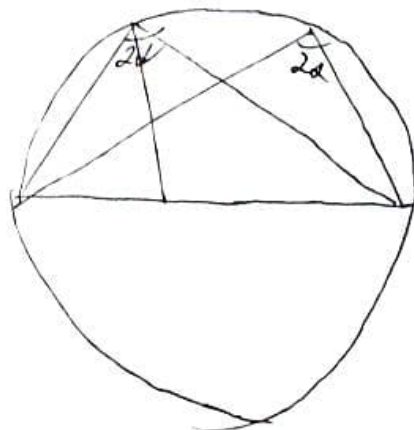
$$84 \cdot \operatorname{tg} \alpha = AC^2$$

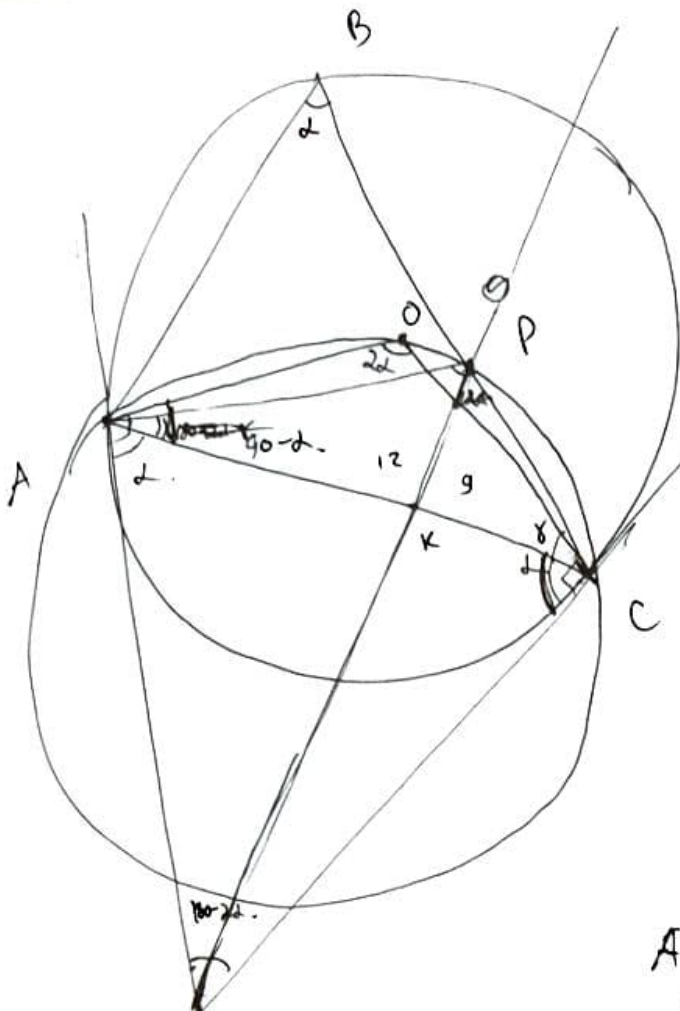
$$AC = \sqrt{84 \cdot \frac{3}{7}} = 6$$



$$\frac{ah}{2} = b \sin \gamma = c \cdot \sin(\alpha + \gamma)$$

$$d^2 = b^2 + c^2 - 2bc \cdot \cos \alpha$$



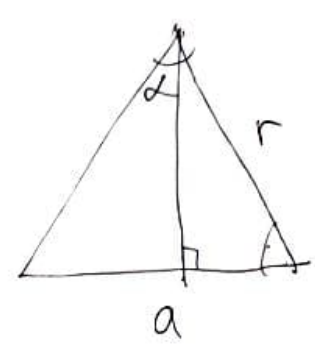
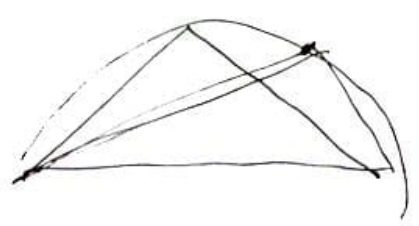
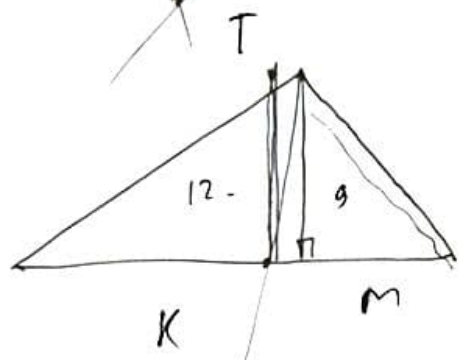
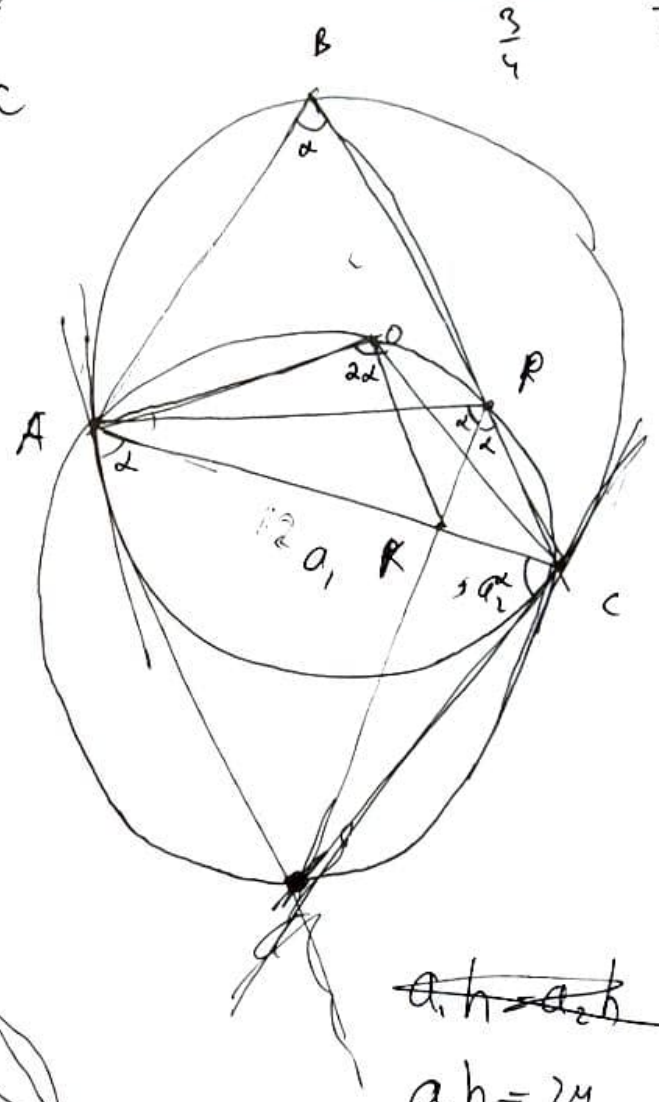


$$180 - 2\alpha - \gamma \quad S \sim K^2$$

$$\frac{a_1}{a_2} = \frac{4}{3} \quad \frac{49}{9} \cdot 9 \sqrt{49}$$

$$a_2 = \frac{3}{4} a_1$$

$$\frac{7}{4} a_1 \quad \frac{3}{4} \quad \frac{7}{3}$$



$$a \cdot h = a \cdot r \sin \alpha \Rightarrow$$

$$\frac{a r \cos \alpha}{2}$$

~~$$a_1 h = a_2 h$$~~

$$a_1 h = 24$$

$$a_2 h = 18$$

$$\frac{a_1}{a_2} = \frac{24}{18} = \frac{4}{3}$$

$$\frac{21}{9} \cdot 9 = 21$$