Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21101289**

ID профиля: **74985**

Вариант 21

Вариант 21. Часть 1. Чистовик

Задача № 1

$$\boxed{\begin{aligned} a_8 &= a_1 + 7d \\ a_{17} &= a_1 + 16d \\ a_{11} &= a_1 + 10d \\ a_{14} &= a_1 + 13d \end{aligned}}$$

$a_8 \cdot a_{17} > S + 27 \Leftrightarrow (a_1 + 7d)(a_1 + 16d) > S + 27$

$a_{11} \cdot a_{14} < S + 60 \Leftrightarrow (a_1 + 10d)(a_1 + 13d) < S + 60$

$\Downarrow$

$(a_1 + 7d)(a_1 + 16d) + S + 60 > S + 27 + (a_1 + 10d)(a_1 + 13d) \Leftrightarrow$

$\cancel{a_1^2} + \cancel{23da_1} + 112d^2 + \cancel{S} + 60 > \cancel{S} + 27 + \cancel{a_1^2} + \cancel{23a_1d} + 130d^2 \Leftrightarrow$

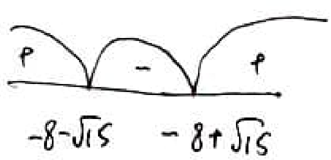
$18d^2 < 33. \Leftrightarrow$ т.к. $d > 0$ и $d \in \mathbb{Z}$ (по условию) $\Rightarrow d < \sqrt{\frac{33}{18}} \Rightarrow$

$\boxed{d = 1} \Rightarrow (a_1 + 7d)(a_1 + 16d) > S + 27;\ S = a_1 + a_2 + \ldots + a_7 = 7a_1 + 21d \Rightarrow$

$(a_1 + 7)(a_1 + 16) > 7a_1 + 48 \Leftrightarrow a_1^2 + 16a_1 + 64 > 0 \Leftrightarrow (a_1 + 8)^2 > 0 \Leftrightarrow \boxed{a_1 \neq -8}$

$\Rightarrow (a_1 + 10d)(a_1 + 13d) < 7a_1 + 21d + 60 \Leftrightarrow a_1^2 + 16a_1 + 49 < 0.\ D = 256 - 196 = 60$

$a_1 = \frac{-16 \pm \sqrt{60}}{2} = -8 \pm \sqrt{15} \Rightarrow$



$\Rightarrow a_1 \in \mathbb{Z}$ (по условию)

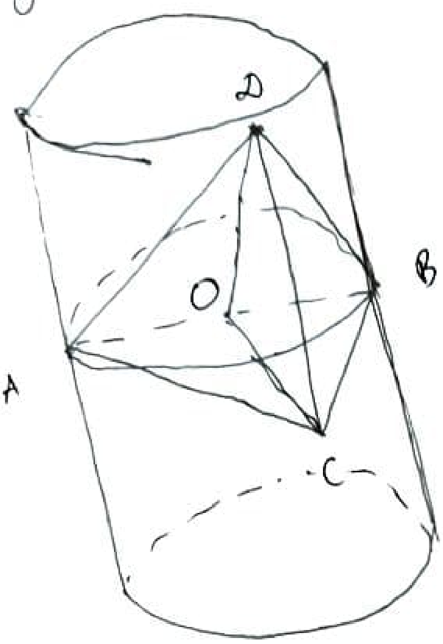
$\Rightarrow$ $a_1 = \{-11, -10, -9, -7, -6, -5\}$

Ответ:

$$\boxed{\begin{array}{lll} a_1 = -11 & a_1 = -9 & a_1 = -6 \\ a_1 = -10 & a_1 = -7 & a_1 = -5 \end{array}}$$

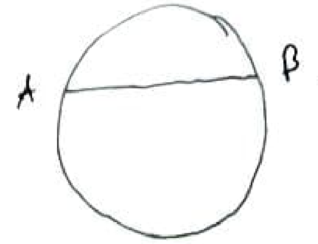
Чистовик. Вариант 21. Часть 1

Задача № 2



Замечаем, что AB ~~хорда~~ отрезок лежащий внутри окружности.

$AB = 4$.



$AC = CB = 5$; $AD = DB = 6$; CD параллельна оси ~~цилиндра~~ цилиндра. $\Rightarrow$ AB лежит на окружности параллельной основанию цилиндра. Тогда радиус цилиндра будет наименьшим если AB - диаметр. $\Rightarrow r = \frac{AB}{2} = 2$. ~~ADC~~ Рассмотрим $\triangle ADC$:

$AD + AC > CD \Leftrightarrow CD < 11$; $AC + CD > AD \Leftrightarrow CD > 1$.

Рассмотрим $\triangle ODC$, где O - середина AB. $\Rightarrow OC \perp AB$ т.к. $\triangle AOC = \triangle OBC$ ($AC = CB$, OC - общая сторона, $AO = OB$ (радиусы)). $\Rightarrow OC = \sqrt{AC^2 - AO^2} = \sqrt{25-4} = \sqrt{21}$.

$\triangle AOD = \triangle BOD$ (OD - общая сторона, $AO = OB$, $AD = DB$) $\Rightarrow OD \perp AB$. $\Rightarrow$ $\Rightarrow$

$OD = \sqrt{AD^2 - AO^2} = \sqrt{32} \Rightarrow CD + \sqrt{21} > \sqrt{32}$, $CD < \sqrt{32} + \sqrt{21}$.

Сравним $\sqrt{32} + \sqrt{21} \vee 11 \Rightarrow 32 + 2\cdot\sqrt{32\cdot 21} + 21 \vee 121 \Leftrightarrow \boxed{34 > \sqrt{32\cdot 21}}$

$\Rightarrow CD < \sqrt{32} + \sqrt{21}$. Сравним $1 \vee \sqrt{32} - \sqrt{21} \Leftrightarrow 26 \vee \sqrt{32\cdot 21} \Leftrightarrow$

$26 > \sqrt{32\cdot 21} \Rightarrow CD > 1 \Rightarrow CD \in (1; \sqrt{32} + \sqrt{21})$

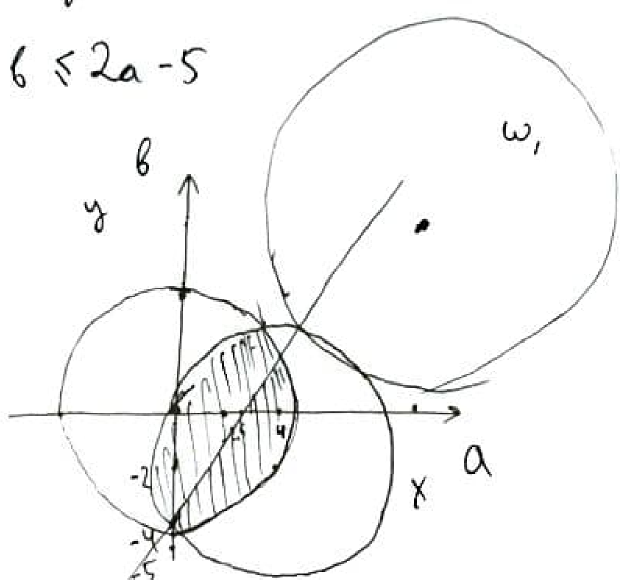
$\boxed{\text{Ответ: } CD \in (1; \sqrt{32} + \sqrt{21})}$

Чистовик. Вариант 21. Часть 1.

Задача №3

$$\begin{cases}(x-a)^2+(y-b)^2 \leq 20 \\ a^2+b^2 \leq \min(8a-4b, 20)\end{cases} \Leftrightarrow \begin{cases}(a-x)^2+(b-y)^2 \leq 20 \\ a^2+b^2 \leq 8a-4b, \text{ при } 8a-4b<20 \\ a^2+b^2 \leq 20, \text{ при } 8a-4b \geq 20\end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases}(a-x)^2+(b-y)^2 \leq 20 \\ (a-4)^2+(b+2)^2 \leq 20, \text{ при } b>2a-5 \\ a^2+b^2 \leq 20, \text{ при } b \leq 2a-5\end{cases}$$

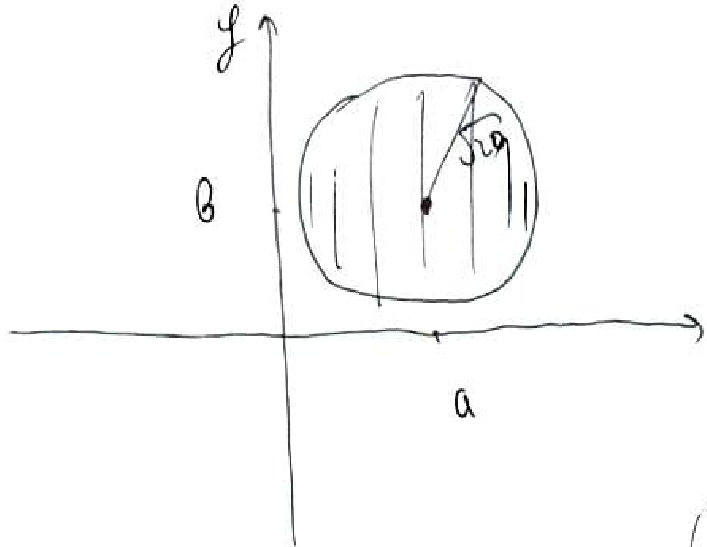


a, b - будут существовать, если ω_1 будет хотя бы ~~каса~~ касаться или пересекать закрашенную часть: $\boxed{(x-4)^2+(y+2)^2 \leq (\sqrt{20})^2}$ - ~~окруж~~

Площадь фигуры M - будет площадь окружности радиусом $\sqrt{20}$.

$S = \pi R^2 = 20\pi$

$\boxed{\text{Ответ: } S = 20\pi.}$

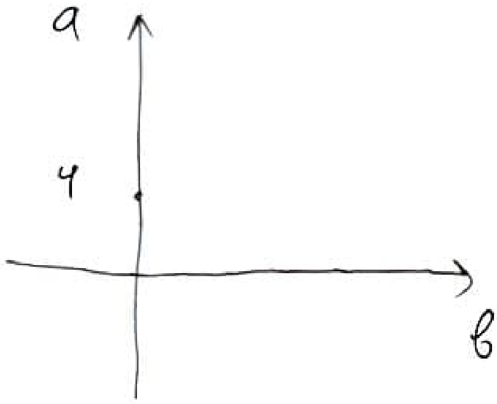


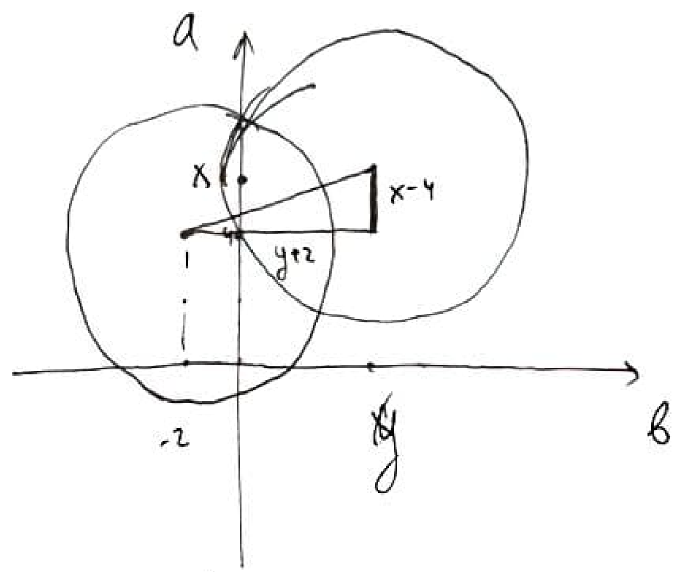
] $8a - 4b < 20$

$a^2 + b^2 < 8a - 4b$.

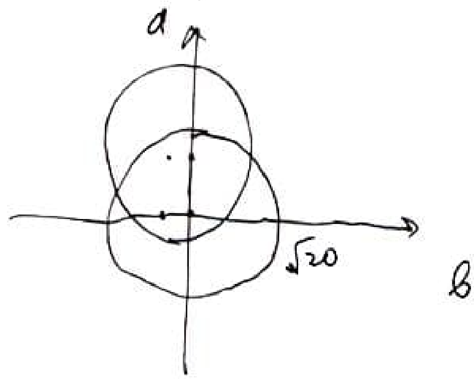
$a - 8a + 16 + b^2 + 4b + 4 < 20$

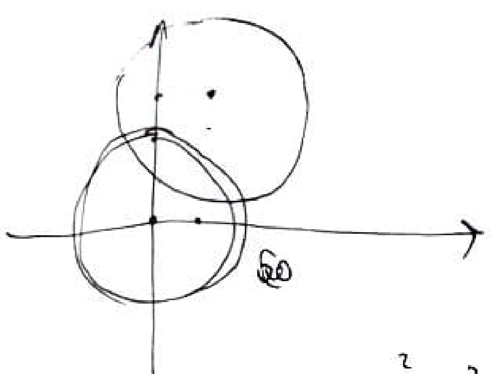
$\cancel{a - 8a + 16} = \boxed{(a-4)^2 + (b+2)^2 < 20.}$





$(2\sqrt{20})^2 \leq (x-4)^2 + (y+2)^2$



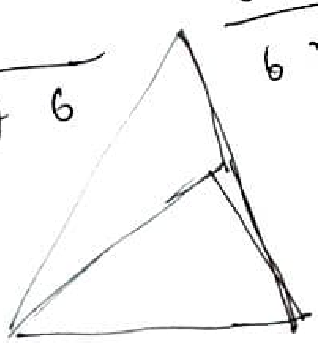


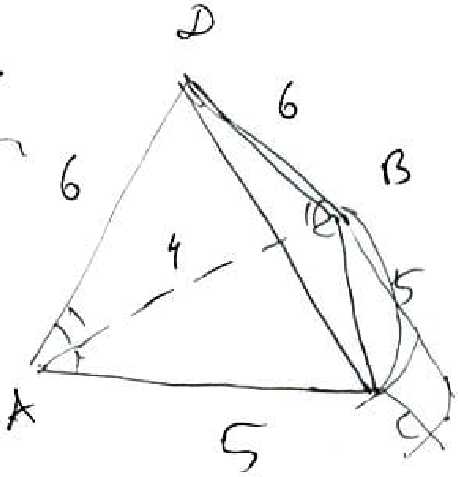
$x^2 + y^2 < (2\sqrt{20})^2$

4,6.

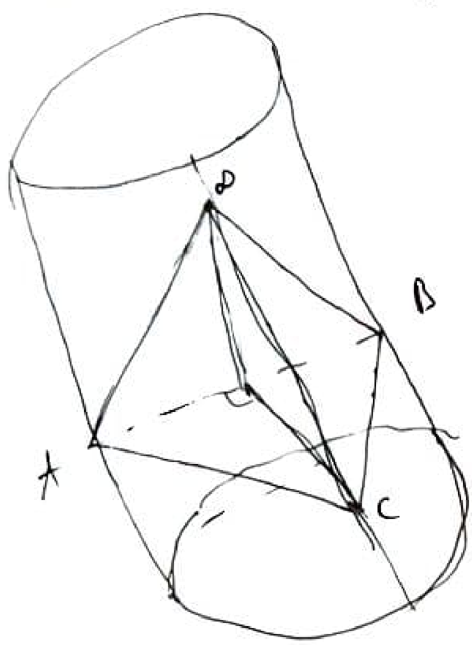
$$\begin{array}{r} 26 \\ \times\ 26 \\ \hline 156 \\ 52 \\ \hline 676 \end{array}$$

$$\begin{array}{r} 32 \\ \times\ 21 \\ \hline 32 \\ 64 \\ \hline 672 \end{array}$$





$$\begin{array}{r} 57 \\ \times\ 57 \\ \hline 399 \\ 285 \\ \hline 3249 \end{array}$$



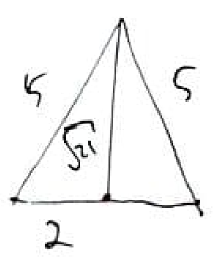
$\boxed{CD < 11}$

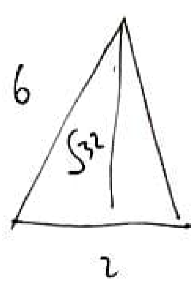
$\sqrt{32}$

$1 \vee$

$53 - 2\sqrt{32 \cdot 21}$

$26 \vee \sqrt{32 \cdot 21}$





$\boxed{CD < \sqrt{32} + \sqrt{21}}$

$5 + CD > 6$

$CD > 1$

$CD + \sqrt{21} > \sqrt{32}$

$CD > \sqrt{32} - \sqrt{21}$

$\sqrt{32} + \sqrt{21} \vee 11$

$53 + 2\sqrt{32 \cdot 21} \vee 121$

$34 \vee \sqrt{32 \cdot 21}$

$a_1 + a_2 + \ldots + a_7 = S$

$a_1 + (a_2 + d) \ldots = \cancel{7 \cdot a_1 +}$ $n \cdot a_1 + \frac{(1+n-1)\cdot n-1}{2} = \frac{n(n-1)d}{2} + n \cdot a_1$

$1 + 3 + 5 + 7 = 1 \cdot 4 + \frac{4(3)\cdot 2}{2} = 4 + 12 = 16$

$a_8 \cdot a_{17} > S + 27$
$a_{11} \cdot a_{14} < S + 60$

$(a_1 + 7d)(a_1 + 16d) > S + 27$

$(a_1 + 10d)(a_1 + 13d) < S + 60$

$a_1^2 + 23a_1 d + 112d^2 > 7a_1 \cancel{+ \frac{7 \cdot 6 \cdot d}{2}} = 7a_1 + 21d + 27$

$a_1^2 + 23a_1 d + 130d^2 < 7a_1 + 21d + 60$

$\cancel{a_1^2} + 23\cancel{a_1 d} + 1\cancel{12}d^2 + \cancel{7a_1} + \cancel{21d} + 60 > \cancel{7a_1} + \cancel{21d} + 27 + \cancel{a_1^2} + 23\cancel{a_1 d} + 13\cancel{0}d^2$

$18d^2$

$60 > 27 + 18d^2 \quad 33 > 18d^2$

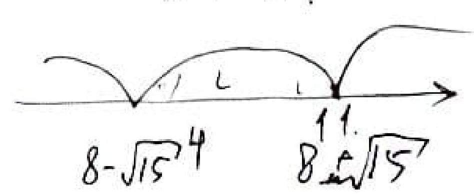
$d > 0 \Rightarrow \quad d < \sqrt{\frac{33}{18}} \Rightarrow \boxed{d = 1}$

$\begin{array}{r} \times 16 \\ 16 \\ \hline 96 \\ 16 \\ \hline 256 \end{array}$

$4\ 5\ 6 \ldots 11.$

$a_1^2 + 23a_1 + 112 > 27a_1 + 21 + 27.$

$\cancel{a_1^2 - 4a_1 + 64 > 0}.$



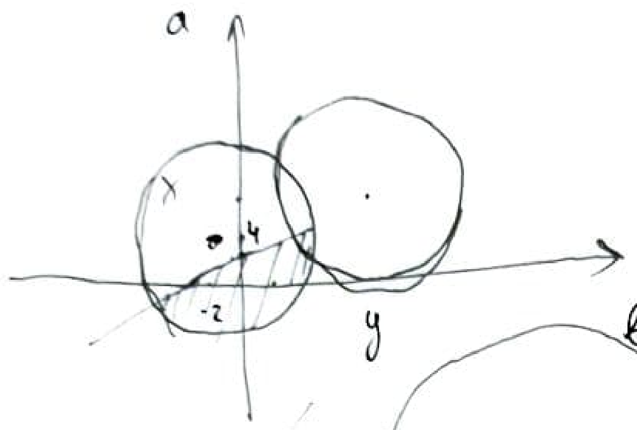
$a_1^2 + 23a_1 + 130 < 7a_1 + 21 + 60.$

$a_1^2 + 16a_1 + 49 < 0.$

$a_{1,2} = \frac{-16 \pm \sqrt{60}}{2} = -8 \pm \sqrt{15}.$

$D = 256 - 196 = 60.$

$$\begin{cases}(x-a)^2+(y-b)^2 \le 20\\ a^2+b^2 \le \min(8a-4b,20)\end{cases} \Leftrightarrow \begin{cases}(x-a)^2+(y-b)^2 \le 20\\ a^2+b^2 \le 8a-4b, \text{ при } 8a-4b<20\\ a^2+b^2 \le 20, \text{ при } 8a-4b \ge 20.\end{cases}$$



$$a^2+b^2 \le 8a-4b$$

$$\Updownarrow$$

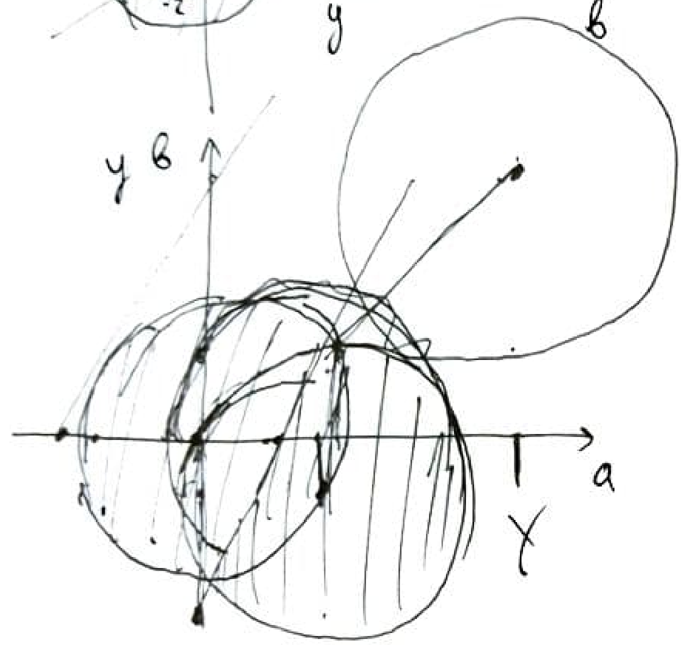
$$(a-4)^2+(b+2)^2 \le 20.$$

$$a < \frac{b}{2}+\frac{5}{2}.$$

$$\cancel{4b > 8}$$

$$\boxed{b > 2a - \cancel{20}\,5.}$$

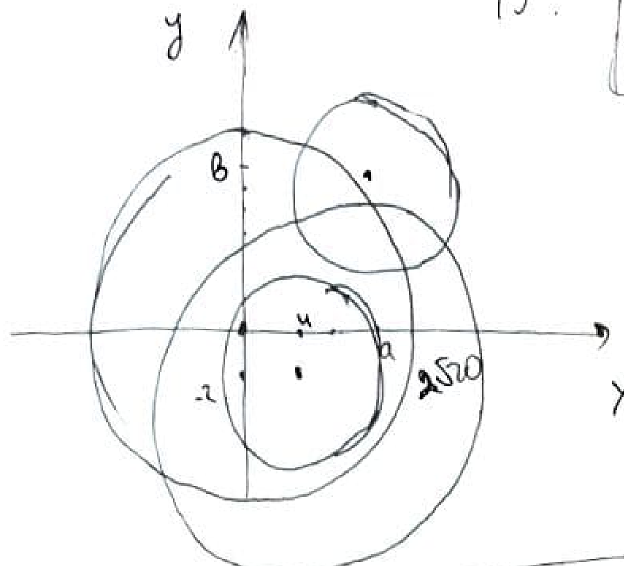
$$b \le 2a-5.$$



$$\begin{cases}\cancel{(y+2)^2+(x-4)^2 \le (2\sqrt{20})^2}\\ \cancel{x^2+y^2 \le 2\sqrt{20}}\end{cases}$$

$$\cancel{(a-x)^2+(b-y)^2}$$

$$(x-4)^2+(y+2)^2 \le (\sqrt{20})^2$$



if: $\boxed{8a-4b<20.}$ $\boxed{2a-b<5}$

elseif $8a-4b>20.$

$2a-b>\cancel{20}\ 5$

$\begin{array}{r} \times 16 \\ 7 \\ \hline 112. \end{array}$

$S=a_1+a_2+\dots+a_7=7a_1+\frac{7d\cdot 6}{2}=\boxed{7a_1+21d.}$

$\begin{cases} a_8=a_1+7d. \\ a_{17}=a_1+16d. \\ a_{11}=a_1+10d \\ a_{14}=a_1+13d, \end{cases} \Rightarrow \begin{array}{l} (a_1+7d)(a_1+16d)>S+27. \\ (a_1+10d)(a_1+13d)< S+60. \end{array}$

$\boxed{\begin{array}{l} a_1\in\mathbb{Z}. \\ a_1\in(-11;-5)\setminus\{-8\} \end{array}}$

$a_1^2+16da_1+7da_1+112d^2>S+27=\cancel{7a_1+21d+27}$

$a_1^2+23a_1d+130d^2<S+60$

$\cancel{a_1^2}+\cancel{23a_1d}+\cancel{112d^2}+\cancel{S}+60>\cancel{a_1^2}+\cancel{23a_1d}+\cancel{130d^2}+\cancel{S}+27$

$33 \qquad 18d^2$

$d^2<\frac{33}{18} \Leftrightarrow d^2<2 \Rightarrow \boxed{d=1}$

$a_1^2+23a_1+112>7a_1+21+27.$

$a_1^2+16a_1+64>0.$

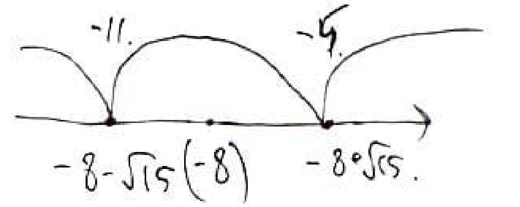
$(a_1+8)^2>0. \Leftrightarrow \boxed{a_1\neq-8.}$

$a_1^2+23a_1+130<7a_1+21+60$

$a_1^2+16a_1+49<0.$

$D=256-196=60.$

$a_{1,2}=\frac{-16\pm\sqrt{60}}{2}=-8\pm\sqrt{15}.$



Вариант 21. Часть 1. Чистовик

Задача № 1

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$a_8 \cdot a_{17} > S + 27 \Leftrightarrow (a_1+7d)(a_1+16d) > S+27$

$a_{11} \cdot a_{14} < S + 60 \Leftrightarrow (a_1+10d)(a_1+13d) < S+60$

$\Downarrow$

$(a_1+7d)(a_1+16d) + S + 60 > S + 27 + (a_1+10d)(a_1+13d) \Leftrightarrow$

$\cancel{a_1^2} + \cancel{23da_1} + 112d^2 + \cancel{S} + 60 > \cancel{S} + 27 + \cancel{a_1^2} + \cancel{23a_1d} + 130d^2 \Leftrightarrow$

$18d^2 < 33 \Leftrightarrow$ т.к. $d > 0$ и $d \in \mathbb{Z}$ (по условию) $\Rightarrow d < \sqrt{\frac{33}{18}} \Rightarrow$

$\boxed{d = 1} \Rightarrow (a_1+7d)(a_1+16d) > S+27;\ S = a_1 + a_2 + \ldots + a_7 = 7a_1 + 21d \Rightarrow$

$(a_1+7)(a_1+16) > 7a_1 + 48 \Leftrightarrow a_1^2 + 16a_1 + 64 > 0 \Leftrightarrow (a_1+8)^2 > 0 \Leftrightarrow \boxed{a_1 \neq -8}$

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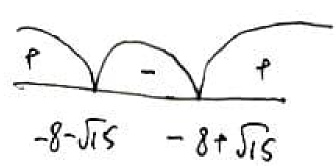
$a_1 = \frac{-16 \pm \sqrt{60}}{2} = -8 \pm \sqrt{15} \Rightarrow$



$\Rightarrow a_1 \in \mathbb{Z}$ (по условию)

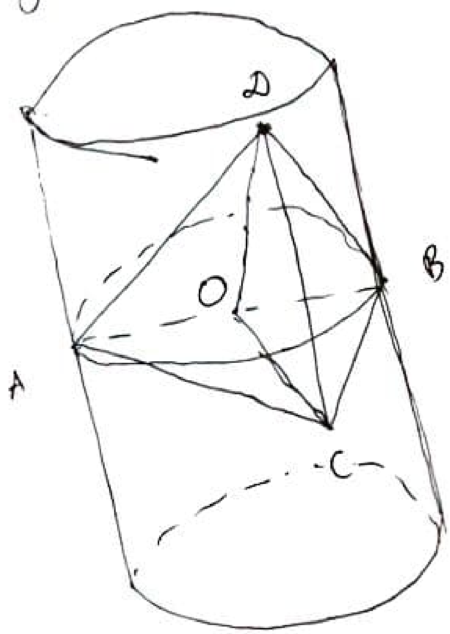
$a_1 \Rightarrow$ ~~$a_1 \in [-11; -5]$~~ $a_1 = \{-11, -10, -9, -7, -6, -5\}$

Ответ:

$a_1 = -11$	$a_1 = -9$	$a_1 = -6$
$a_1 = -10$	$a_1 = -7$	$a_1 = -5$

Чистовик. Вариант 21. Часть 1

Задача № 2



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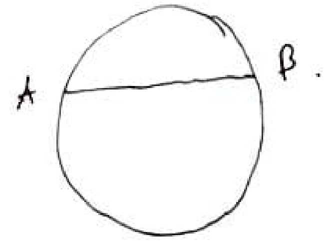

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$\triangle AOD = \triangle BOD$. ($OD$ - общая сторона, $AO = OB$, $AD = DB$) $\Rightarrow OD \perp AB$. $\Rightarrow$ ~~OD~~ $\Rightarrow$

$OD = \sqrt{AD^2 - AO^2} = \sqrt{32} \Rightarrow CD + \sqrt{21} > \sqrt{32}$, $CD < \sqrt{32} + \sqrt{21}$.

Сравним $\sqrt{32} + \sqrt{21} \vee 11 \Rightarrow 32 + 2\sqrt{32 \cdot 21} + 21 \vee 121 \Leftrightarrow$ $\boxed{34 > \sqrt{32 \cdot 21}}$

$\Rightarrow CD < \sqrt{32} + \sqrt{21}$. Сравним $1 \vee \sqrt{32} - \sqrt{21} \Leftrightarrow 26 \vee \sqrt{32 \cdot 21} \Leftrightarrow$

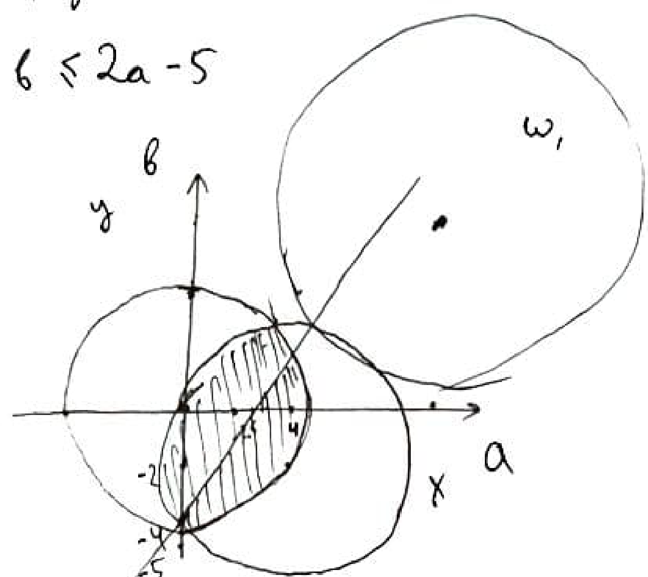
$26 > \sqrt{32 \cdot 21} \Rightarrow CD > 1 \Rightarrow CD \in (1; \sqrt{32} + \sqrt{21})$

$\boxed{\text{Ответ: } CD \in (1; \sqrt{32} + \sqrt{21})}$

Задача №3

$$\begin{cases}(x-a)^2+(y-b)^2\le 20\\ a^2+b^2\le\min(8a-4b,20)\end{cases}\Leftrightarrow\begin{cases}(a-x)^2+(b-y)^2\le 20\\ a^2+b^2\le 8a-4b,\ \text{при } 8a-4b<20\\ a^2+b^2\le 20,\ \text{при } 8a-4b\ge 20\end{cases}\Leftrightarrow$$

$$\Leftrightarrow\begin{cases}(a-x)^2+(b-y)^2\le 20\\ (a-4)^2+(b+2)^2\le 20,\ \text{при } b>2a-5\\ a^2+b^2\le 20,\ \text{при } b\le 2a-5\end{cases}$$

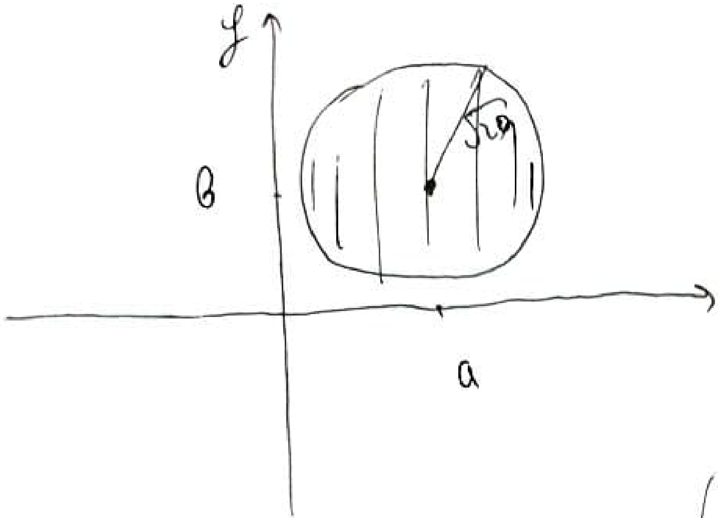


a, b - будут существовать, если ω_1 будет хотя бы ~~каса~~ касаться или пересекать закрашенную часть: $\boxed{(x-4)^2+(y+2)^2\le(\sqrt{20})^2}$ - ~~окружн~~

Площадь фигуры M - будет площадь окружности радиусом $\sqrt{20}$.

$$S=\pi R^2=20\pi$$

$$\boxed{\text{Ответ: } S=20\pi.}$$

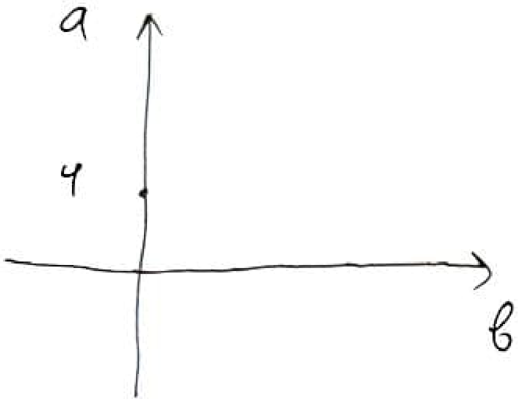


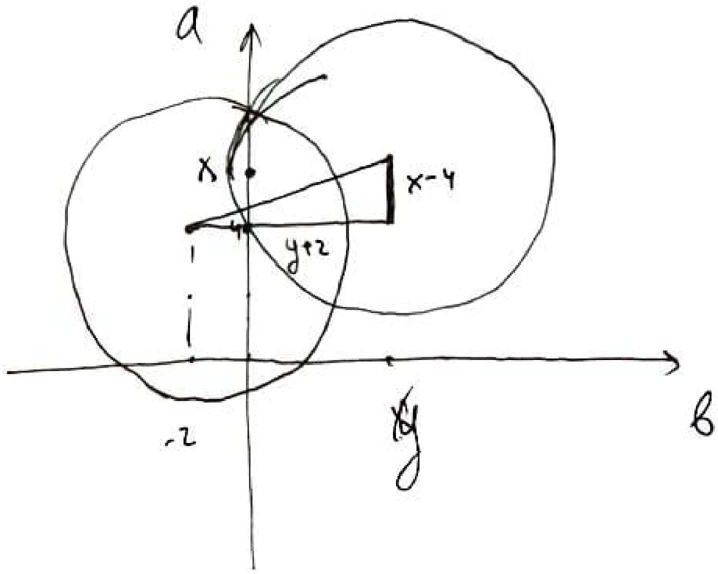
] $8a - 4b < 20$

$a^2 + b^2 < 8a - 4b.$

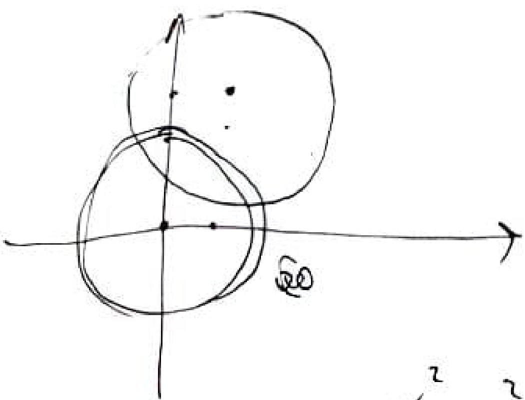
$a - 8a + 16 + b^2 + 4b + 4 < 20$

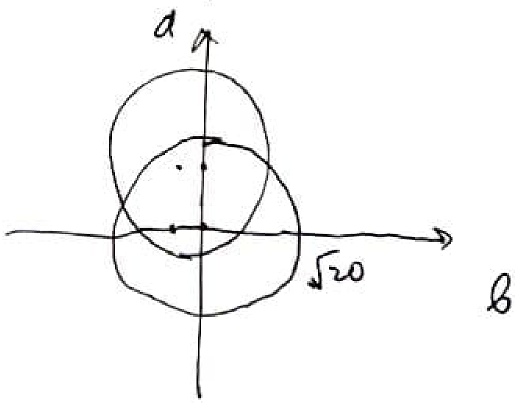
$\boxed{\cancel{a - 8a + 16} = (a-4)^2 + (b+2)^2 < 20.}$





$(2\sqrt{20})^2 \leq (x-4)^2 + (y+2)^2$





$x^2 + y^2 < (2\sqrt{20})^2$

$a_1+a_2+...+a_7=S$

$a_1+(a_2+d) \ldots = \cancel{7\cdot a_1+}$

$n\cdot a_1+\frac{(1+n-1)\cdot n-1}{2} = \frac{n(n-1)d}{2}+n\cdot a_1$

$\cancel{1}+3+5+7 = 1\cdot 4+\frac{4(3)\cdot 2}{2} = 4+12=16$

$a_8\cdot a_{17} > S+27$

$a_{11}\cdot a_{14} < S+60$

$(a_1+7d)(a_1+16d) > S+27$

$(a_1+10d)(a_1+13d) < S+60$

$a_1^2+23a_1d+112d^2 > 7a_1+\cancel{\frac{7\cdot 6\cdot d}{2}} = 7a_1+21d+27$

$a_1^2+23a_1d+130d^2 < 7a_1+21d+60$

$\cancel{a_1^2}+23\cancel{a_1d}+11\cancel{2}d^2+\cancel{7a_1}+\cancel{21d}+60 > \cancel{7a_1}+\cancel{21d}+27+\cancel{a_1^2}+23\cancel{a_1d}+13\cancel{0}d^2$

$18d^2$

$60>27+18d^2 \quad 33>18d^2$

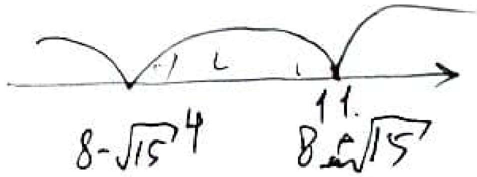
$d>0 \Rightarrow d<\sqrt{\frac{33}{18}} \Rightarrow \boxed{d=1}$

$$\begin{array}{r} \times\ 16 \\ 16 \\ \hline 96 \\ 16\ \ \\ \hline 256 \end{array}$$

4 5 6 ... 11

$a_1^2+23a_1+112>27a_1+21+27.$

$\cancel{a_1^2-4a_1+64>0}.$



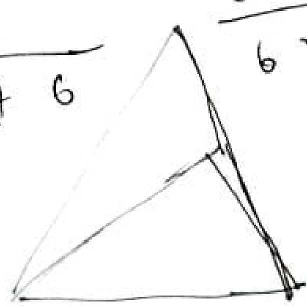
$a_1^2+23a_1+130<7a_1+21+60.$

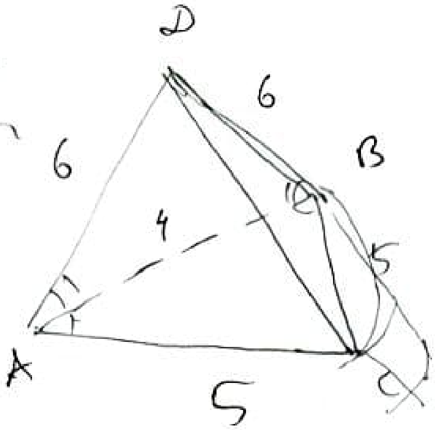
$a_1^2+16a_1+49<0. \qquad a_{1,2}=\frac{16\pm\sqrt{60}}{2}=8\pm\sqrt{15}.$

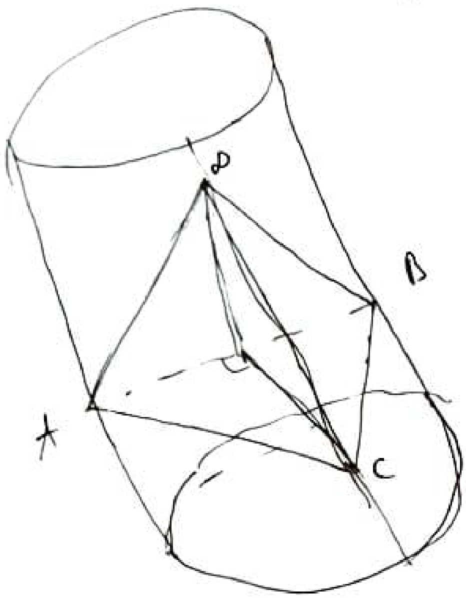
$D=256-196=60.$

$$\begin{array}{r} 26 \\ \times\ 26 \\ \hline 156 \\ 52\ \ \\ \hline 676 \end{array} \qquad \begin{array}{r} 32 \\ \times\ 21 \\ \hline 32 \\ 64\ \ \\ \hline 672 \end{array}$$

4,8.







$$\begin{array}{r} 57 \\ \times\ 57 \\ \hline 399 \\ 285\ \ \\ \hline 3249 \end{array}$$

$\boxed{CD < 11}$

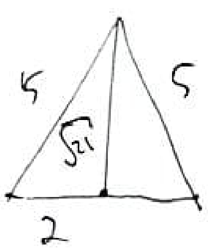
$\sqrt{32}$

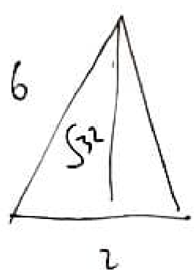
$1 \vee$

$53 - 2\sqrt{32 \cdot 21}$

$26 > \sqrt{32 \cdot 21}$

$\boxed{CD < \sqrt{32} + \sqrt{21}}$





$5 + CD > 6$

$CD > 1$

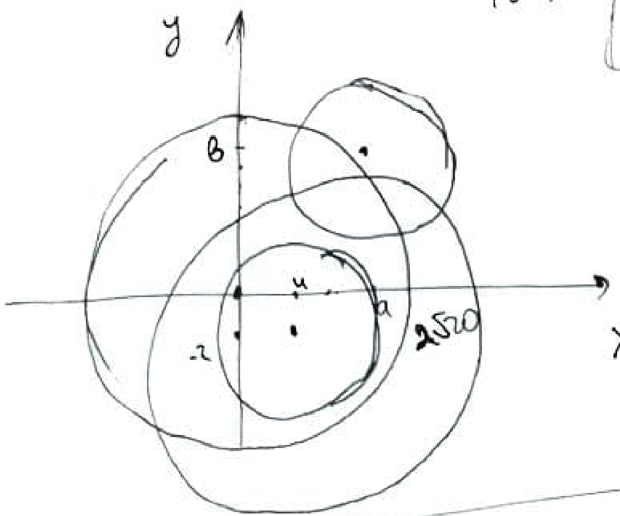
$CD + \sqrt{21} > \sqrt{32}$

$CD > \sqrt{32} - \sqrt{21}$

$\sqrt{32} + \sqrt{21} \vee 11$

$53 + 2\sqrt{32 \cdot 21} \vee 121$

$34 \vee \sqrt{32 \cdot 21}$



if: $\boxed{8a-4b<20}$ $\boxed{2a-b<5}$

elseif $8a-4b>20$

$2a-b>\cancel{20}\,5$

$16 \times 7 = 112$

$S=a_1+a_2+\dots+a_7=7a_1+\frac{7d\cdot 6}{2}=\boxed{7a_1+21d}$

$$\begin{cases} a_8=a_1+7d \\ a_{17}=a_1+16d \\ a_{11}=a_1+10d \\ a_{14}=a_1+13d \end{cases} \Rightarrow \begin{array}{l} (a_1+7d)(a_1+16d)>S+27 \\ (a_1+10d)(a_1+13d)<S+60 \end{array}$$

$\boxed{a_1\in\mathbb{Z}. \quad a_1\in(-11;-5)\setminus\{-8\}}$

$a_1^2+16da_1+7da_1+112d^2>S+27=\cancel{7a_1+21d+27}$

$a_1^2+23a_1d+130d^2<S+60$

$\cancel{a_1^2}+\cancel{23a_1d}+\cancel{112d^2}+\cancel{S}+\cancel{60}>\cancel{a_1^2}+\cancel{23a_1d}+\cancel{130d^2}+\cancel{S}+2\cancel{7}$

$33 \quad 18d^2$

$d^2<\frac{33}{18} \Rightarrow d^2<2 \Rightarrow \boxed{d=1}$

$a_1^2+23a_1+112>7a_1+21+27$

$a_1^2+16a_1+64>0$

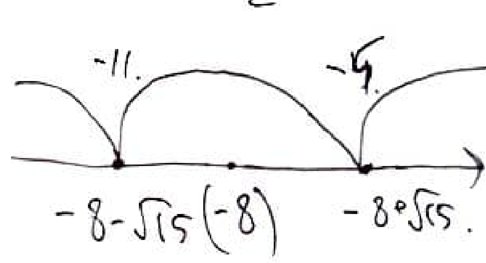
$(a_1+8)^2>0 \Leftrightarrow \boxed{a_1\neq-8}$

$a_1^2+23a_1+130<7a_1+21+60$

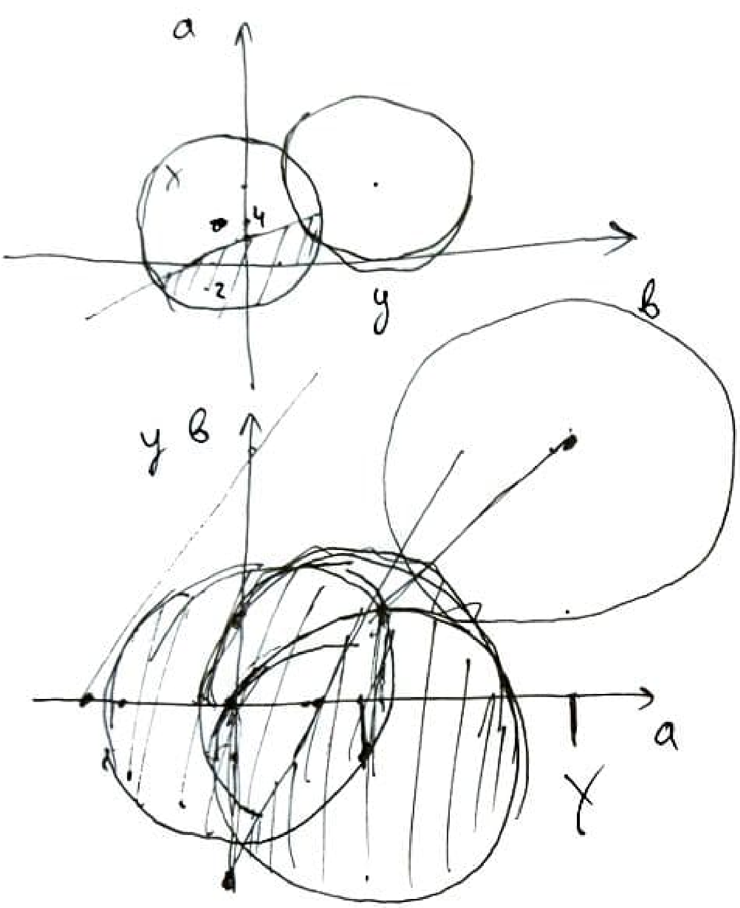
$a_1^2+16a_1+49<0$

$D=256-196=60$

$a_{1,2}=\frac{-16\pm\sqrt{60}}{2}=-8\pm\sqrt{15}$



$$\begin{cases}(x-a)^2+(y-b)^2 \leqslant 20\\ a^2+b^2 \leqslant \min(8a-4b,\,20)\end{cases} \Leftrightarrow \begin{cases}(x-a)^2+(y-b)^2 \leqslant 20\\ a^2+b^2 \leqslant 8a-4b, \text{ при } 8a-4b<20\\ a^2+b^2 \leqslant 20, \text{ при } 8a-4b \geqslant 20\end{cases}$$



$$a^2+b^2 \leqslant 8a-4b$$

$$\Updownarrow$$

$$(a-4)^2+(b+2)^2 \leqslant 20$$

$$a < -\frac{b}{2}+\frac{5}{2}$$

$$\cancel{4b > 8}$$

$$\boxed{b > 2a - \cancel{20}\,5}$$

$$b \leqslant 2a-5$$

$$\cancel{\begin{cases}(y+2)^2+(x-4)^2 \leqslant (2\sqrt{20})^2\\ x^2+y^2 \leqslant 2\sqrt{20}\end{cases}}$$

$$\cancel{(a-x)^2+(b-y)^2}$$

$$(x-4)^2+(y+2)^2 \leqslant (\sqrt{20})^2$$

Вариант 21. Часть 1. Чистовик

Задача № 1

$a_8 = a_1 + 7d$

$a_{17} = a_1 + 16d$

$a_{11} = a_1 + 10d$

$a_{14} = a_1 + 13d$

$a_8 \cdot a_{17} > S + 27 \Leftrightarrow (a_1 + 7d)(a_1 + 16d) > S + 27$

$a_{11} \cdot a_{14} < S + 60 \Leftrightarrow (a_1 + 10d)(a_1 + 13d) < S + 60$

$\Downarrow$

$(a_1+7d)(a_1+16d) + S + 60 > S + 27 + (a_1+10d)(a_1+13d) \Leftrightarrow$

$\cancel{a_1^2} + \cancel{23da_1} + 112d^2 + \cancel{S} + 60 > \cancel{S} + 27 + \cancel{a_1^2} + \cancel{23a_1d} + 130d^2 \Leftrightarrow$

$18d^2 < 33 \Leftrightarrow$ т.к. $d > 0$ и $d \in \mathbb{Z}$ (по условию) $\Rightarrow d < \sqrt{\frac{33}{18}} \Rightarrow$

$\boxed{d = 1} \Rightarrow (a_1+7d)(a_1+16d) > S + 27;\ S = a_1 + a_2 + \ldots + a_7 = 7a_1 + 21d \Rightarrow$

$(a_1+7)(a_1+16) > 7a_1 + 48 \Leftrightarrow a_1^2 + 16a_1 + 64 > 0 \Leftrightarrow (a_1+8)^2 > 0 \Leftrightarrow \boxed{a_1 \neq -8}$

$\Rightarrow (a_1+10d)(a_1+13d) < 7a_1 + 21d + 60 \Leftrightarrow a_1^2 + 16a_1 + 49 < 0.\ D = 256 - 196 = 60$

$a_1 = \frac{-16 \pm \sqrt{60}}{2} = -8 \pm \sqrt{15} \Rightarrow$

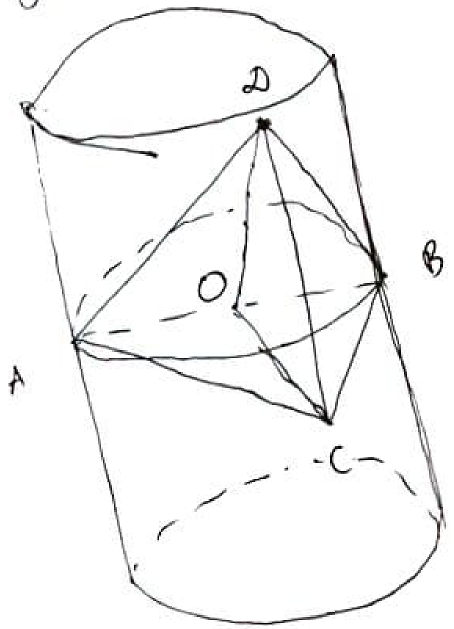
$+ \quad -8-\sqrt{15} \quad - \quad -8+\sqrt{15} \quad +$

$\Rightarrow a_1 \in \mathbb{Z}$ (по условию)

$\cancel{a_1} \Rightarrow \cancel{a_1 \in [-11; -5]}$ $a_1 = \{-11, -10, -9, -7, -6, -5\}$

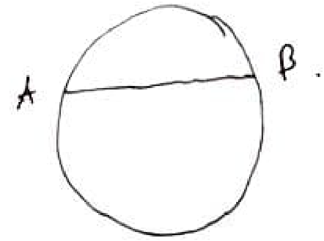
Ответ: $\boxed{\begin{array}{lll} a_1 = -11 & a_1 = -9 & a_1 = -6 \\ a_1 = -10 & a_1 = -7 & a_1 = -5 \end{array}}$

Задача № 2



Замечаем, что AB ~~хорда~~ отрезок лежащий внутри окружности.

$AB=4.$



$AC=CB=5$; $AD=DB=6.$; CD параллельна оси ~~цилиндра~~ цилиндра. $\Rightarrow$ AB лежит на окружности параллельной основанию цилиндра. Тогда радиус цилиндра будет наименьшим если AB - диаметр. $\Rightarrow r=\frac{AB}{2}=2$. ~~$ADC$~~ Рассмотрим $\triangle ADC$:

$AD+AC>CD \Leftrightarrow CD<11$; $AC+CD>AD \Leftrightarrow CD>1.$

Рассмотрим $\triangle ODC$, где O - середина AB. $\Rightarrow OC\perp AB$. т.к. $\triangle AOC=\triangle OBC$ ($AC=CB$, OC - общая сторона, $AO=OB$ (радиусы)). $\Rightarrow OC=\sqrt{AC^2-AO^2}=\sqrt{25-4}=\sqrt{21}.$

$\triangle AOD=\triangle BOD$. ($OD$ - общая сторона, $AO=OB$, $AD=DB$) $\Rightarrow OD\perp AB.\Rightarrow$ ~~ODC~~ $\Rightarrow$

$OD=\sqrt{AD^2-AO^2}=\sqrt{32}\Rightarrow CD+\sqrt{21}>\sqrt{32}$, $CD<\sqrt{32}+\sqrt{21}.$

Сравним $\sqrt{32}+\sqrt{21}\vee 11\Rightarrow 32+2\cdot\sqrt{32\cdot 21}+21\vee 121\Leftrightarrow$ $\boxed{34>\sqrt{32\cdot 21}}$

$\Rightarrow CD<\sqrt{32}+\sqrt{21}$. Сравним $1\vee\sqrt{32}-\sqrt{21}\Leftrightarrow 26\vee\sqrt{32\cdot 21}\Leftrightarrow$

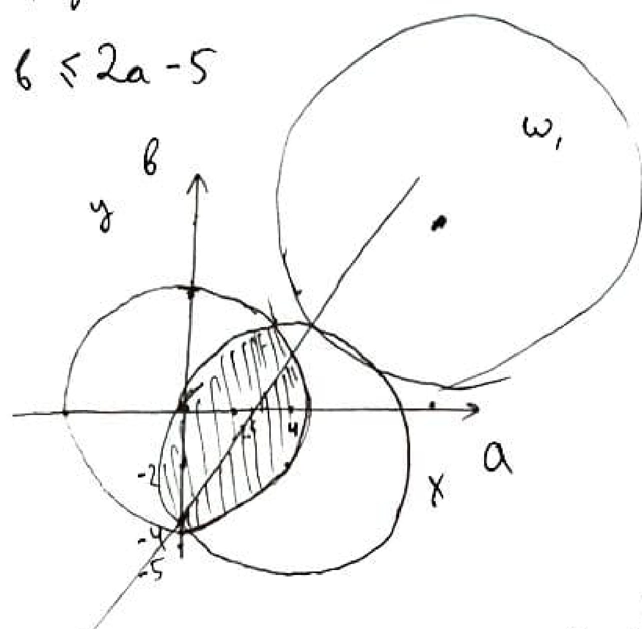
$26>\sqrt{32\cdot 21}\Rightarrow CD>1\Rightarrow CD\in(1;\sqrt{32}+\sqrt{21})$

$\boxed{\text{Ответ: } CD\in(1;\sqrt{32}+\sqrt{21})}$

Задача №3

$$\begin{cases}(x-a)^2+(y-b)^2\leqslant 20\\ a^2+b^2\leqslant \min(8a-4b,20)\end{cases} \Leftrightarrow \begin{cases}(a-x)^2+(b-y)^2\leqslant 20\\ a^2+b^2\leqslant 8a-4b,\text{ при } 8a-4b<20\\ a^2+b^2\leqslant 20,\text{ при } 8a-4b\geqslant 20\end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases}(a-x)^2+(b-y)^2\leqslant 20\\ (a-4)^2+(b+2)^2\leqslant 20,\text{ при } b>2a-5\\ a^2+b^2\leqslant 20,\text{ при } b\leqslant 2a-5\end{cases}$$

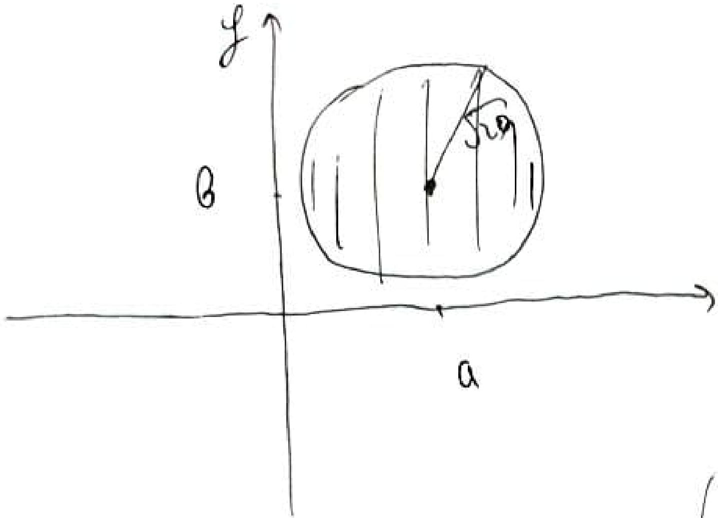


a, b – будут существовать, если ω_1 будет хотя бы ~~каса~~ касаться или пересекать заштрихованную часть: $\boxed{(x-4)^2+(y+2)^2\leqslant(\sqrt{20})^2}$ – ~~окружн~~

Площадь фигуры M – будет площадь окружности радиусом $\sqrt{20}$.

$S=\pi R^2=20\pi$

Ответ: $\boxed{S=20\pi}$.

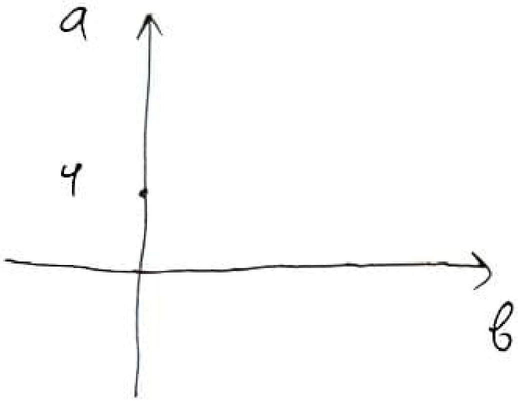


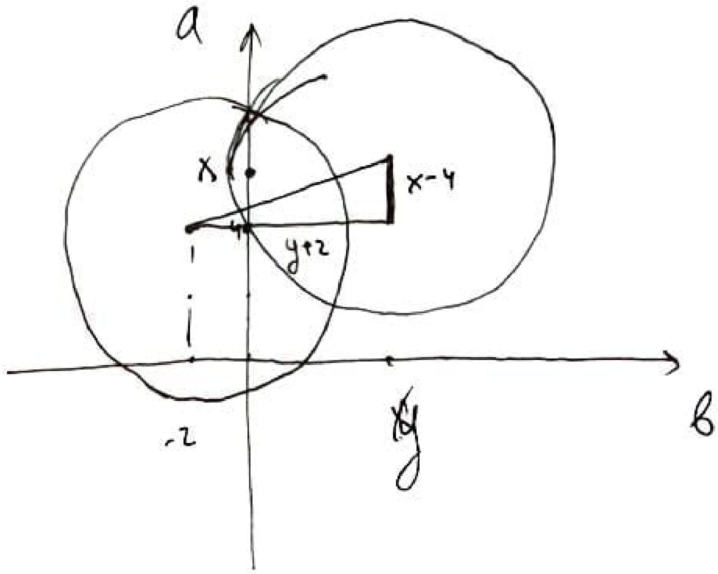
$] \quad 8a - 4b < 20$

$a^2 + b^2 < 8a - 4b.$

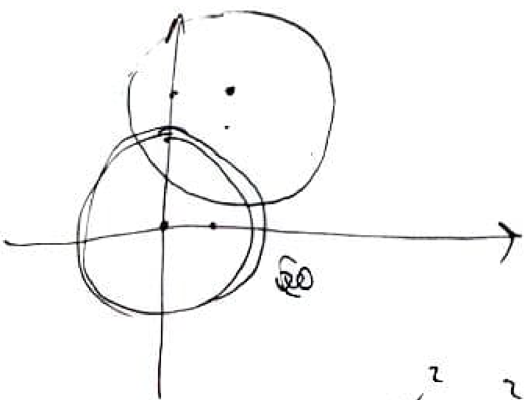
$a - 8a + 16 + b^2 + 4b + 4 < 20$

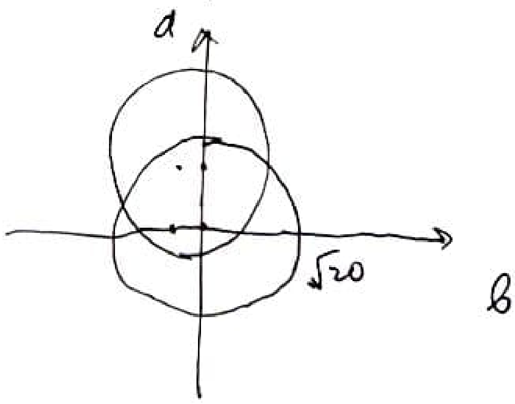
$\cancel{a - 8a + 16 =} \boxed{(a-4)^2 + (b+2)^2 < 20.}$





$(2\sqrt{20})^2 \leq (x-4)^2 + (y+2)^2$





$x^2 + y^2 < (2\sqrt{20})^2$

$a_1 + a_2 + \dots + a_7 = S$

$a_1 + (a_2 + d) \dots = \cancel{7 \cdot a_1 +}$

$n \cdot a_1 + \frac{(1+n-1) \cdot n-1}{2} = \frac{n(n-1)d}{2} + n \cdot a_1$

$\cancel{1} + 3 + 5 + 7 = 1 \cdot 4 + \frac{4(3) \cdot 2}{2} = 4 + 12 = 16$

$a_8 \cdot a_{17} > S + 27$

$a_{11} \cdot a_{14} < S + 60$

$(a_1 + 7d)(a_1 + 16d) > S + 27$

$(a_1 + 10d)(a_1 + 13d) < S + 60$

$a_1^2 + 23a_1d + 112d^2 > 7a_1 + \cancel{\frac{7 \cdot 6 \cdot d}{2}} = 7a_1 + 21d + 27$

$a_1^2 + 23a_1d + 130d^2 < 7a_1 + 21d + 60$

$\cancel{a_1^2} + 23\cancel{a_1d} + 11\cancel{2}d^2 + \cancel{7a_1} + \cancel{21d} + 60 > \cancel{7a_1} + \cancel{21d} + 27 + \cancel{a_1^2} + 23\cancel{a_1d} + 13\cancel{0}d^2$

$18d^2$

$60 > 27 + 18d^2 \quad 33 > 18d^2$

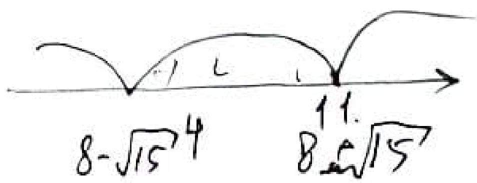
$d > 0 \Rightarrow \quad d < \sqrt{\frac{33}{18}} \Rightarrow \boxed{d = 1}$

$\times \begin{array}{r} 16 \\ 16 \\ \hline 96 \\ 16 \\ \hline 256 \end{array}$

4 5 6 ... 11.

$a_1^2 + 23a_1 + 112 > 27a_1 + 21 + 27.$

$\cancel{a_1^2 - 4a_1 + 64 > 0}.$



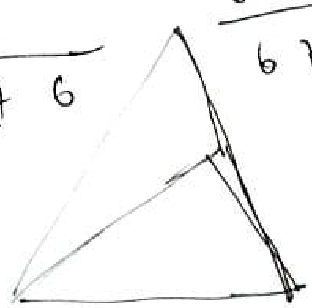
$a_1^2 + 23a_1 + 130 < 7a_1 + 21 + 60.$

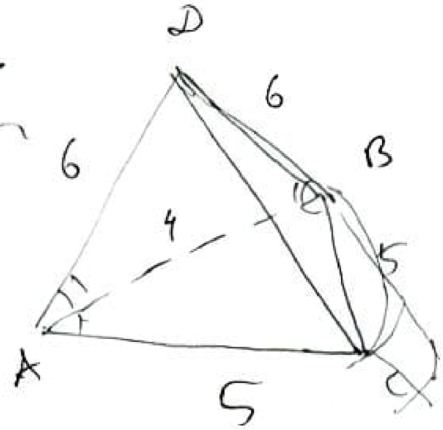
$a_1^2 + 16a_1 + 49 < 0.$ $\quad a_{1,2} = \frac{-16 \pm \sqrt{60}}{2} = 8 \pm \sqrt{15}.$

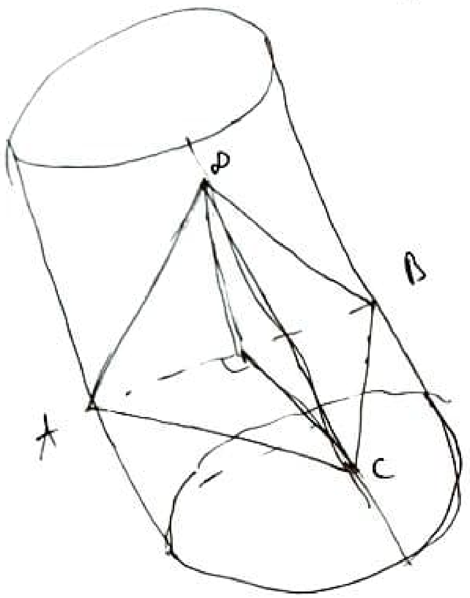
$D = 256 - 196 = 60.$

$$\begin{array}{r} \times\,26 \\ 26 \\ \hline 156 \\ 52 \\ \hline 676 \end{array} \qquad \begin{array}{r} \times\,32 \\ 21 \\ \hline 32 \\ 64 \\ \hline 672 \end{array}$$

4,8.







$\boxed{CD < 11}$

$$\begin{array}{r} \times\,57 \\ 57 \\ \hline 399 \\ 285 \\ \hline 3249 \end{array}$$

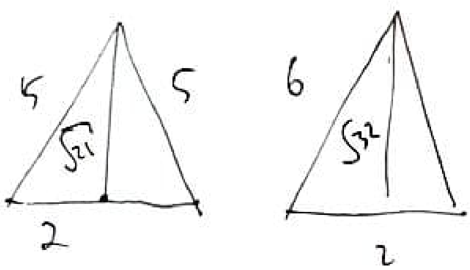
$\sqrt{32}$

$1 \vee$

$53 - 2\sqrt{32\cdot 21}$

$26 > \sqrt{32\cdot 21}$

$\boxed{CD < \sqrt{32} + \sqrt{21}}$



$5 + CD > 6$

$CD > 1$

$CD + \sqrt{21} > \sqrt{32}$

$CD > \sqrt{32} - \sqrt{21}$

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$53 + 2\sqrt{32\cdot 21} \vee 121$

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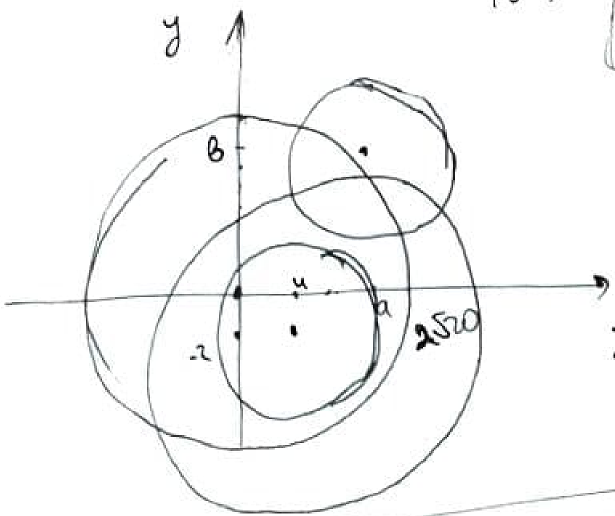

if: $\boxed{8a-4b<20}$ $\boxed{2a-b<5}$

elseif $8a-4b>20$

$2a-b>\cancel{20}\,5$

$\times\begin{array}{r}16\\7\\\hline 112\end{array}$

$S=a_1+a_2+\dots+a_7=7a_1+\dfrac{7d\cdot 6}{2}=\boxed{7a_1+21d}$

$\begin{cases} a_8=a_1+7d \\ a_{17}=a_1+16d \\ a_{11}=a_1+10d \\ a_{14}=a_1+13d \end{cases} \Rightarrow \begin{cases}(a_1+7d)(a_1+16d)>S+27 \\ (a_1+10d)(a_1+13d)<S+60\end{cases}$

$\boxed{\begin{array}{l} a_1\in\mathbb{Z} \\ a_1\in(-11;-5)\setminus\{-8\}\end{array}}$

$a_1^2+16da_1+7da_1+112d^2>S+27=\cancel{7a_1+21d+27}$

$a_1^2+23a_1d+130d^2<S+60$

$\cancel{a_1^2}+\cancel{23a_1d}+\cancel{112d^2}+\cancel{S}+\cancel{60}>\cancel{a_1^2}+\cancel{23a_1d}+\cancel{130}d^2+\cancel{S}+\cancel{27}$

$33 \qquad 18d^2$

$d^2<\dfrac{33}{18} \quad \Rightarrow d^2<2 \Rightarrow \boxed{d=1}$

$a_1^2+23a_1+112>7a_1+21+27.$

$a_1^2+16a_1+64>0.$

$(a_1+8)^2>0. \Leftrightarrow \boxed{a_1\neq-8.}$

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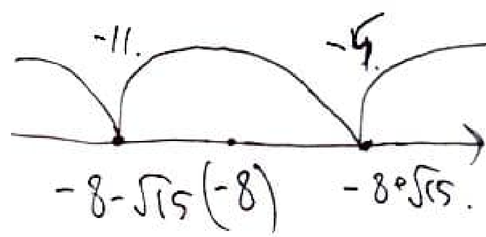
$D=256-196=60.$

$a_{12}=\dfrac{-16\pm\sqrt{60}}{2}=-8\pm\sqrt{15}.$



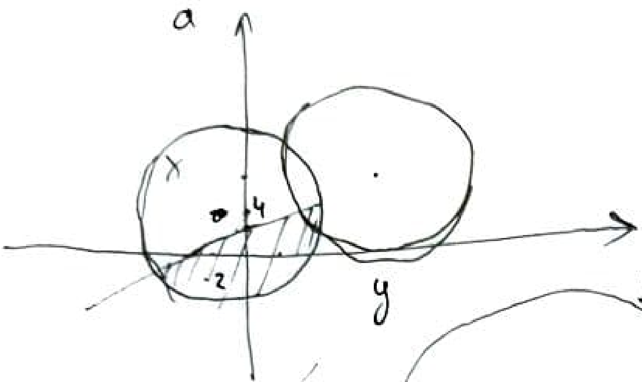
$$\begin{cases} (x-a)^2+(y-b)^2 \leqslant 20 \\ a^2+b^2 \leqslant \min(8a-4b, 20) \end{cases} \Leftrightarrow \begin{cases} (x-a)^2+(y-b)^2 \leqslant 20 \\ a^2+b^2 \leqslant 8a-4b, \text{ при } 8a-4b<20 \\ a^2+b^2 \leqslant 20, \text{ при } 8a-4b \geqslant 20 \end{cases}$$

$a^2+b^2 \leqslant 8a-4b$

$\Updownarrow$

$(a-4)^2+(b+2)^2 \leqslant 20$

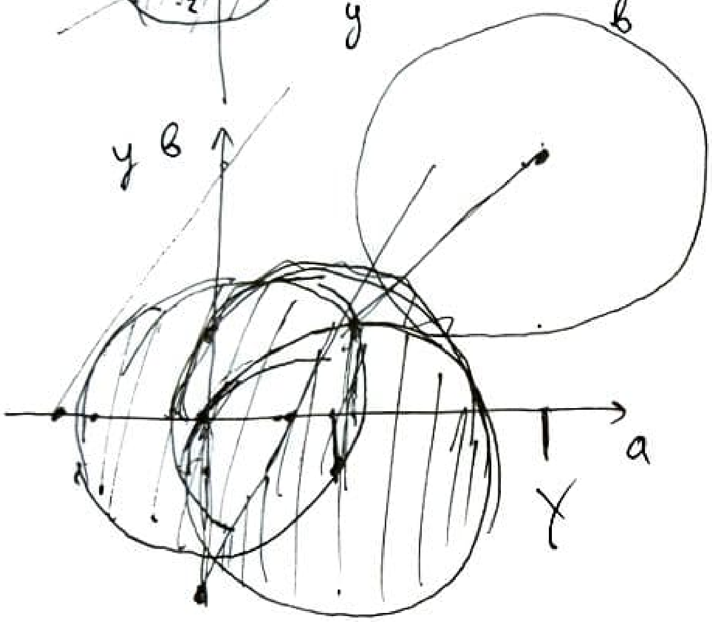
$a < -\frac{b}{2}+\frac{5}{2}$



$\cancel{4b > 8}$

$\boxed{b > 2a - \cancel{20}\,5}$

$b \leqslant 2a-5$



$\cancel{\begin{cases} (y+2)^2+(x-4)^2 \leqslant (2\sqrt{20})^2 \\ x^2+y^2 \leqslant 2\sqrt{20} \end{cases}}$

$\cancel{(a-x)^2+(b-y)^2}$

$(x-4)^2+(y+2)^2 \leqslant (\sqrt{20})^2$

Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21101289**

ID профиля: **74985**

Вариант 21

Задача №4

$$\begin{cases} НОД(a,b,c)=35 \\ НОК(a,b,c)=5^{18}\cdot 7^{36} \end{cases} \Rightarrow \begin{matrix} a=35\cdot p \\ b=35q \\ c=35\cdot k \end{matrix}$$, причем $НОД(p,q,k)=1$

$\Rightarrow НОК(a,b,c)=5^{18}\cdot 7^{36}=35\cdot p\cdot q\cdot k \Leftrightarrow \boxed{p\cdot q\cdot k=5^{17}\cdot 7^{35}}$

$\underline{5^{17}}$ $\quad \underline{7^{i}}$ $\quad \underline{7^{35-i}}$ $\rightarrow$ 36 вариантов

$\underline{5^{16}}$ $\quad \underline{7^{i}\cdot 5^{1}}$ $\quad \underline{7^{35-i}}$ $\rightarrow$ $36\cdot 2$ вар.

$\underline{5^{15}}$ $\quad \underline{7^{i}\cdot 5^{2}}$ $\quad \underline{7^{35-i}}$ $\rightarrow$ $36\cdot 3$ вар.

$\vdots \quad \vdots \quad \vdots$

$\underline{5}$ $\cdot$ $\underline{7^{i}\cdot 5^{16}}$ $\quad \underline{\cancel{35}\,7^{35-i}}$ $\rightarrow$ $36\cdot 17$ вар.

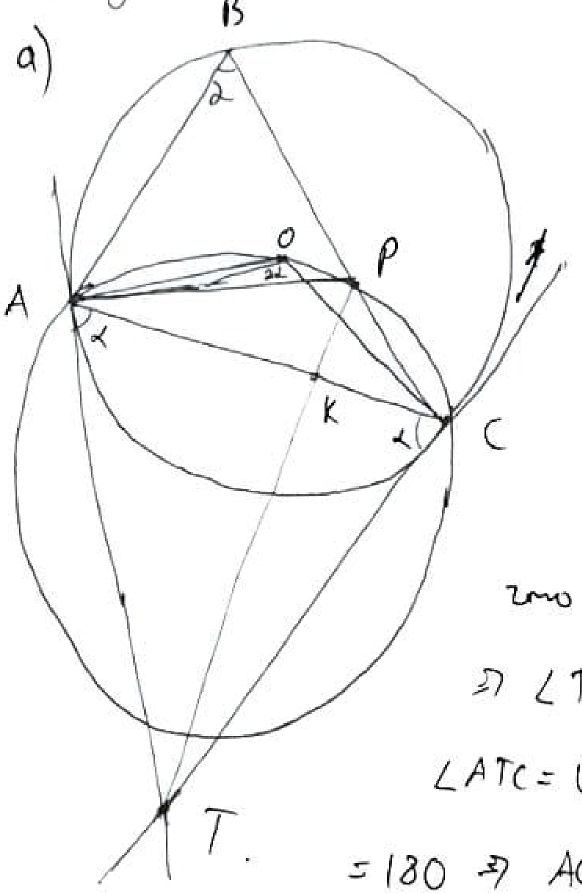
Всего вариантов: $36\cdot(1+2+\ldots+17)\cdot 6$ (умножаем на 6, т.к. $(a,b,c)\,(b,a,c)\,(b,c,a)\,(a,c,b)\,(c,a,b)\,(c,b,a)$ – разные)

$\boxed{\text{Ответ: } 36\,(1+2+\ldots+17)\cdot 6=33048}$

Вариант 21. Часть 2. Чистовик

Задача № 6

а)



$S_{APK} = 12$

$S_{CPK} = 9$

Докажем, что T - лежит на окр-ти описанной вокруг $\triangle AOC$. Пусть $\angle ABC = \alpha \Rightarrow$

$\angle AOC = 2\alpha$ (центральный угол)

$\triangle AOC$ - равнобедренный ($AO = OC$ радиусы) $\Rightarrow$

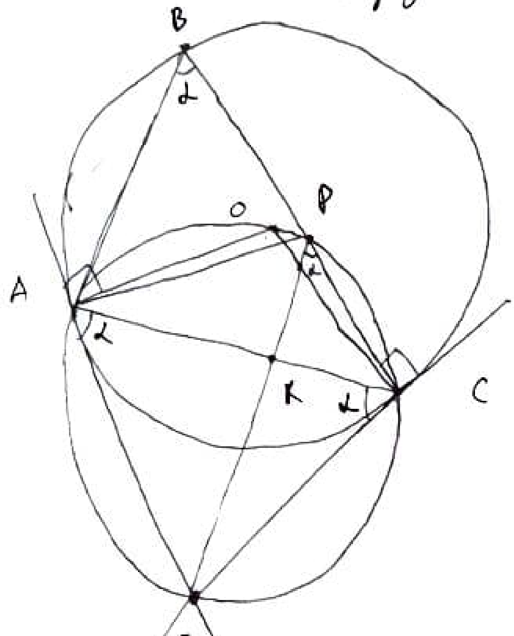
$\angle OCA = \angle OAC = \frac{180 - 2\alpha}{2} = 90 - \alpha$. Заметим, что $OC \perp CT$ и $AO \perp AT$ (радиус к касательной)

$\Rightarrow \angle TAC = \angle TCA = 90 - \angle OCA = 90 - \angle OAC = \alpha \Rightarrow$

$\angle ATC = 180 - \alpha - \alpha = 180 - 2\alpha \Rightarrow \angle AOC + \angle ATC = 2\alpha + 180 - 2\alpha =$

$= 180 \Rightarrow AOCT$ - вписанный четырехугольник $\Rightarrow$

T - лежит на этой окружности, перерисуем:



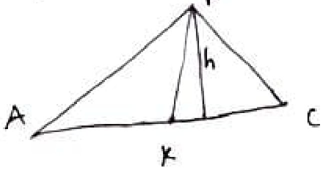
$\angle TPC = \angle TAC$ (вписанные углы) $\Rightarrow$

рассмотрим $\triangle ABC$ и $\triangle KPC$:

$\angle KCP$ - общий $\angle KPC = \alpha = \angle ABC \Rightarrow$

$\triangle ABC \sim \triangle KPC \Rightarrow S_{ABC} = S_{KPC} \cdot k^2$.

рассмотрим $\triangle APC$:



$S_{KPC} = h \cdot \frac{KC}{2} = 9$

$S_{APK} = h \cdot \frac{AK}{2} = 12$ $\Rightarrow$

$\frac{KC}{AK} = \frac{9}{12} \Leftrightarrow \boxed{KC = AK \cdot \frac{9}{12}} \Rightarrow AC = AK + KC = \frac{7}{4} AK \Rightarrow k = \frac{AC}{KC} = \frac{7}{3} \Rightarrow$

$$\boxed{S_{ABC} = S_{CPK} \cdot k^2 = 9 \cdot \frac{49}{9} = 49}$$

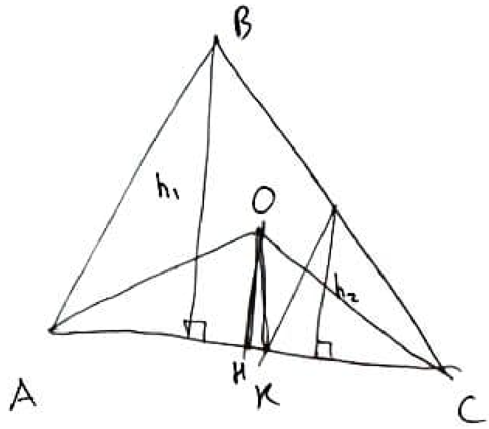
Вариант 21. Часть. Чистовик

Задача № 6 (продолжение)

б) $\angle ABC = \alpha = \operatorname{arctg} \frac{3}{7}$ $S_{ABC} = \cancel{8}\ S_{ABK} + S_{CBK} = 21$

$h_1 = \frac{AC}{2} = 49$

$h_2 = \frac{KC}{2} = 9$ $\Rightarrow \frac{h_1}{h_2} = \frac{7}{3}$



OH – высота $\Rightarrow \angle HOC = \alpha$ (биссектриса).

$\operatorname{tg}\alpha = \frac{AC}{2h_2} \Rightarrow \frac{AC}{2\cdot\frac{18}{\frac{3}{7}AC}} \Rightarrow AC = \sqrt{\frac{7}{3}\operatorname{tg}\alpha\cdot 2\cdot 18} = 6$

Ответ: а) $S_{ABC} = 49$

б) $AC = 6$

Вариант 21. Часть 2. Чистовик

Задача № 5

$\log_{\sqrt{2x-3}}(x+1)$, $\log_{2x^2-3x+5}(2x-3)^2$, $\log_{x+1}(2x^2-3x+5)$

ОДЗ:
$x>\frac{3}{2}$
$2x^2-3x+5>0$
$2x^2-3x+5\neq 1$
$x\neq 0$
$x>-1$
$x\neq 2$

$a=2x-3$
$b=x+1$
$c=2x^2-3x+5$

$\Rightarrow 2\log_a b,\ 2\log_c a,\ \log_b c$

Пусть $\begin{cases}2\log_a b=2\log_c a\\ \log_b c+1=2\log_c a\end{cases} \Leftrightarrow \begin{cases}\frac{\ln b}{\ln a}=\frac{\ln a}{\ln c}\\ \frac{\ln c+\ln b}{\ln b}=2\frac{\ln a}{\ln c}\end{cases} \Rightarrow$

$(\ln b+\ln c)\cdot\ln c=2\ln a\cdot\ln b \Leftrightarrow \ln^2 a\cdot\ln c+\ln^3 c=2\ln^3 a$

$\ln a\equiv t$, $\ln c\equiv m$ $\Rightarrow t^2m+m^3=2t^3$. $\boxed{t=m}$ – является корнем $\Rightarrow$

$2t^3-t^2m-m^3=(t-m)(2t^2+mt+m^2)=0 \Leftrightarrow \left[\begin{array}{l}t=m\\ 2t^2+mt+m^2=0\end{array}\right. \Rightarrow$

$D=m^2-8m^2=-7m^2<0 \Rightarrow \boxed{t=m} \Leftrightarrow \ln a=\ln c \Leftrightarrow a=c \Leftrightarrow$

$2x-3=2x^2-3x+5 \Leftrightarrow 2x^2-5x+8=0$ $D=25-64<0$. нет решений

Пусть $\begin{cases}2\log_a b=\log_b c\\ \log_b c=2\log_c a+1\end{cases} \Leftrightarrow \begin{cases}\frac{2\ln b}{\ln a}=\frac{\ln c}{\ln b}\\ \frac{\ln c}{\ln b}=\frac{2\ln a+\ln c}{\ln c}\end{cases} \Rightarrow$

$\ln^2 c=2\ln a\cdot\ln b+\ln c\cdot\ln b \Leftrightarrow \ln^3 c=4\ln^3 b+\ln^2 c\ln b$ | $t=\ln c$, $m=\ln b$.

$t=2m$ – корень $\Rightarrow t^3-mt^2-4m^3=(t-2m)(t^2+mt+2m^2)=0 \Leftrightarrow$

$\left[\begin{array}{l}t=2m\\ t^2+mt+2m^2\end{array}\right. \Rightarrow D=m^2-8m^2<0 \Rightarrow t=2m \Leftrightarrow \ln c=\ln b^2 \Leftrightarrow 2x^2-3x+5=x^2+2x+1 \Leftrightarrow$

$x^2-5x+4=0 \Rightarrow D=25-16=9$ $x_{1,2}=\frac{5\pm 3}{2}=\left[\begin{array}{l}4\\ 1<\frac{3}{2}\end{array}\right. \Rightarrow \boxed{x=4}$.

Задача №5 (продолжение)

Пусть

$$\begin{cases} 2\log_c a = \log_b c \\ \log_b c = 2\log_a b + 1 \end{cases} \Leftrightarrow \begin{cases} \dfrac{2\ln a}{\ln c} = \dfrac{\ln c}{\ln b} \\ \dfrac{\ln c}{\ln b} = \dfrac{2\ln b}{\ln a} + \dfrac{\ln a}{\ln a} \end{cases} \Rightarrow$$

$$\ln c \cdot \ln a = (2\ln b + \ln a)\ln b = 2\ln^2 b + \frac{\ln^2 c}{2} = \ln c \cdot \frac{\ln^2 c}{2\ln b} \Leftrightarrow$$

$$4\ln^3 b + \ln^2 c \ln b = \ln^3 c \quad \Big| \; t = \ln c \; ; \; m = \ln b \Rightarrow$$

$$t^3 - mt^2 - 4m^3 = 0 \quad t = 2m - \text{корень} \Rightarrow t^3 - mt^2 - 4m^3 = (t-2m)(t^2 + mt + 2m^2) =$$

$$= 0 \Leftrightarrow \left[\begin{array}{l} t = 2m \\ t^2 + mt + 2m^2 = 0 \end{array}\right. \Rightarrow D = m^2 - 8m^2 < 0 \Rightarrow t = 2m \quad \ln c = 2\ln b \Leftrightarrow \boxed{x = 4}$$

(См. предыдущий ответ).

Ответ: $\boxed{x = 4}$

$$\frac{2\ln a}{\ln c} = \frac{\ln c}{\ln b}$$

$$\frac{\ln c}{\ln b} = \frac{2\ln b + \ln a}{\ln a}$$

$$\ln c \cdot \ln a = 2\ln^2 b + \ln a \cdot \ln b = 2\ln^2 b + \frac{\ln^2 c}{2} = \ln c \cdot \frac{\ln^2 c}{2\ln b}$$

$$2\ln a \cdot \ln b = \ln^2 c$$

$$4\ln^2 b + \ln^2 c = \frac{\ln^3 c}{\ln b}$$

$$4\ln^2 b \cdot \ln b + \ln^2 c \cdot \ln b = \ln^3 c$$

$$t^3 - mt^2 - 4m^3 = 0 \qquad \boxed{t = 2m}$$

$$\begin{array}{l|l} t^3 - mt^2 - 4m^3 & t - 2m \\ t^3 - 2mt^2 & (t^2 + mt + 2m^2)(t - 2m) = 0 \\ \hline mt^2 - 4m^3 & \\ mt^2 - 2m^2t & \\ \hline 2m^2t - 4m^3 & \\ 2m^2t - 4m^3 & \\ \hline \end{array}$$

$$h_1 \cdot \frac{AC}{2} = 49$$

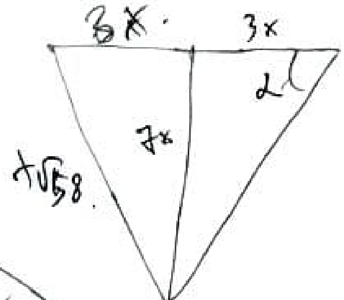
$$h_2 \cdot \frac{KC}{2} = 9 \qquad \frac{h_1}{h_2} \cdot \frac{7}{3} = \frac{49}{9}$$

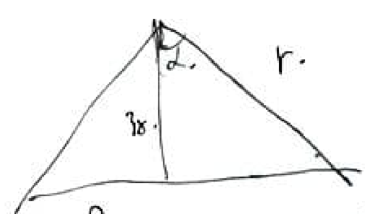
$$\frac{h_1}{h_2} = \frac{7}{3}$$

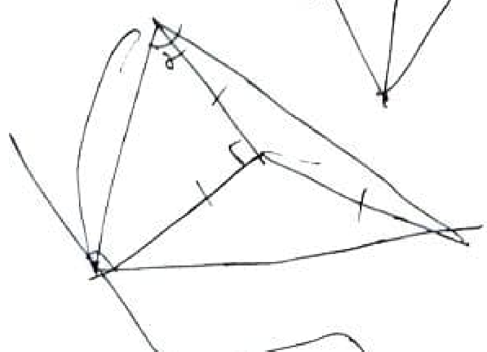
$$AC = \frac{7}{3} KC$$

$$\ln c = 2\ln b$$

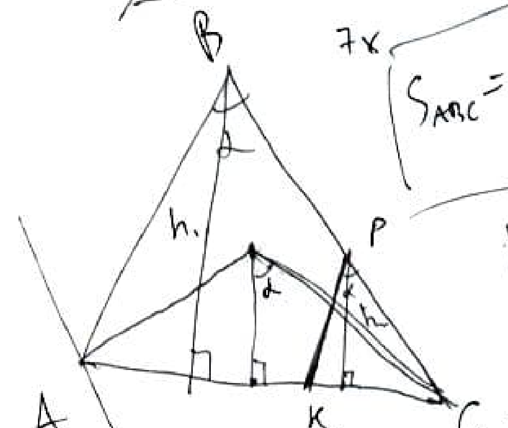
$$r = \sqrt{58}$$







$$\boxed{S_{ABC} = 49}$$



$$h_2 = \frac{18}{\frac{3}{7}AC}$$

$$\boxed{r = \frac{AC}{2\sin\alpha}}$$

$$tg\,\alpha = \frac{AC}{2h_2}$$

$$tg\,\alpha = \frac{AC}{2 \cdot \frac{18}{\frac{3}{7}AC}} \Rightarrow AC^2 = \frac{7\,tg\,\alpha \cdot 2 \cdot 18}{3} =$$

$2\log_a b = \log_b c = \frac{2\ln b}{\ln a} = \frac{\ln c}{\ln b}$ $\quad \ln a = \frac{2\ln^2 b}{\ln c}$

$\log_b c = 2\log_c a + 1 = \frac{\ln c}{\ln b} = \frac{2\ln a + \ln c}{\ln c}$

$\ln^2 c = 2\ln a \cdot \ln b + \ln c \cdot \ln b$

$\ln^2 c = 2 \cdot \frac{2\ln^3 b}{\ln c} + \ln^2 c \cdot \ln b$ $\quad \ln^3 c = 4\ln^3 b + \ln^2 c \ln b$

$\ln c = 2\ln b$

$\ln^3 c - 4\ln^3 b - \ln^2 c \ln b$

$8 - 4 - 4$

$t^3 - 4m^3 - t^2 m \mid t - 2m$

$t^3 - 2mt^2$

$(t^2 + mt + 2m^2)(t - 2m) = 0$

$mt^2 - 4m^3$

$mt^2 - 2m^2 t$

$2m^2 t - 4m^3$

$2m^2 t - 4m^3$

$\cancel{D = m^2}$ $\quad t = 2m$

$\ln c = \ln b^2$

$c = b^2$ $\quad 2x^2 - 3x + 5 = x^2 + 2x + 1$

$x^2 - 5x + 4 = 0$ $\quad D = 25 - 16 = 9$

$x_{1,2} = \frac{5 \pm 3}{2} = 4$, 1

$\log_{\sqrt{2x-3}}(x+1)$ $\log_{2x^2-3x+5}(2x-3)^2$ $\log_{x+1}(2x^2-3x+5)$.

$x>-1$ $x\neq 0$

$x\neq\frac{3}{2}$

$\sqrt{2x-3}=a.$

$x+1=b.$

$2x^2-3x+5=c.$

$\ln a\neq 0$

$\ln c\neq 0.$

$2x^2-3x+5>0.$

$D=9-80$

$$\begin{cases} 2\log_a b=\log_c a^2 \\ \log_b c+1=2\log_c a. \end{cases} \Leftrightarrow$$

$\log_a b=\log_c a.=\frac{\ln b}{\ln a}=\frac{\ln a}{\ln c}\Rightarrow$

$\ln^2 a=\ln b\cdot\ln c.$

$\frac{\ln c}{\ln b}+\frac{\ln b}{\ln b}=2\frac{\ln a}{\ln c}=\frac{\ln(bc)}{\ln b}=\frac{2\ln a}{\ln c}$

$(\ln b+\ln c)\ln\cdot c=2\ln a\cdot\ln b.$

$\ln^2 a+\ln^2 c=2\ln a\cdot\frac{\ln^2 a}{\ln c}$

$\ln^2 a\ln c+\ln^3 c=2\ln^3 a.$

$t=\ln a$

$m=\ln c.$

$t^2\cdot m+m^3=2t^3$

$(2t^2+mt+m^2)(t-m)=0.$

$$\begin{array}{l|l} 2t^3-t^2m-m^3 & t-m \\ \hline 2t^3-2t^2m & \boxed{2t^2+mt+m^2} \\ \hline t^2m-m^3 & \\ t^2m-m^2t & \\ \hline m^2t-m^3 & \end{array}$$

$\cancel{D=m^2-8m^2}<0.$

$\boxed{t=m}$

$\ln a=\ln c.$

$a=c.$

$2x-3=2x^3-3x+5$

$2x^2-5x+8=0$

$D=25-$

~~a · b · c~~

$a = 35p$ $\quad 35p \cdot kq = 5^{\cancel{18}} \cdot 7^{36}$

$b = 35k$

$c = 35q$ $\quad p \cdot k \cdot q = 5^{17} \cdot 7^{35}$

18 · ___ ___ ___

НОД$(p, k, q) = 1$

$1 + \_ \quad 17 =$

$\boxed{18 \cdot 36 \cdot 3} = 6^3 \cdot 3^2$ $\quad \frac{(1+17) \cdot 17}{2} = 9 \cdot 17$

$5^{17} \cdot \underline{7^{35}}$ ___ ... $\quad 36 \cdot$ $\quad \begin{array}{r} \times 17 \\ 9 \\ \hline 153 \end{array}$

$5^{16} \quad 7^k \cdot 5 \quad \underline{7^{35-k}} \quad 36 \cdot 2$

$5^{15} \quad 7^k \cdot 5^2 \quad 7^{35-k} \quad 36 \cdot 3$

$5^{14} \quad 7^k \cdot 5^3 \quad 7^{35-k} \quad 36 \cdot 4 \dots$

$\vdots$

$5 \quad 7^k \cdot 5^{16} \cdot 7^{35-k}. \quad 36 \cdot 17.$

~~$1 \quad 7^k \cdot 5^{17}$~~

$36 \cdot 18.$

$\begin{array}{r} \times 36 \\ 6 \\ \hline 216 \end{array}$ $\quad \begin{array}{r} \times 153 \\ 216 \\ \hline 918 \\ 153 \\ 306 \\ \hline 33048 \end{array}$

$\boxed{36 \cdot \sum_{1}^{17} i \cdot 3.}$

$\alpha = \operatorname{arctg}\frac{3}{7}$.

$2\alpha = 2\operatorname{arctg}\frac{3}{7}$.

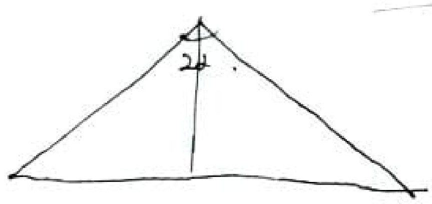
$S_{ADC} = 21$!

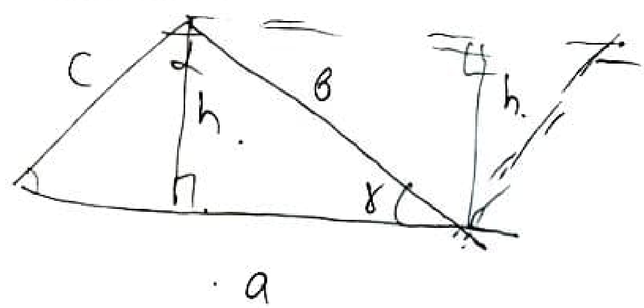
$$\boxed{r = \frac{AC}{2\sin\alpha}}$$

$$S_{ADC} = \frac{AC\cdot h}{2} = \frac{AC\cdot r\cos\alpha}{2} = 21 = \frac{AC\cdot\frac{AC}{2\sin\alpha}\cdot\cos\alpha}{2} = 21.$$

$$AC^2\cdot\frac{\operatorname{ctg}\alpha}{4} = 21 \qquad 84\cdot\operatorname{tg}\alpha = AC^2$$

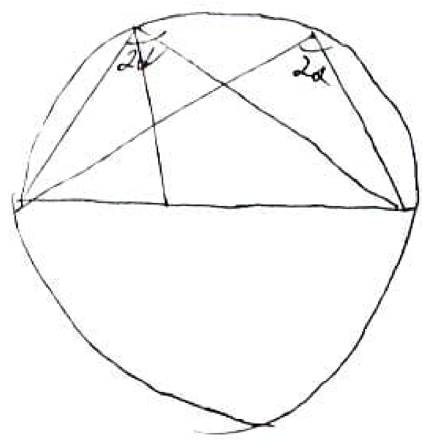
$$\boxed{AC = \sqrt{84\cdot\tfrac{3}{7}} = 6.}$$

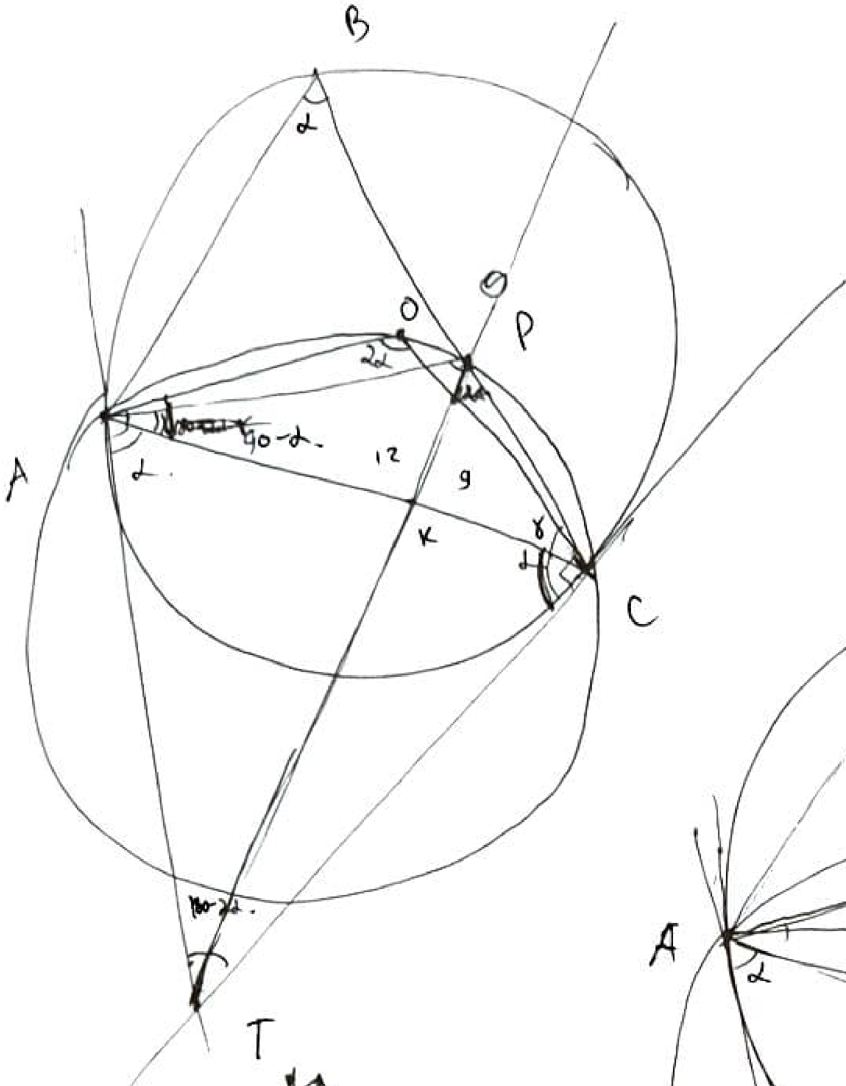




$$\frac{ah}{2} = b\,\cancel{\cos}\sin\gamma = c\cdot\sin(\alpha+\gamma).$$

$$a^2 = b^2 + c^2 - 2bc\cdot\cos\alpha.$$



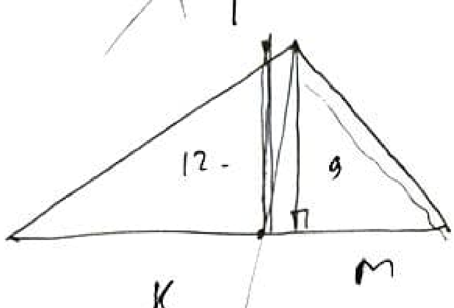


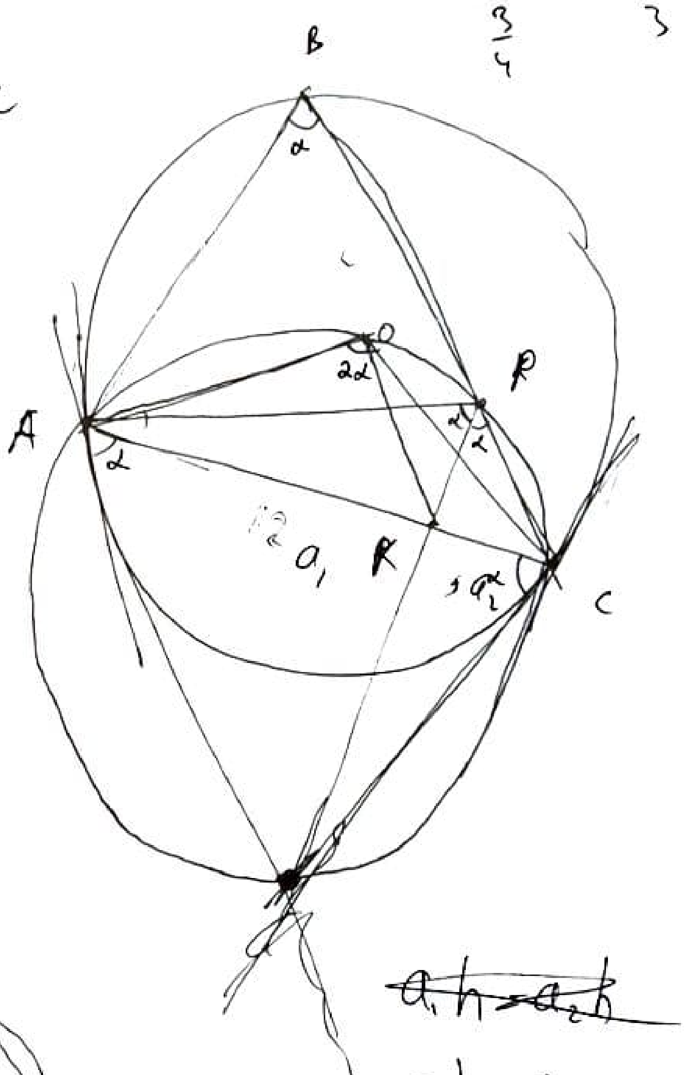
$180 - 2\alpha - \gamma$ $\quad S \sim k^2$

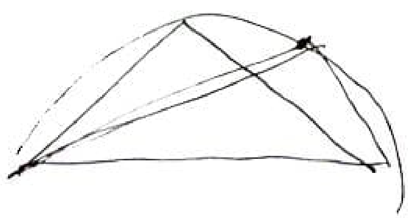
$\frac{a_1}{a_2} = \frac{4}{3}$ $\quad \frac{49}{9} \cdot 9 = \boxed{49}$

$a_2 = \frac{3}{4} a_1$

$\frac{\boxed{\frac{7}{4} a_1}}{\frac{3}{4}} = \frac{7}{3}$



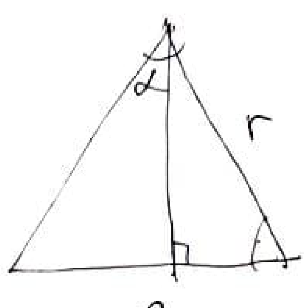




$\cancel{a_1 h = a_2 h}$

$a_1 h = 24$

$a_2 h = 18$



$a \cdot h = a \cdot r \sin\alpha$

$\boxed{\frac{\arccos\alpha}{2}}$

$\frac{a_1}{a_2} = \frac{24}{18} = \frac{12}{9}$

$\frac{21}{9} \cdot 9 = 21$