Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21104090**

ID профиля: **136785**

Вариант 19

Чистовик

① S - сумма п. 14 членов $\Rightarrow S_{14} = \frac{2a_1 + 13r}{2} \cdot 14 = 14a_1 + 91r$

$$\begin{cases} a_9 a_{17} > S + 12 \\ a_{11} a_{15} < S + 47 \end{cases} \Leftrightarrow \begin{cases} (a_1 + 8r)(a_1 + 16r) > S + 12 \\ (a_1 + 10r)(a_1 + 14r) < S + 47 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} a_1^2 + 24a_1 r + 128r^2 > S + 12 \\ a_1^2 + 24a_1 r + 140r^2 < S + 47 \end{cases} \Rightarrow \left.\begin{matrix} 12r^2 < 35 \\ r > 0 \\ r \in Z \end{matrix}\right\} \Rightarrow r = 1$$

$$\begin{cases} a_1^2 + 24a_1 + 128 > 14a_1 + 91 + 12 \\ a_1^2 + 24a_1 + 140 < 14a_1 + 91 + 47 \end{cases} \Leftrightarrow \begin{cases} a_1^2 + 10a_1 + 25 > 0 \\ a_1^2 + 10a_1 + 2 < 0 \end{cases}$$

$a_1^2 + 10a_1 + 25 > 0$

$\Delta = 100 - 100 = 0$

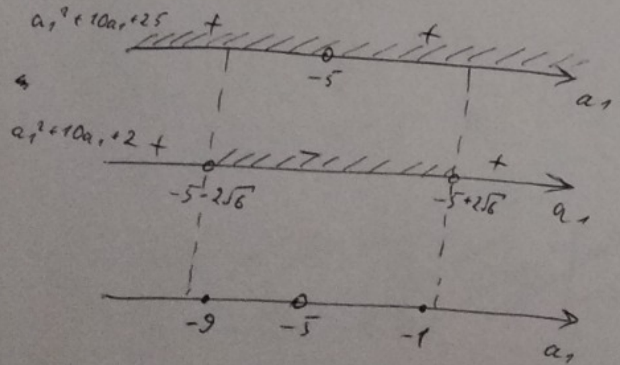
$a_1 = -\frac{10}{2} = -5$

$a_1^2 + 10a_1 + 2 < 0$

$\Delta = 100 - 8 = 92$

$$\left[\begin{matrix} a_1 = \frac{-10 - 4\sqrt{6}}{2} \\ a_1 = \frac{-10 + 4\sqrt{6}}{2} \end{matrix}\right. \Leftrightarrow \left[\begin{matrix} a_1 = -5 - 2\sqrt{6} \\ a_1 = -5 + 2\sqrt{6} \end{matrix}\right.$$

$$\begin{cases} -10 < -5 - 2\sqrt{6} < -9 \\ -1 < -5 + 2\sqrt{6} < 0 \end{cases}$$

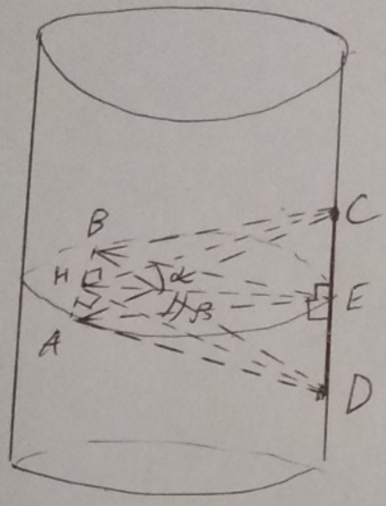


$a_1 \in \{-9; -8; -7; -6; -4; -3; -2; -1\}$

Ответ: $a_1 \in \{-9; -8; -7; -6; -4; -3; -2; -1\}$

Чистовик.

2



ΔCAB – равноб. $\Rightarrow AH = BH$ (H – сер. AB)
ΔDAB – равноб. $\Rightarrow H$ – сер. AB $\Bigg\} \Rightarrow$

$\Rightarrow AB \perp CD$ ($CH \perp AB$; $DH \perp AB$; $CH, DH \in$ одной пл.)

$$\begin{cases} CH \cdot \cos\alpha = DH \cdot \cos\beta \\ R \text{ опис. окр. около } \Delta ABE - \min \end{cases}$$

$$\left.\begin{array}{l} \cancel{R} = \frac{ab^2}{4R} = \frac{ah}{2} \Rightarrow R = \frac{b^2}{2h} \\ h^2 = b^2 - \frac{a^2}{4} \end{array}\right\} \Rightarrow R = \frac{h^2 + \frac{a^2}{4}}{2h} = \frac{h}{2} + \frac{a^2}{8h}$$

Найдем минимум функции $R(h) = \frac{h}{2} + \frac{a^2}{8h}$

$$R'(h) = \frac{1}{2} - \frac{a^2}{8h^2} = 0$$

$$\frac{a^2}{8h^2} = \frac{1}{2}$$

$$\left.\begin{array}{l} \frac{a^2}{4} = h^2 \\ a = 2 \end{array}\right\} \Rightarrow \left[\begin{array}{l} h = 1 - \checkmark \ (HE) \\ h = -1 \ \varnothing \ (h > 0) \end{array}\right.$$

$$\cancel{\left[\begin{array}{l} CD = CE + ED \\ CD = |CE - ED| \end{array}\right.} \Leftrightarrow \cancel{\left[\begin{array}{l} CD = HE \sin\alpha + HE \sin\beta \\ CD = |HE \sin\alpha - HE \sin\beta| \end{array}\right.} \Leftrightarrow \cancel{\left[\begin{array}{l} CD = HE(\sin\alpha + \sin\beta) \\ CD = HE\,|\sin\alpha - \sin\beta| \end{array}\right.}$$

$$\cos\alpha = \frac{HE}{HC} = \frac{1}{\sqrt{6^2 - 1^2}} = \frac{1}{\sqrt{35}} \qquad \sin\alpha = \sqrt{1 - \cos^2\alpha} = \sqrt{\frac{34}{35}}$$

$$\cos\beta = \frac{HE}{HD} = \frac{1}{\sqrt{7^2 - 1^2}} = \frac{1}{\sqrt{48}} \qquad \sin\beta = \sqrt{1 - \cos^2\beta} = \sqrt{\frac{47}{48}}$$

$$\left[\begin{array}{l} CD = CE + ED \\ CD = |CE - ED| \end{array}\right. \Leftrightarrow \left[\begin{array}{l} CD = CH \sin\alpha + DH \cdot \sin\beta \\ CD = |CH \sin\alpha - DH \sin\beta| \end{array}\right. \qquad \begin{array}{l} CH = \sqrt{6^2 - 1^2} = \sqrt{35} \\ DH = \sqrt{7^2 - 1^2} = \sqrt{48} \end{array}$$

$$\left[\begin{array}{l} CD = \sqrt{34} + \sqrt{47} \\ CD = \sqrt{47} - \sqrt{34} \end{array}\right.$$

Ответ: $CD = \sqrt{34} + \sqrt{47}$
$CD = \sqrt{47} - \sqrt{34}$

Черновик

14

1. $\begin{cases} a_9 a_{17} > S + 12 \\ a_{11} a_{15} < S + 47 \end{cases}$ $\begin{cases} a_1^2 + 24a_1 r + 128 r^2 > S + 12 \\ a_1^2 + 24a_1 r + 140 r^2 < S + 47 \end{cases}$

c=

$a_1^2 + 24a_1 + 128 > 14a_1 + 103$

$a_1^2 + 24a_1 + 140 < 14a_1 + 138$

128

153 >

$S = \frac{2a_1 + 13r}{2} \cdot 14 = 14a_1 + 91r$

$7 \cdot 15 = 105$

140

$12r^2 < 35$

$8 \cdot 16 > 103$

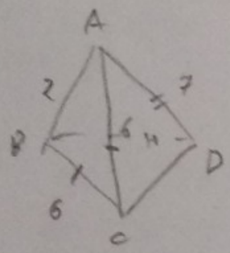
$r = 1$

103

$a_9 a_{17} = (a_1 + 8r)(a_1 + 16r) = a_1^2 + 24a_1 r + 128 r^2 > 14a_1 + 91r + 12$

R -

$a_1^2 - 14a_1 + 24a_1 r + 128r^2 - 91r - 12 > 0$



$\sqrt{35} \cdot \sin\alpha + \sqrt{48} \cdot \sin\beta$ - min

$\sqrt{35}$ $\frac{35\cos^2\alpha + 1}{2\sqrt{35}\cos\alpha} = \frac{48\cos^2\beta + 1}{8\sqrt{3}\cos\beta}$

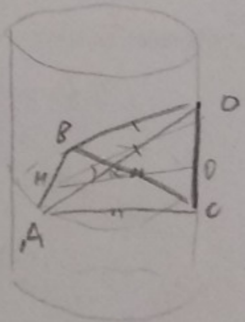
OD

$\frac{ha}{2} = \frac{ab^2}{4R}$

$\sin\alpha = \frac{OD}{HD} \Rightarrow OD = HD \cdot \sin\alpha$

$h = \frac{b^2}{2R}$

$a_1^2 + 10a_1 + 25 > 0$



$2R = \frac{b^2}{h}$ $R = \frac{h^2 + 1}{2h}$

$a_1^2 + 10a_1 + 2 < 0$

$AB \perp CD$

$R = \frac{b^2}{2h}$ $\frac{h}{2} + \frac{1}{2h}$

$(a_1 + 5)^2 > 0$

$\frac{\sqrt{35}\cos\alpha}{2} + \frac{1}{2\sqrt{35}\cos\alpha}$ - min

$\sqrt{48}$ $4\sqrt{3}$

$h = \sqrt{36 - 1} = \sqrt{35}$

$a_1^2 + 10a_1 + 2 < 0$

$\Delta = 100 - 8 = 92 = 48 \cdot 2 = 4\sqrt{6}$

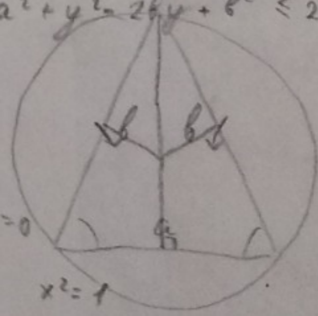
$h^2 = b^2 - \frac{a^2}{4}$ $b^2 = h^2 + \frac{a^2}{4}$

$\frac{4\sqrt{6} - 10}{2}$

$x^2 - 2ax + a^2 + y^2 - 2by + b^2 \le 25$

$\begin{cases} (x-a)^2 + (y-b)^2 \le 25 \\ a^2 + b^2 \le \min(-8a - 6b, 25) \end{cases}$

$\sqrt{35}\cos\alpha + \frac{1}{\sqrt{35}\cos\alpha}$



$\frac{ab^2}{2 \cdot 4R} =$

$\left(x + \frac{1}{x}\right)' = 1 - \frac{1}{x^2} = 0$

$\sin = \sqrt{1 - \frac{1}{35}}$

$1 = \frac{1}{x^2}$ $x^2 = 1$

$\sqrt{35} \cdot \sqrt{\frac{34}{35}} + \sqrt{48}\sqrt{\frac{47}{48}} =$

$a^2 + b^2 \le 25 - x^2 - y^2 + 2ax + 2by$

$x = 1$

$x = -1$

$= \sqrt{\left(\frac{a}{2} + b\right)\left(b - \frac{a}{2}\right)\frac{a^2}{4}} =$

$= \sqrt{34} + \sqrt{47}$

$\sqrt{24} - \sqrt{25} \approx -0,\ldots$

$\sqrt{24} + \sqrt{25} \approx 9,\ldots$

$\left[\begin{array}{l} \sqrt{35}\cos\alpha = 1 \\ \sqrt{35}\cos\alpha = -1 \end{array}\right.$

$\sqrt{47} - \sqrt{34}$

$p = \frac{a}{2} + b$

$= \frac{a}{2}\sqrt{b^2 - \frac{a^2}{4}}$

$\cos\alpha = \frac{\sqrt{35}}{35}$

$2\sqrt{6} - 5$

$\cos\alpha = -\frac{\sqrt{35}}{35}$

$\frac{b^2}{R} = \sqrt{b^2 - \frac{a^2}{4}} \Rightarrow R = \frac{b^2}{\sqrt{b^2 - \frac{a^2}{4}}}$

$-2\sqrt{6} - 5$

$R = \frac{36}{\sqrt{36 - 1}} = \frac{36}{\sqrt{35}} = \frac{36}{35}\sqrt{35}$

$2\sqrt{6} - 5$

$2\sqrt{6} + 5$

Черновик

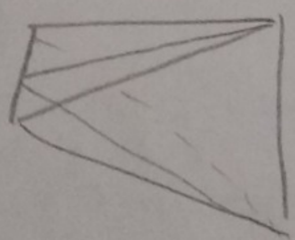
$$\begin{cases}(x-a)^2+(y-b)^2 \le 25 \\ a^2+b^2 \le \min(-8a-6b, 25)\end{cases}$$

$$x^2+y^2+a^2+b^2-2ax-2by \le 25$$

$$\left[\begin{array}{l}\begin{cases}x^2+y^2-2ax-2by \le 0 \\ -8a-6b \ge 25\end{cases} \\ \\ \begin{cases}x^2+y^2-8a-6b-2ax-2by \le 25 \\ -8a-6b \le 25\end{cases}\end{array}\right.$$

$x=-4$
$y=-3$?

$$x^2+y^2-2a(4+x)-2b(3+y) \le 25$$



Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

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Вариант 19

Чистовик

1) НОД $-$ 21 $\Rightarrow$ хотя бы одно из чисел можно представить как $3 \cdot 7^n$, $n \geq 1$

и хотя бы одно — как $3^m \cdot 7$, $m \geq 1$

НОК $- 3^{17} \cdot 7^{15}$ — хотя бы одно из чисел можно представить как $3^{17} \cdot 7^n$, $n \geq 1$,

и хотя бы одно — как $3^m \cdot 7^{15}$, $n \geq 1$

Распределение степеней числа 3:

$15 \cdot P_3 + 2 \frac{P_3}{P_2 \cdot P_1} = 15 \cdot 3! + 2 \frac{3!}{2! \cdot 1!} = 96$ $\quad$ ("1", "n", "17")

$1 \leq n \leq 17$

Распред. степеней числа 7:

$13 \cdot P_3 + 2 \frac{P_3}{P_2 \cdot P_1} = 13 \cdot 3! + 2 \frac{3!}{2! \cdot 1!} = 84$ $\quad$ ("1", "m", "15")

$1 \leq m \leq 15$

$96 \cdot 84 = 8064$ — все возможные комбинации

Ответ: 8064

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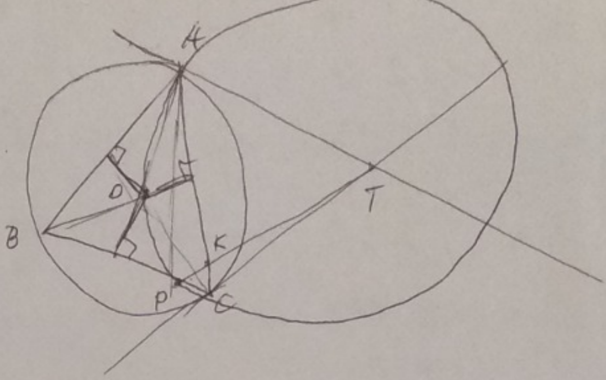
Черновик

$3 \cdot 7$

$3^{17} \cdot 7^{15}$



$3 \cdot 7$

$\angle AOC \equiv \angle APC$

$\log_{(\frac{x}{2}-1)^2}\left(\frac{x}{2}-\frac{1}{4}\right) = \log_{\sqrt{x-\frac{11}{4}}}\left(\frac{x}{2}-1\right)$

$\log_{(\frac{x}{2}-\frac{1}{4})}\left(x-\frac{11}{4}\right)^2 = \log_{\sqrt{x-\frac{11}{4}}}\left(\frac{x}{2}-1\right)+1$

$\frac{AP \cdot PC \cdot AC}{4R} = \frac{AO^2\,AC}{4R}$

$\log_{(\frac{x}{2}-\frac{1}{4})}\left(x-\frac{11}{4}\right)^2 = \log_{(\frac{x}{2}-1)^2}\left(\frac{x}{2}-\frac{1}{4}\right)+1$

$AP \cdot PC = AO^2$

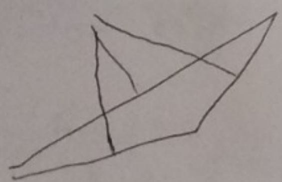
$\frac{1}{2}\log_{(\frac{x}{2}-1)}\left(\frac{x}{2}-\frac{1}{4}\right) = \frac{\log_{(\frac{x}{2}-1)} \cancel{\left(\frac{x}{2}-1\right)}\ 1}{\frac{1}{2}\log_{(\frac{x}{2}-1)}\sqrt{x-\frac{11}{4}}}$

$\frac{CK}{AK} = \frac{3}{5}$

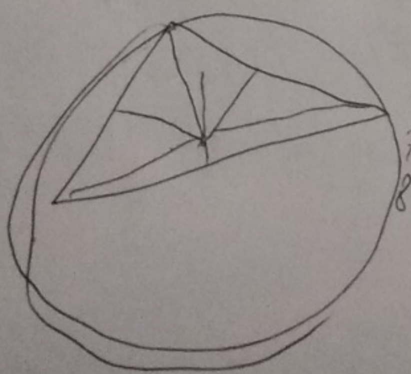
$\frac{1}{4}\log_{(\frac{x}{2}-1)}\left(\frac{x}{2}-\frac{1}{4}\right) \cdot \log_{(\frac{x}{2}-1)}\left(x-\frac{11}{4}\right) = 1$

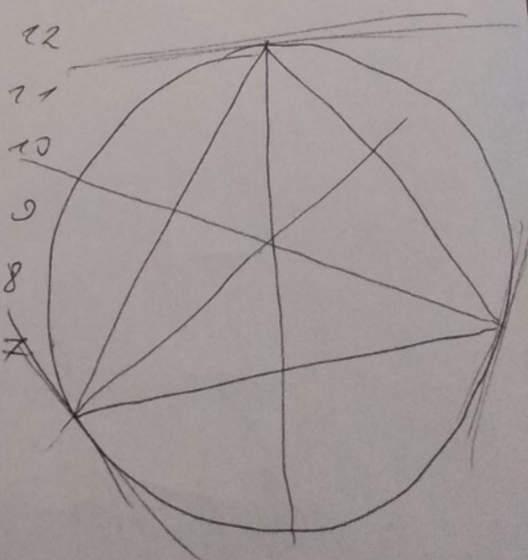
$$\begin{array}{r} 96 \\ \times\ 84 \\ \hline 384 \\ 768\ \ \\ \hline 8064 \end{array}$$

$\frac{1}{4}\left(\log_{(\frac{x}{2}-1)}(2x-1) - \log_{(\frac{x}{2}-1)}4\right)\left(\log_{(\frac{x}{2}-1)}(4x-11) - \log_{(\frac{x}{2}-1)}4\right) = 1$



1	15		1	1	13
2	14		1	2	12
3	13		1	3	11
4	12		1	4	10
5	11		1	5	9
6	10		1	6	8
7	9		1	7	7
8	8				





Черновик

1 1 15
1 2 14
1 3 13
1 4 12
1 5 11
1 6 10
1 7 9
1 8 8

42 × 36

$$\begin{array}{r} 42 \\ \times 36 \\ \hline 72 \\ 144 \\ \hline 1512 \end{array}$$

1 1 13
1 2 12
1 3 11
1 4 10
1 5 9
1 6 8
1 7 7

$tg\,\alpha = 2$

$\frac{\sin\alpha}{\cos\alpha} = 2$

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$S = \frac{1}{2} bc \sin\alpha$

$a^2 = b^2 + c^2 - 2S$

1 8 8 | 8 1 8 | 8 8 1
1 7 7 | 1 7 7 | 1 7 7

1 8 8 | 8 1 8 | 8 8 1
7 1 7 | 7 1 7 | 7 1 7

1 8 8 | 8 1 8 | 8 8 1
7 7 1 | 7 7 1 | 7 7 1

$\log_a b \cdot \log_a c$

$$\log_{(\frac{x}{2}-1)^2}\left(\frac{x}{2}-\frac{1}{4}\right) = \log_{\sqrt{x-\frac{11}{4}}}\left(\frac{x}{2}-1\right)$$

$a^2 = b^2 - h^2 + c^2 - h^2$

$a^2 = b^2 + c^2 - 2h^2$

$$\log_{(\frac{x}{2}-1)}\left(\frac{x}{2}-\frac{1}{4}\right) = 4\log_{(x-\frac{11}{4})}\left(\frac{x}{2}-1\right)$$

$$\frac{4}{\log_{(\frac{x}{2}-1)}\left(x-\frac{11}{4}\right)}$$

$2S = 2h^2$

$S = h^2$

$S = \frac{1}{2} ah$

$S = \frac{1}{2} a\sqrt{S}$

$a = \frac{2S}{\sqrt{S}}$

$$\log_{(\frac{x}{2}-1)}\left(\frac{x}{2}-\frac{1}{4}\right)$$

$$\log_{(\frac{x}{2}-1)}(2x-1) - \log_{(\frac{x}{2}-1)} 4 = 4\log_{(x-\frac{11}{4})}(x-2) - 4\log_{(x-\frac{11}{4})} 2$$

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