Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21103654**

ID профиля: **211218**

Вариант 19

$$\frac{-175 \pm 90\sqrt{2}}{8} = -21\tfrac{7}{8} \pm 11{,}25\sqrt{2}$$

-37 $\quad$ -6

$$\boxed{a_1 \in [-37; -6].}$$

$15-21$

11,25 × 1,42 = 15,9750

11,25 × 1,41 = 15,8625

48 × 91 = 4368 $\quad$ 91 × 91 = 8281

140 × 4 = 560 $\quad$ 560 × 14 = 7840

$$140d^2 + d(24a_1 - 91) + a_1^2 - 14a_1 - 47 < 0$$

$$D = 576a_1^2 - 4368a_1 + 8281 - 560a_1^2 + 7840a_1 + 26320 = 16a_1^2 + 3472a_1 + 34601$$

560 × 47 = 26320 $\quad$ 7840 − 4368 = 3472 $\quad$ 26320 + 8281 = 34601

$$d = \frac{91 - 24a_1 \pm \sqrt{16a_1^2 + 3472a_1 + 34601}}{280}$$

34601 : 7 = 4943

4943 : 13 = 37 ... 33

$$\begin{cases} a_9 a_{17} > S + 12 \\ a_{11} a_{15} < S + 47 \end{cases}$$

$$(a_1+8d)(a_1+16d) > \frac{2a_1+d\cdot 13}{2}\cdot 14 + 12$$

$$a_1^2 + 24a_1 d + 128d^2 > 14a_1 + 91d + 12$$

$$128d^2 + d(24a_1 - 91) + a_1^2 - 14a_1 - 12 > 0$$

1) $D < 0 \Rightarrow a_1 \in [-37; -6]$

2) $D \geq 0$

$$D = 64a_1^2 + 2800a_1 + 14425 \geq 0$$

$$d = \frac{91 - 24a_1 \pm \sqrt{64a_1^2 + 2800a_1 + 14425}}{256} > 0$$

$$\begin{array}{r} 14425 \\ -\ 8281 \\ \hline 6144 \end{array}$$

$$\pm\sqrt{64a_1^2 + 2800a_1 + 14425} > 24a_1 - 91$$

1) $24a_1 > 91 \Rightarrow a_1 > \frac{91}{24} = 3\frac{19}{24}$

$$\begin{array}{r} 4368 \\ +\ 2800 \\ \hline 7168 \end{array}$$

$$64a_1^2 + 2800a_1 + 14425 > 576a_1^2 - 4368a_1 + 8281$$

$$512a_1^2 - 7168a_1 - 6144 < 0$$

$$64a_1^2 - 896a_1 - 768 < 0$$

$$16a_1^2 - 224a_1 - 192 < 0$$

$$4a_1^2 - 56a_1 - 48 < 0$$

$$a_1^2 - 14a_1 - 12 < 0$$

$$\begin{array}{r|l} 244 & 2 \\ 122 & 2 \\ 61 & \end{array}$$

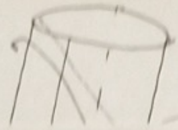
$$D = 196 + 48 = 244$$

$$a_1 = \frac{14 \pm 2\sqrt{61}}{2} = 7 \pm \sqrt{61}$$

$$a_1 \in \left(3\frac{19}{24}; 7 + \sqrt{61}\right)$$

шестовых (1)

5.



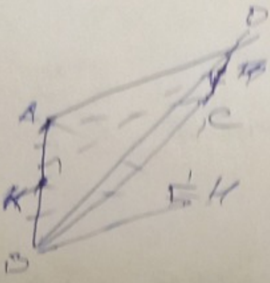
Дано:

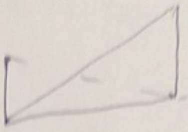
ABCD

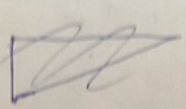
= 7

6

$\angle BAP$







в цилиндр.

ж пов-ти

цилиндра

ндра

цилиндра.

$CD = DH - CH = \boxed{\sqrt{47} - \sqrt{34}}$

$$x^2 + y^2 - 2ax - 2by + \min(-8a - 6b; 25) \le 25$$

$$x^2 + y^2 - 2ax - 2by \le 0$$

$$(x-a)^2 + (y-b)^2 \le a^2 + b^2$$

ов - ву

-, тогда

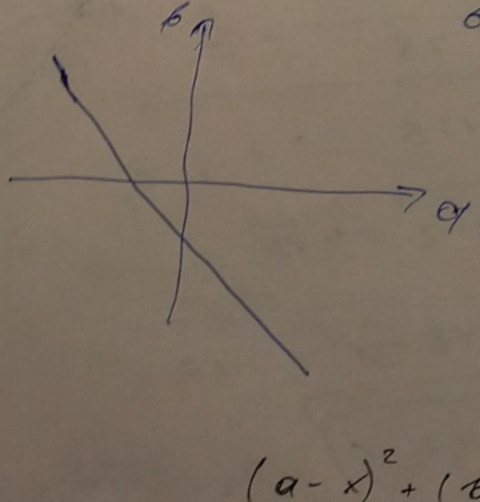
$$-8a - 6b \vee 25$$

дра,

и на CD.

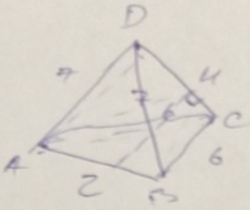
$$b \vee \cancel{25} - \frac{8}{6}a - \frac{25}{6}$$

т. к.

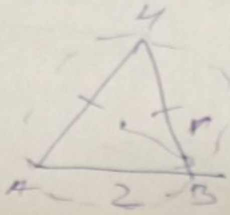


$$(a-x)^2 + (b-y)^2 \le 25$$

Чистовик (1)



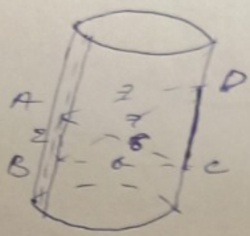
$\Rightarrow AB \perp CD \Rightarrow AB \parallel \alpha$

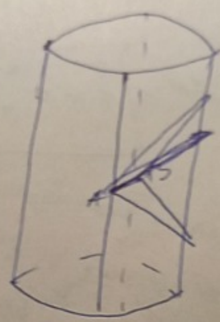


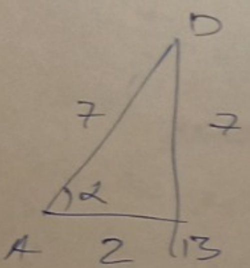
может ли AB быть α?

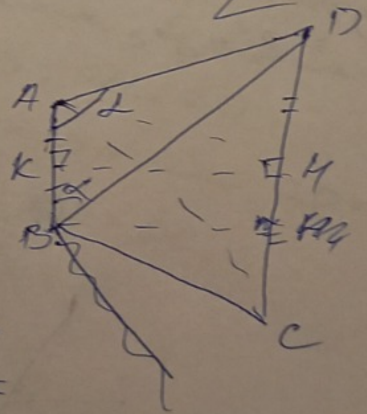
$\Downarrow$

да, тогда 2 вар-та









$KH = 1$

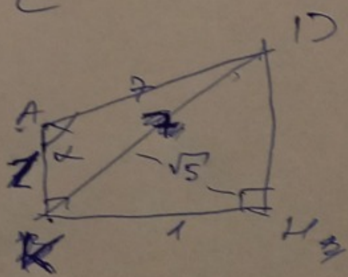
$$49 = 4 + 49 - 2\cdot 14\cos\alpha$$

$$\cos\alpha = \frac{53-49}{28} = \frac{1}{7}$$

$$KD = \sqrt{1+49-2\cdot 1\cdot 7\cdot \frac{1}{7}} = \sqrt{50-2} = 4\sqrt{3}$$

$$DH = \sqrt{48-1} = \sqrt{47}$$

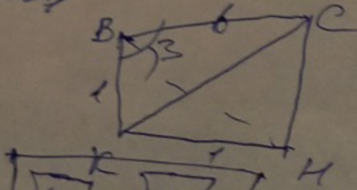
$$\cancel{CD = 2\sqrt{47}}$$



$$36 = 4 + 36 - 2\cdot 12\cos\beta$$

$$\cos\beta = \frac{40-36}{24} = \frac{1}{6}$$

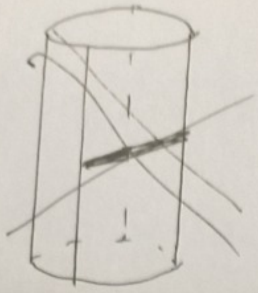
~~KD~~

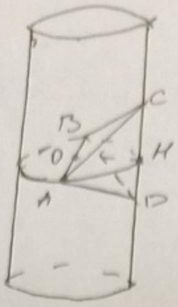


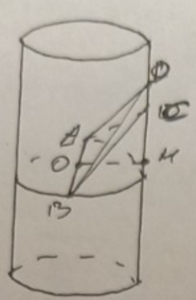
$$KC = \sqrt{36+1-2\cdot 1\cdot 6\cdot \frac{1}{6}} = \sqrt{35}$$

$$CH = \sqrt{35-1} = \sqrt{34} \Longrightarrow CD = \sqrt{47}+\sqrt{34}$$

Чистовик (1)







Дано:

тэт-р $ABCD$

$AD = BD = 7$

$AC = BC = 6$

$AB = 2$

$ABCD$ впис в цилиндр.

т. $A, B, C, D \in$ бок пов-ти цилиндра

$CD \parallel$ ось цилиндра

Найти:

CD при min r цилиндра.

Решение:

Т.к. $\triangle ABC$ и $\triangle ABD$ равнобедренные, то по св-ву тэт-ра $AB \perp CD \Rightarrow AB \parallel$ диаметру основания, тогда $r \geqslant \frac{AB}{2} \Rightarrow \min(r) = \frac{AB}{2} = 1$.

Тогда пусть т. O - центр окр-ти сечения цилиндра, где AB яв-ся диаметром. Опустим пер-р OH на CD. Тогда $AO = BO$; $OH \perp AB$; $OH \perp CD$.

Тогда имеем: $CD = CH + DH$ или $CD = DH - CH$, т.к. $AD > AC$ и $BD > BC \Rightarrow OD > OC$.

Пусть $\angle BAD = \alpha$; $\angle BAC = \beta$

$$\cos\alpha = \frac{AD^2 + AB^2 - BD^2}{2AB \cdot AD} = \frac{1}{7}$$

$$\cos\beta = \frac{AC^2 + AB^2 - BC^2}{2 \cdot AB \cdot AC} = \frac{1}{6}$$

$$OD = \sqrt{OA^2 + AD^2 - 2\,OA \cdot AD \cdot \cos\alpha} = 4\sqrt{3}$$

$$OC = \sqrt{OA^2 + AC^2 - 2\,OA \cdot AC \cdot \cos\beta} = \sqrt{35}$$

$$DH = \sqrt{OD^2 - OH^2} = \sqrt{47}$$

$$CH = \sqrt{OC^2 - OH^2} = \sqrt{34}.$$

Ответ: $\sqrt{47} + \sqrt{34}$; $\sqrt{47} - \sqrt{34}$.

$)^2+(4-b)^2 \le 25$ – ур-е окр-ти ... т. $(a;b)$ и

Чистовик (2)

6

$$\begin{cases} a_9 a_{17} > S+12 \\ a_{11} a_{15} < S+47 \\ d>0 \\ a_1, a_2, \dots a_{14} \in \mathbb{Z} \Rightarrow d \in \mathbb{N} \end{cases} \Rightarrow \begin{cases} (a_1+8d)(a_1+16d) > \frac{2a_1+13d}{2}\cdot 14+12 \\ (a_1+10d)(a_1+14d) < \frac{2a_1+13d}{2}\cdot 14+47 \end{cases}$$

$a_1^2+24a_1d+128d^2-12 > a_1^2+24a_1d+140d^2-47$

$12d^2 < 35$

$d^2 < \frac{35}{12} = 2\frac{11}{12}$

$d < \sqrt{\frac{35}{12}} = \frac{1}{2}\cdot\sqrt{\frac{35}{3}} < \frac{1}{2}\cdot\sqrt{\frac{36}{3}} = 3\cdot\frac{1}{\sqrt{3}} = \sqrt{3}$

$d = \underline{1}.$

$(a_1+8)(a_1+16) > \frac{2a_1+13}{2}\cdot 14+12$

$a_1^2+24a_1+128 > 14a_1+91+12$

$a_1^2+10a_1+25>0$

$(a_1+5)^2>0$

$a_1 \ne -5$

$(a_1+10)(a_1+14) < \frac{2a_1+13}{2}\cdot 14+47$

$a_1^2+24a_1+140 < 14a_1+91+47$

$a_1^2+10a_1+2<0$

$D = 100-8 = 92$

$a_1 = \frac{-10 \pm 2\sqrt{23}}{2}$

$a_1 \in (-5-\sqrt{23}; -5+\sqrt{23})$

$-5 < -\sqrt{23} < -4 \Rightarrow -10 < -5-\sqrt{23} < -9$

$\Downarrow$

$-1 < -5+\sqrt{23} < 0$

$a_1 \in \{-9; -8; -7; -6; -4; -3; -2; -1\}$

Ответ: $a_1 \in \{-9; -8; -7; -6; -4; -3; -2; -1\}$.

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Чистовик (3)

$$\begin{cases}(x-a)^2+(y-b)^2\le 25 \\ a^2+b^2\le \min(-8a-6b,\ 25)\end{cases}$$

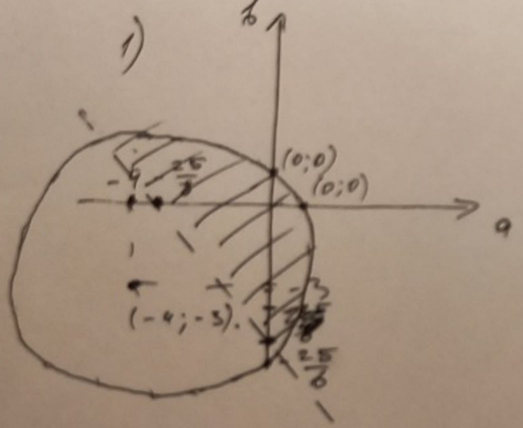
$(x-a)^2+(y-b)^2\le 25$ — ур-е окр-ти с центром в т. $(a; b)$ и рад. 5

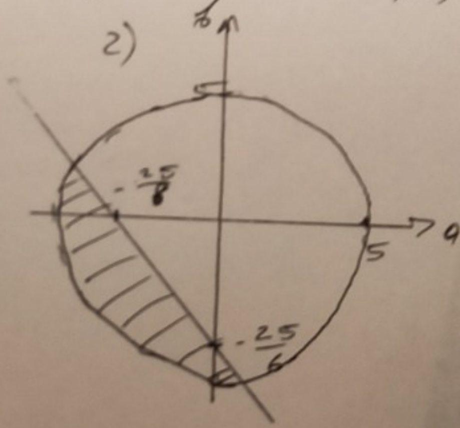
1) $a^2+b^2\le -8a-6b \iff -8a-6b<25$

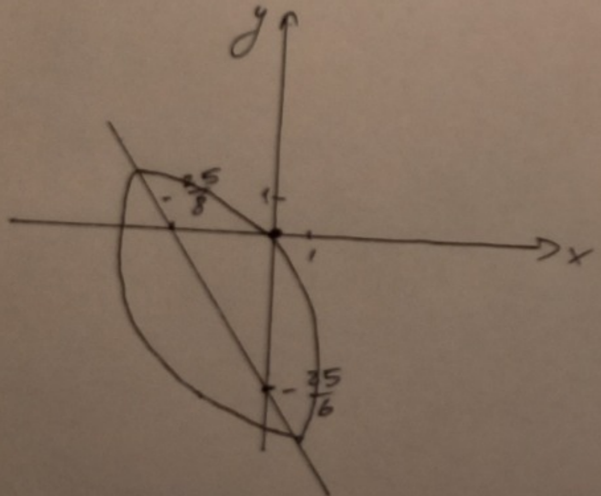
$(a+4)^2+(b+3)^2\le 25$ — ур-е окр-ти с центром в т. $(-4; -3)$ и рад. 5.

2) $a^2+b^2\le 25 \iff -8a-6b\geq 25$

ур-е окр-ти с центром в т. $(0;0)$ и рад. 5.







$a_9 a_{17} > S+12$

$a_{11} a_{15} < S+47$

$a_1^2 + a_1(24d - 14) + 128d^2 - 91d - 12 > 0$

$D = 576d^2 - 672d + 196 - 512d^2 + 364d + 48 =$

$= 64d^2 - 308d + 244 = 4(16d^2 - 77d + 61)$

$$\begin{array}{r} 24 \\ 28 \\ \hline 1\,92 \\ 48 \\ \hline 672 \end{array}$$

180

1) $D < 0$

$16d^2 - 77d + 61 < 0$

$D_1 = 77^2 - 64 \cdot 61$

$d = \dfrac{77 \pm \sqrt{77^2 - 64 \cdot 61}}{32}$

$a_9 a_{17} > S+12$

$a_{11} a_{15} < S+47$

$(a_1+8d)(a_1+16d) > \frac{2a_1+d(14-1)}{2}\cdot 14+12$

$a_1^2+24a_1d+128d^2 > \frac{28a_1+13\cdot 14d}{2}+12$

$a_1^2+24a_1d+128d^2 > 14a_1+91d+12$

$(a_1+10d)(a_1+14d) < \frac{28a_1+13\cdot 14d}{2}+47$

$a_1^2+24a_1d+140d^2 < 14a_1+91d+47$

$128d^2+d(24a_1-91)+a_1^2-14a_1-12>0$

$d=(24a_1-91)^2-512(a_1^2-14a_1-12)<0$

$576a_1^2-4368a_1+8281-512a_1^2+7168a_1+6144=$

$=64a_1^2+2800a_1+14425<0$

$d=2800^2-256\cdot 14425=7840000-$

$2800=100\cdot 2^2\cdot 7$ $-3692800=4147200=2^{11}\cdot 3^4\cdot 5^2$

$$\begin{array}{r} 7840000 \\ 3692800 \\ \hline 4147200 \end{array}$$

4147200	100
41472	2
20736	2
10368	2
5184	2
2592	2
1296	2
648	2
324	2
162	2
81	3^4

$a_1=\frac{-2800\pm 2^5\cdot 9\cdot 5\sqrt{2}}{128}=$

$=\frac{-175\pm 90\sqrt{2}}{8}$

$$\begin{array}{r} 7168 \\ 4368 \\ \hline 2800 \end{array}\qquad \begin{array}{r} 512 \\ 12 \\ \hline 1024 \\ 512 \\ \hline 6144 \end{array}\qquad \begin{array}{r} 512 \\ 14 \\ \hline 2048 \\ 512 \\ \hline 7168 \end{array}\qquad \begin{array}{r} 48 \\ 91 \\ \hline 48 \\ 432 \\ \hline 4368 \end{array}\qquad \begin{array}{r} 91 \\ 91 \\ \hline 91 \\ 819 \\ \hline 8281 \end{array}$$

$$\begin{array}{r} 8281 \\ 6144 \\ \hline 14425 \end{array}\qquad \begin{array}{r} 28 \\ 28 \\ \hline 224 \\ 56 \\ \hline 784 \end{array}\qquad \begin{array}{r} 25 \\ 7 \\ \hline 175 \end{array}\qquad \begin{array}{r} 14425 \\ 256 \\ \hline 86550 \\ 72125 \\ 28850 \\ \hline 3692800 \end{array}$$

32
45

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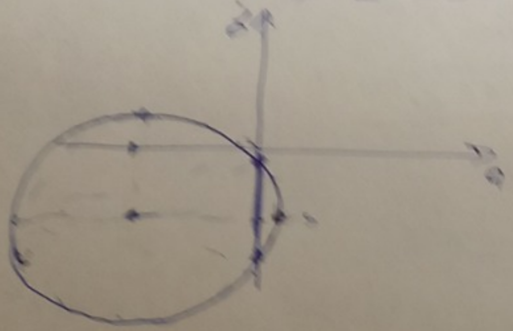
$(x-a)^2+(y-b)^2 \le 25$

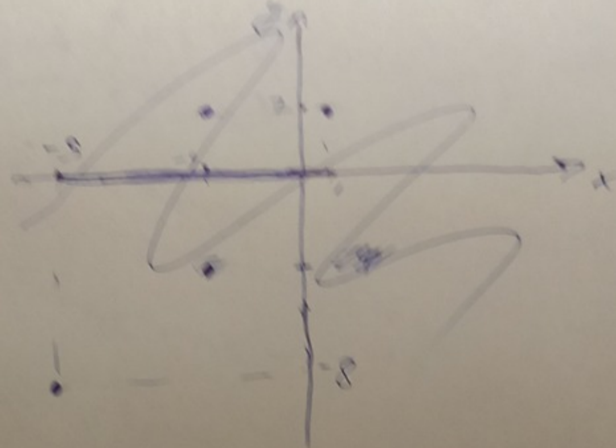
$a^2+b^2 \le \min(-8a-6b;\ 25)$

1) $-8a-6b \le 25$

$a^2+b^2 \le -8a-6b$

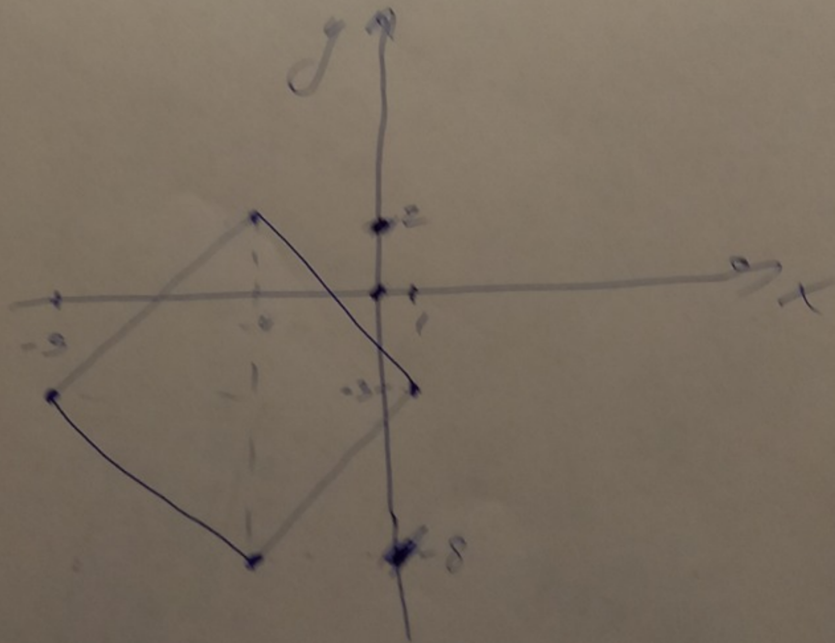
$(a+4)^2+(b+3)^2 \le 25$





$a \in [-9;1]$

$b \in [-8;2]$



Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21103654**

ID профиля: **211218**

Вариант 19

$\frac{1}{2}\log_b a$

$2\log_c b$

$2\log_a c$

$$\frac{t-1}{t}=0$$

$$t=1$$

$$\log_a c=1$$

$$a=c$$

$$\frac{x}{2}-\frac{1}{4}=x-\frac{11}{4}$$

$$\frac{x}{2}=\frac{10}{4}$$

$$\boxed{x=5}$$

② = ③

$$\cancel{2\log_c b\cdot 2\log_a c=4\frac{\log_c b}{\log_c a}=}$$

$$\cancel{=4\log_a b=\left(\frac{1}{2}\log_b a-1\right)^2=}$$

$$\cancel{=\frac{1}{4}\log_b^2 a-\log_b a+1}$$

$$\cancel{\frac{1}{4}t^2-t+1=\frac{4}{t}}$$

$$\cancel{t^2-4t+4-\frac{1}{t}=0}$$

① = ③

$$\frac{1}{2}\log_b a\cdot 2\log_a c=\frac{\log_a c}{\log_a b}=\log_b c$$

$$(2\log_c b-1)^2=4\log_c^2 b-4\log_c b+1$$

$$4t^2-4t+1=\frac{1}{t}$$

$$4t^2\left(1-\frac{1}{t}\right)+\left(1-\frac{1}{t}\right)=0$$

$$(4t^2+1)\left(1-\frac{1}{t}\right)=0$$

$$t=1$$

$$b=c\Rightarrow\frac{x}{2}-1=x-\frac{11}{4}$$

$$\frac{x}{2}=\frac{7}{4}$$

$$x=\frac{14}{4}=\frac{7}{2}=\boxed{3,5}$$

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② = ③

$4\log_c b \cdot \log_a c = 4\frac{\log_c b}{\log_c a} = 4\log_a b$

$\left(\frac{1}{2}\log_b a - 1\right)^2 = \frac{1}{4}\log_b^2 a - \log_b a + 1 = 4\log_a b$

$\frac{1}{4}t^2 - t + 1 = \frac{4}{t}$

$t^2 - 4t + 4 - \frac{16}{t} = 0$

$t^2\left(1 - \frac{4}{t}\right) + 4\left(1 - \frac{4}{t}\right) = 0$

$(t^2+4)\left(\frac{t-4}{t}\right) = 0$

$t = 4$

$\log_b a = 4$

$a = b^4$

$\frac{x}{2} - \frac{1}{4} = \left(\frac{x}{2} - 1\right)^4$

~~$\frac{1}{2}\left(x - \frac{1}{2}\right) = \left(\frac{x}{2} - 1\right)$~~

$\frac{2x-1}{4} = \left(\frac{x-2}{2}\right)^4$

$4(2x-1) = (x-2)^4$

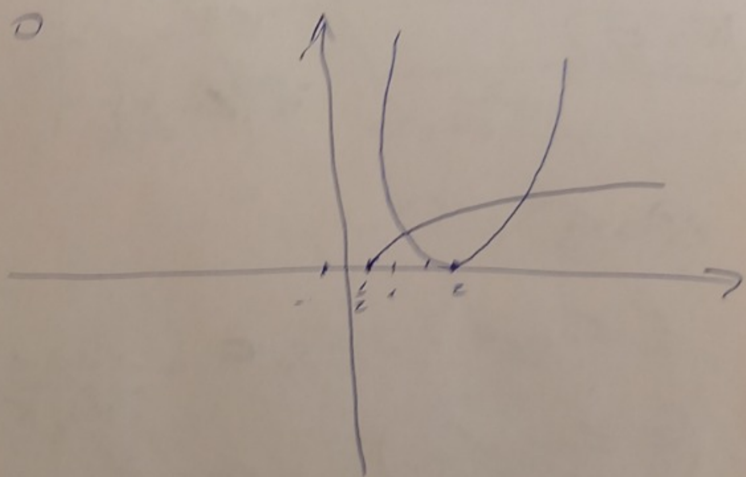
$8x - 4 = x^4 - 12x^3 + 24x^2 - 32x - 16$

$x^4 - 12x^3 + 24x^2 - 40x - 12 = 0$

$-\frac{-4}{2} = 2$

$x^2 - 4x + 4 = 2\sqrt{2x-1}$

$(x-2)^2 = 2\sqrt{2x-1}$



① $\frac{1}{2}\log_b a = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$

② $2\log_c b$

③ $2\log_{b^4} c$

~~$\log_c b = \frac{1}{4}\log_b c = \frac{1}{8} - 1 = -\frac{7}{8}$~~

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$НОД(a,b,c) = 21$

$НОК(a,b,c) = 3^{17} \cdot 7^{15}$

одно из чисел содержит $3 \cdot 7^{n}$, и одно из них — $3^{m} \cdot 7$

1 из чисел содержит 3^{17}

1 из чисел содержит 7^{15}

~~1) если $a = 21$~~

~~b и c имеют 3^{16} и 7^{14}~~

~~$b \geq 21$, $c \geq 21$~~

~~ост — 3^{14} и 7^{12}~~

~~в могут иметь 3^{k} и 7^{l}~~

$k \in \{0, 1, \dots 14\}$

$l \in \{$

$\overbrace{0, 0, 12}^{13}$

$0, 1, 11$

$\overbrace{1, 0, 11}^{12}$

...

$\overbrace{12, 0, 0}^{1}$

$\frac{13 \cdot 14}{2} = 91$

~~a, b, c =~~

$a = 3^{\alpha_1} \cdot 7^{\alpha_2}$

$b = 3^{\beta_1} \cdot 7^{\beta_2}$

$c = 3^{\gamma_1} \cdot 7^{\gamma_2}$

$\frac{15}{7}$

$$\begin{array}{r} 15 \\ \times\ 17 \\ \hline 105 \\ 15 \\ \hline 255 \end{array}$$

$\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2 \geq 1$

$a = 21 \cdot \dots$

$b = 21 \cdot \dots$

$c = 21 \cdot \dots$

ост 3^{14}, 7^{12}

$$\begin{array}{r} 120 \\ \times\ 91 \\ \hline 120 \\ 108 \\ \hline 10920 \end{array}$$

$14 = 0 + \overbrace{0 + 14}^{15 сл}$

$0 + 1 + 13$

$0 + 2 + 12$

...

$0 + 14 + 0$

$1 + \overbrace{0 + 13}^{14}$

$2 + \overbrace{0 + 12}^{13}$

...

$14 + \overbrace{0 + 0}^{1}$

$S_3 = \frac{15 \cdot 16}{2} = 120$

$$\begin{array}{r} 255 \\ \times\ 36 \\ \hline 1530 \\ 765 \\ \hline 9180 \end{array}$$

$\boxed{10920}$

НОД (а,

ОК (а,

Пусть

Из ус

α_1, β

$\alpha_2,$

числ

Тогда

из α

из α

кот-

ло

2 · 3

От

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$\log_{(\frac{x}{2}-1)^2}\left(\frac{x}{2}-\frac{1}{4}\right)$ $\Rightarrow a$

$\log_{\sqrt{x-\frac{11}{4}}}\left(\frac{x}{2}-1\right) = b$

$\log_{\frac{x}{2}-\frac{1}{4}}\left(x-\frac{11}{4}\right)^2 = c$

$\frac{x}{2}-\frac{1}{4}=a$

$\frac{x}{2}-1=b$

$x-\frac{11}{4}=c$

$a=b=c-1$

$a+1=b+1=c$

$a+b+c=3a+1=3b+1=3c-2$

$\log_b a \cdot \log_a c =$

$= \frac{\log_a c}{\log_a b} = \log_b c$

$\log_{(\frac{x}{2}-1)^2}\left(\frac{x}{2}-\frac{1}{4}\right) \cdot \log$

$\log_c a \cdot \log_c b$

1) $\log_{b^2} a = \frac{1}{2}\log_b a$

2) $\log_{c^{\frac{1}{2}}} b = 2\log_c b$

3) $\log_a c^2 = 2\log_a c$

1=2

$\frac{1}{2}\log_b a \cdot 2\log_c b =$

$= \frac{\log_b a}{\log_b c} = \log_c a$

$\log_c a = (\log_a c^2 - 1)^2 =$

$4\log_a^2 c - 4\log_a c + 1 = \log_c a$ $= \cancel{\log_a c^A}\ \log_a^2 \frac{c^2}{a} = 4\log_a^2 \frac{c}{a}$

$4t^2-4t+1-\frac{1}{t}=0$

$4t^2(t-1)+(1-\frac{1}{t})=0 \Rightarrow (4t^2+1)(1-\frac{1}{t})=0$

$2\log_c b - \frac{2}{?}\log_b c = \frac{?}{5} - 5$

$16\log_c b - 4\log_b c = -7$

$16t - \frac{4}{t} = -7$

$t^2 = \frac{1}{4} \Rightarrow t = \pm\frac{1}{2}$

$t = -\frac{4}{7} \Rightarrow t \in \varnothing$

если $x = 5$, то

$\frac{1}{2}\log_{1,5}\left(\frac{5}{2} - \frac{1}{4}\right) = \frac{1}{2}\log_{\frac{3}{2}}\frac{9}{4} = \frac{1}{2}\cdot 2 = 1$

$2\log_{\frac{9}{4}}(1,5) = 2\cdot\frac{1}{2} = 1$

~~log~~ $2\log_{\frac{9}{4}}\left(\frac{9}{4}\right) = 2$

если $x = 3,5$

$\frac{1}{2}\log_{\frac{3}{4}}\left(\frac{7}{4} - \frac{1}{4}\right) = \frac{1}{2}\log_{\frac{3}{4}}\frac{3}{2}$

$2\log_{\frac{3}{4}}\left(\frac{7}{4} - 1\right) = 2\log_{\frac{3}{4}}\frac{3}{4} = 2$

$2\log_{\frac{3}{2}}\left(\frac{3}{4}\right)$

① = ②

~~$a\sqrt{b} = \sqrt{c^3}$~~

~~$\sqrt{ab} = |c|$~~

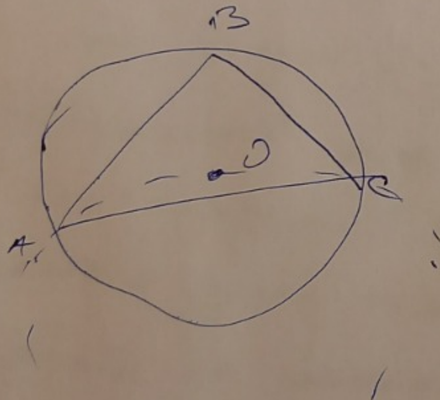
$\log_3 9 \cdot \log_3 27 =$

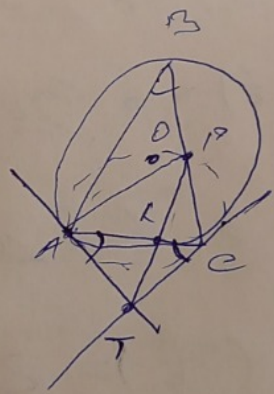
$= 2 \cdot 3 = 6$

$\frac{1}{2} \log_6 9 = 2 \log_c 6$

$\log_6 9 = \frac{4}{\log_6 c}$

~~$\log_6 9 \cdot \log_6 c = 4$~~

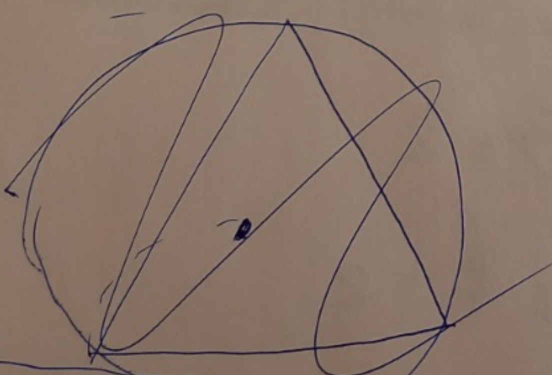


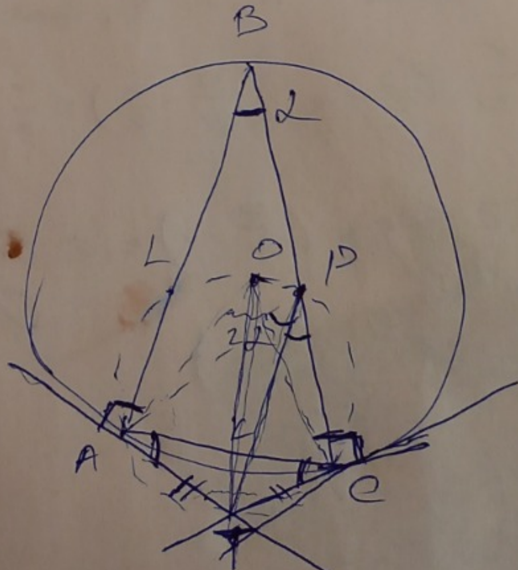


$S_{APK} = 10$

$S_{CPK} = 6$

S_{ABC} — ?





$\boxed{\angle ABC = \angle CAT = \angle ACT = \alpha}$

~~$\angle ABC = \alpha$~~

$\boxed{\angle ATC = 180 - 2\alpha \text{ из } \triangle ACT}$

$\boxed{\angle AOC = 2\alpha}$

Чистовик (1)

№ 4

НОД $(a, b, c) = 21$

НОК $(a, b, c) = 3^{17} \cdot 7^{15}$

Пусть $a = 3^{\alpha_1} \cdot 7^{\alpha_2}$

$b = 3^{\beta_1} \cdot 7^{\beta_2}$

$c = 3^{\gamma_1} \cdot 7^{\gamma_2}$.

Из условия следует, что α_1, или β_1, или γ_1 равна 1; α_1, β_1 или $\gamma_1 = 17$; α_2, β_2 или γ_2 равна 1; α_2, β_2 или γ_2 равна 15. Оставшиеся числа находятся на отрезке $[1; 17]$ и $[1; 15]$. Тогда всего $3 \cdot 2 = 6$ способов выбрать 1 и 17 из $\alpha_1, \beta_1, \gamma_1$ и $3 \cdot 2 = 6$ способов выбрать 1 и 15 из $\alpha_2, \beta_2, \gamma_2$. Остаётся одно число из $\alpha_1, \beta_1, \gamma_1$, кот-е можно выбрать 17 способами и 1 число из $\alpha_2, \beta_2, \gamma_2$ — 15 способов. Итого способов $2 \cdot 3 \cdot 17 \cdot 2 \cdot 3 \cdot 15 = 9180$.

Ответ: 9180.

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Чистовик (2)

№ 5

$$\log_{(\frac{x}{2}-1)^2}\left(\frac{x}{2}-\frac{1}{4}\right) = \frac{1}{2}\log_{(\frac{x}{2}-1)}\left(\frac{x}{2}-\frac{1}{4}\right)$$

$$\log_{\sqrt{x-\frac{11}{4}}}\left(\frac{x}{2}-1\right) = 2\log_{(x-\frac{11}{4})}\left(\frac{x}{2}-1\right)$$

$$\log_{(\frac{x}{2}-\frac{1}{4})}\left(x-\frac{11}{4}\right)^2 = 2\log_{(\frac{x}{2}-\frac{1}{4})}\left(x-\frac{11}{4}\right)$$

Пусть $\frac{x}{2}-\frac{1}{4}=a$; $\frac{x}{2}-1=b$; $x-\frac{11}{4}=c$

Имеем 3 числа:

① $\frac{1}{2}\log_b a$

② $2\log_c b$

③ $2\log_a c$

1) ① = ②

$$\frac{1}{2}\log_b a = 2\log_a c - 1 \ |^2$$

$$\frac{1}{2}\log_b a \cdot \frac{1}{2}\log_b a = \frac{1}{2}\log_b a \cdot 2\log_c b = \log_c a$$

$$(2\log_a c - 1)^2 = 4\log_a^2 c - 4\log_a c + 1$$

Пусть $\log_a c = t$

$$4t^2 - 4t + 1 = \frac{1}{t}$$

$$4t^2\left(1-\frac{1}{t}\right) + \left(1-\frac{1}{t}\right) = 0$$

$$(4t^2+1)\left(1-\frac{1}{t}\right) = 0$$

$t = 1$

$\log_a c = 1 \Rightarrow \log_{(\frac{x}{2}-\frac{1}{4})}\left(x-\frac{11}{4}\right) = 1$

$$x - \frac{11}{4} = \frac{x}{2} - \frac{1}{4}$$

$x = 5$

Чистовик (3)

если $x=5$, то

$\frac{1}{2}\log_{(2,5-1)}(2,5-\frac{1}{4})=\frac{1}{2}\log_{\frac{3}{2}}\frac{9}{4}=1$

$2\log_{(5-\frac{11}{4})}(1,5)=2\log_{\frac{9}{4}}\frac{3}{2}=1$

$2\log_{(2,5-\frac{1}{4})}(5-\frac{11}{4})=2$

2) ① = ③

$\frac{1}{2}\log_b a=2\log_c b-1\,|\,{}^{\wedge}2$

$\frac{1}{2}\log_b^2 a=\frac{1}{2}\log_b a\cdot 2\log_a c=\log_b c$

$(2\log_c b-1)^2=4\log_c^2 b-4\log_c b+1$

Пусть $\log_c b=t$

$4t^2-4t+1=\frac{1}{t}$

$4t^2(1-\frac{1}{t})+(1-\frac{1}{t})=0$

$(4t^2+1)(1-\frac{1}{t})=0$

$t=1 \Rightarrow \log_c b=1 \Rightarrow b=c \Rightarrow \frac{x}{2}-1=x-\frac{11}{4}$

$\frac{x}{2}=\frac{7}{4} \Rightarrow x=3,5$

если $x=3,5$, то

$\frac{1}{2}\log_{(\frac{7}{4}-1)}(\frac{7}{4}-\frac{1}{4})=\frac{1}{2}\log_{\frac{3}{4}}\frac{3}{2}$

$2\log_{(\frac{7}{2}-\frac{11}{4})}(\frac{7}{4}-1)=2\log_{\frac{3}{4}}\frac{3}{4}=2 \Rightarrow x\in\varnothing$

$2\log_{(\frac{7}{4}-\cancel{\frac{1}{4}})}(\frac{7}{2}-\frac{11}{4})=2\log_{\frac{3}{2}}\frac{3}{4}$

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Чистовик 4

(2) = (3)

~~$\frac{1}{2}\log_b a = 2$~~

$2\log_c b = \frac{1}{2}\log_b a - 1 \;\Big|\,{}^{\wedge}2$

$2\log_c^2 b = 4\log_c b \cdot \log_a c = 4\log_a b$

$(\frac{1}{2}\log_b a - 1)^2 = \frac{1}{4}\log_b^2 a - \log_b a + 1 =$

Пусть $t = \log_b a$

$\frac{1}{4}t^2 - t + 1 = \frac{4}{t}$

$t^2 - 4t + 4 - \frac{16}{t} = 0$

$t^2(1 - \frac{4}{t}) + 4(1 - \frac{4}{t}) = 0$

$(t^2 + 4)(1 - \frac{4}{t}) = 0$

$t = 4 \Rightarrow \log_b a = 4 \Rightarrow a = b^4$

(1) $\frac{1}{2}\log_b a = \frac{1}{8}$

(2) $2\log_c b$

(3) $2\log_{b^4} c = \frac{1}{2}\log_b c$

$2\log_c b = \frac{1}{2}\log_b c = \frac{1}{8} - 1 = -\frac{7}{8}$

Пусть $t = \log_c b$

$4t = \frac{1}{t} \Rightarrow t = \pm\frac{1}{2}$

$2t = -\frac{7}{8} \Rightarrow t = -\frac{7}{16} \Rightarrow t \in \varnothing \Rightarrow x \in$ ~~⌀~~

Ответ: $x = 5$.

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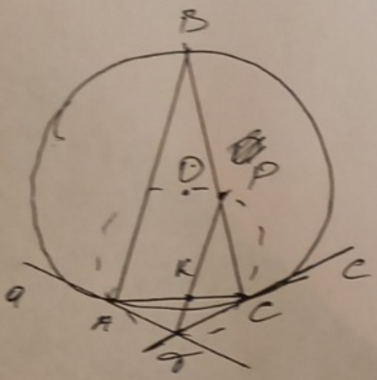
Чистовик (5)

№6



Дано:

ост. $\triangle ABC$

ω опис около $\triangle ABC$

$\omega(O; OA)$; $\omega_1 \ni$ т. A, O, C.

$\omega_1 \cap BC =$ т. P

a и c - кас-е к ω

т. $A \in a$; т. $C \in c$

$a \cap c =$ т. T

$PT \cap AC =$ т. K

$S_{APK} = 10$

$S_{CPK} = 6$

Найти:

1) $S_{\triangle ABC}$ - ?

2) AC - ? если $\angle ABC = \operatorname{arctg} 2$

Решение:

По св-ву кас-ой. $\angle CAT = \angle ABC$; $\angle ACT = \angle ABC$

Пусть $\angle ABC = \alpha$, тогда $\angle AOC = 2\alpha$

Из $\triangle ACT$: $\angle ATC = 180^\circ - 2\alpha$

Тогда $\angle AOC + \angle ATC = 180^\circ \Rightarrow$ т. $A, O, C, T \in \omega_2$,

но т. $A, O, P, C \in \omega_1 \Rightarrow \omega_1 = \omega_2$, т.е. т. $A, O, P, C, T \in \omega_1$.

Тогда $\angle APT = \angle TPC = \angle TAC = \angle CAT = \alpha$, т.к. опираются на равные хорды.

$$\begin{cases} S_{APK} = \dfrac{AK \cdot PK \cdot \sin\angle AKP}{2} \\ S_{CPK} = \dfrac{CK \cdot PK \cdot \sin\angle CKP}{2} \end{cases} \Rightarrow \frac{AK}{CK} = \frac{S_{APK}}{S_{CPK}} = \frac{5}{3}.$$

a

б

c

4

2

$$\frac{8x \cdot 8}{\sin 2\alpha} = 2r$$

$$r = \frac{4x}{\sin 2\alpha} = \frac{\sqrt{R^2 + (2r \sin\alpha)^2}}{2}$$

$$\frac{64x^2}{\sin^2 2\alpha} = R^2 + 4r^2 \sin^2\alpha$$

$$\frac{4r^2 \sin 2\alpha}{\sin^2 2\alpha} = R^2 + 4r^2 \sin^2\alpha$$

$$4r^2\left(\frac{1}{\sin 2\alpha} - \sin^2\alpha\right) = R^2$$

$$R = 2r\sqrt{\frac{1 - 2\sin^3\alpha \cdot \cos\alpha}{2\sin\alpha \cdot \cos\alpha}}$$

$$\frac{AC}{\sin\alpha} = 2R$$

$$\frac{AC^2}{4\sin^2\alpha} = 4r^2\left(\frac{1}{\sin 2\alpha} - \sin^2\alpha\right)$$

$$AC^2 = 4r^2\left(\frac{2\sin\alpha}{\cos\alpha} - 4\sin^4\alpha\right)$$

$$4r^2 \sin^2 2\alpha$$

$$\sin^2 2\alpha = \frac{2\sin\alpha - 4\sin^4\alpha \cdot \cos\alpha}{\cos\alpha}$$

$$4\sin^2\alpha\cos\alpha\left(\cos^2\alpha + \frac{\sin^2\alpha}{\cos\alpha}\right) = 2\,\mathrm{tg}\,\alpha$$

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$a = 3^l \cdot 7^n$

$b = 3^m \cdot 7^r$

$c = 3^k \cdot 7^p$

$\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2$

$\frac{6 \cdot 5}{2} \cdot 4 \cdot 3$ - расстановка их

4 у них $\in \boxed{\{1, 1, 17, 15\}}$

2 у них $\in \{1, 2, \dots 17\}$ и $\{1, 2, \dots 15\}$

ост - 2 числа 3^9 и 7^6

1) $a = 21$
$b = 3^{17} \cdot 7^{15}$
$c = 3^9 \cdot 7^6 \Rightarrow 17 \cdot 15$ сл - 6 $\Big\}$ 6

2) $a = 21$
$b = 3^{17} \cdot 7^9$
$c = 3^6 \cdot 7^{15} \Rightarrow 17 \cdot 15$ сл - 6 $\Big\}$ 6

3) $a = 3 \cdot 7^9$
$b = 3^6 \cdot 7$
$c = 3^{17} \cdot 7^{15} \Rightarrow 17 \cdot 15$ сл - 6 $\Big\}$ 6

4) $a = 3 \cdot 7^{15}$
$b = 3^{17} \cdot 7$
c

$\alpha_1, \beta_1, \gamma_1$

$\alpha_2, \beta_2, \gamma_2$

5) $3 \cdot 2 \cdot 17$

7) $3 \cdot 2 \cdot 15$

$\times$ $6^2 \cdot 17 \cdot 15$