Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21101112**

ID профиля: **831986**

Вариант 19

ЧИСТОВИК

№1

1) $S_{14} = \frac{a_1 + a_{14}}{2} \cdot 14 = \frac{a_1 + a_1 + (13d)}{2} \cdot 14 = (2a_1 + 13d) \cdot 7$

2) $\begin{cases} (a_1+8d)(a_1+16d) > 7(2a_1+13d)+12, \\ (a_1+10d)(a_1+14d) < 7(2a_1+13d)+47; \end{cases}$

$\begin{cases} a_1^2 + 24a_1d + 128d^2 > 14a_1 + 91d + 12 \\ a_1^2 + 24a_1d + 140d^2 < 14a_1 + 91d + 47 \end{cases} \Big| \cdot (-1)$

$\begin{cases} a_1^2 + 24a_1d + 128d^2 > 14a_1 + 91d + 12 \\ -a_1^2 - 24a_1d - 140d^2 > -14a_1 - 91d - 47 \end{cases}$

3) $-12d^2 > -35$

$d^2 < \frac{35}{12}$

$\begin{cases} |d| < \sqrt{\frac{35}{12}} \\ d \in Z \end{cases}, \quad \begin{cases} |d| < \sqrt{2\frac{11}{12}} \\ d \in Z, \end{cases} \Rightarrow d = \pm 1,$

но по условию прогрессия возрастающая, с.л. $d = -1$ - не подходит.

при $d = 1$

$\begin{cases} (a_1+8)(a_1+16) > 7(a_1+13)+12 \\ (a_1+10)(a_1+14) < 7(a_1+13)+47 \end{cases}$

1

21101112 (U831986 M1297191)

чистовик

$$\begin{cases} a_1^2 + 24a_1 + 128 > 14a_1 + 91 + 12 \\ a_1^2 + 24a_1 + 140 < 14a_1 + 91 + 47 \end{cases}$$

$$\begin{cases} a_1^2 + 10a_1 + 25 > 0 \quad (1) \\ a_1^2 + 10a_1 + 2 < 0 \quad (2) \end{cases}$$

(1) $(a_1+5)^2 > 0$, $a_1 \neq -5$

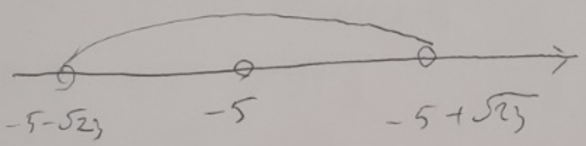
(2) $a_1^2 + 10a_1 + 2 < 0$

$D = 100 - 8 = 4 \cdot 23$

$a_{1,2} = \dfrac{-10 \pm 2\sqrt{23}}{2} = -5 \pm \sqrt{23}$

$4 < \sqrt{23} < 5 \Rightarrow -5 + \sqrt{23} < 0$

$-5 - \sqrt{23} > -10$



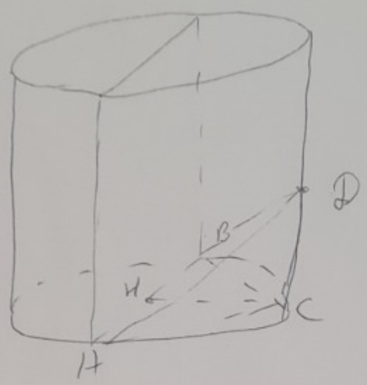
целые $a_1 = \{-9; -8; -7; -6; -4; -3; -2; -1\}$

Ответ: $a_1 = \{-9; -\cancel{8}8; -7; -6; -4; -3; 2; -1\}$

21101112 (U831986 M1297191)

ЧИСТОВИК

№2



$R = \frac{1}{2} AB = 1 = AH$

проведем DH, - высота и мед. т.к $\triangle ABD$ равнобед. $DH = \sqrt{48}$

Аналогично $CH = \sqrt{35}$

$DC = \sqrt{13}$

по т. о неравенств $\triangle$

$\sqrt{35} + x > \sqrt{48}$, $\sqrt{48} + \sqrt{35} > x$

$$\begin{cases} x > \sqrt{48} - \sqrt{35} \\ x < \sqrt{48} + \sqrt{35} \end{cases}$$

Ответ: $x \in (\sqrt{48} - \sqrt{35}; \sqrt{48} + \sqrt{35})$

3

Ответ: $\int_{-\frac{353}{2}}^{+2} \left(-\frac{4}{3}a - \frac{25}{\ldots} + \sqrt{\ldots}\right.$ Чистовик

чистовик

№3

$$\begin{cases} (x-a)^2 + (y-b)^2 = 25 \\ a^2 + b^2 \le \min(-8a-6b, 25) \end{cases}$$

① $\begin{cases} (x-a)^2 + (y-b)^2 \le 5^2 \\ -8a-6b \le 25 \\ a^2+b^2 \le -8a-6b \end{cases}$

② $\begin{cases} (x-a)^2 + (y-b)^2 \le 5 \\ -8a-6b > 25 \\ a^2+b^2 < 25 \end{cases}$

$\Rightarrow$ $6b > 8a - 25$

$b > -\frac{4}{3}a - \frac{25}{6}$

$-\frac{4}{3}a - \frac{25}{6} = 0$

$-\frac{3}{4}b - \frac{25}{8} = 0$

$6b + 25 = 0$

$b = -\frac{25}{6}$

$\Rightarrow$ $a^2 + 8a + 16 + b^2 + 6b + 9 \le 25$

$(a+4)^2 + (b+3)^2 \le 5^2$

$\begin{cases} a = -\frac{25}{8} \\ b = 0 \end{cases}$

$$\begin{cases} (x-a)^2 + (y-b)^2 = 25 \\ a^2 + b^2 = 25 \end{cases}$$

$x(x-2a) + y(y-2b) = 0$

$x^2 - 2ax + y^2 - 2by = 0$

$x^2 - 2ax = y^2 + 2by$

(4)

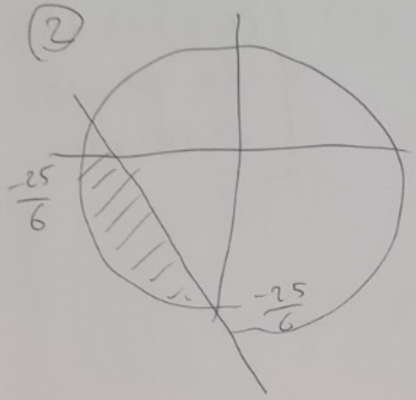
21101112 (U831986 M1297191)

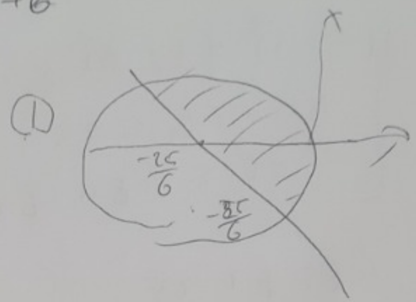
Чистовик

$(y-b)^2 = 25 - (x-a)^2$

$y - b = \pm\sqrt{25-(x-a)^2}$

$y = \pm\sqrt{25-(x-a)^2} + b$





$$\begin{cases} a^2 + b^2 = 25 \\ b = \frac{4}{3}a - \frac{25}{6} \end{cases}$$

$b^2 = 25 - a^2$

$b = \pm\sqrt{25-a^2}$

$(\frac{4}{3}a + \frac{25}{6})^2 + a^2 = 5$

$64a^2 + 16\cdot 25a + 25^2 + 36a^2 = 25\cdot 36$

$4a^2 + 16a + 16 - 27 = 0$

$(a+2)^2 = \frac{27}{4}$

$a = \pm\sqrt{\frac{27}{4}} - 2$

$S = \int_{-\frac{3\sqrt{3}}{2}-2}^{\frac{3\sqrt{3}}{2}-2} \left(-\frac{4}{3}a - \frac{25}{6} + \sqrt{25-a^2}\right) da$

Ответ: $\int_{-\frac{3\sqrt{3}}{2}-2}^{\frac{3\sqrt{3}}{2}-2} \left(-\frac{4}{3}a - \frac{25}{6} + \sqrt{25-a^2}\right) da$ (5)

21101112 (U831986 M1297191)

Исходник

Ответ: $\displaystyle\int\limits_{-\frac{3\sqrt{3}}{2}-2}^{-\frac{3\sqrt{3}}{2}+2}\left(-\frac{4}{3}a-\frac{25}{6}+\sqrt{25-a^2}\right)da$

21101112 (U831986 M1297191)

$a_9 \cdot a_{17} > \quad a_1 + a_{14} \quad 14 \quad \ldots \qquad a_{15} = a_1 + a_{14}$

$(a_1 + 8)(a_1 + 16) > 7(2a_1 + 13) + 12$

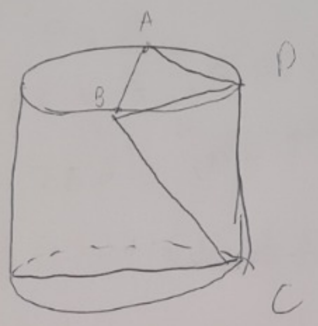
$(a_1 + 10)(a_1 + 14) < 7(2a_1 + 13) + 47$

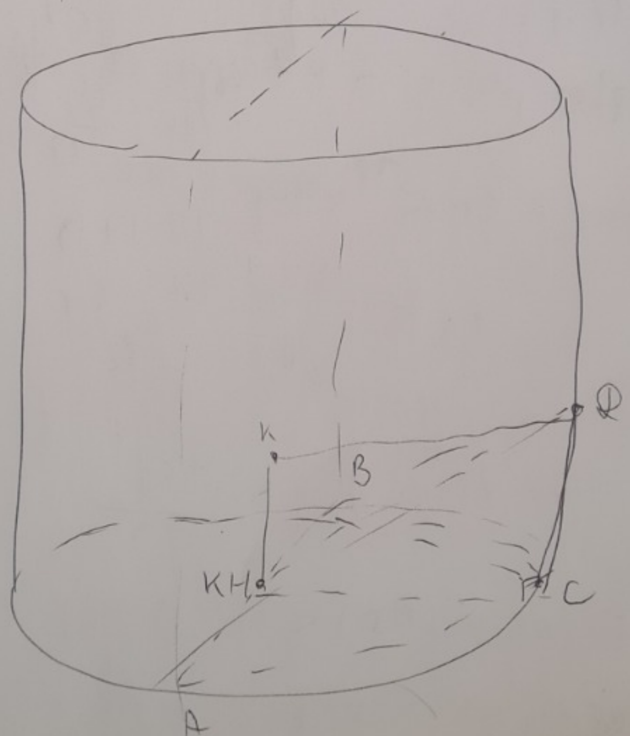
$16 + 12 = 128$

$a_1^2 + 24a_1 + 128 > 14a_1 + 91 + 12$

$a^2 + 24a_1 + 140 > 1$

$58 \,|\, 4 \quad 2\ldots$





21101112 (U831986 M1297191)

74

$a_1^2 + 50a_1 + 48\cdot24 - 39\cdot7 - 12 > 0$

$a_1^2 + 24a_1 d + 128 d^2 > 14a_1 + 13\cdot7\cdot d_1 + 12$

$a_1^2 (24a_1 d - 14a_1) + (128 d^2 - 13\cdot7 d_1 - 12) > 0$

$a_1^2 + (\quad (a_1 + 3\cdot8\cdot) (a_1 + 16\cdot3) > (a_1+13\cdot3)\cdot7 + 12$

$a_1^2 + a_1\cdot72 + 24\cdot48 > a_1\cdot7 + 39\cdot7 + 12$

$a_1^2 + a_1\cdot65 + 1152 > 7a_1 + 285$

$a_1^2 + 58a_1 + 867 > 0$

$D = 58^2 - 4\cdot17^2\cdot3$

$4\cdot29^2 - 4\cdot17^2\cdot3 =$

$= 4(29^2 - 17^2\cdot3)$

$24 \times 48 = 192 + 96 = 1152$

$39 \cdot 7 = 273$

$1152 - 285 = 867$

$867 : 3 = 289$

$900 - 25 = 875$, $- 25 = 850$

$17 \cdot 17 \cdot 3$

$17 \quad 51$

289

$25 \times 25 = 261 + 58 = 841$

$+ 39\cdot7 + 12$

$a_9 \cdot a_{17} > \frac{a_1 + a_{14}}{2} \cdot 14 + 12$ $a_{11} \cdot a_{15} < \frac{a_1+a_{14}}{2} \cdot 14$

$+47$

$a_9 \cdot a_{17} > \frac{a_1+a_{14}}{2} \cdot 14 + 12$ $a_{11} \cdot a_{15} < S + 47 = 12 + 35$

$a_{11} \cdot a_{15} - 35 < S + \cancel{47}\ 12$

$a_9 \cdot a_{17} > S + 12$

$a_{11} \cdot a_{15} - 35 < S + 12$ $a_{11} \cdot a_{15} - 35 > a_9 \cdot a_{17}$

$a_1 + 10d$ $a_9 \cdot a_{17} > (a_1 + a_{14}) \cdot 7 + 12$ $a_1 + d = a_2$

$a_{11} \cdot a_{15} < (a_1 + a_{14}) \cdot 7 + 47$

$$\begin{array}{r} 16 \\ \times 8 \\ \hline 128 \end{array}$$

$(a_1 + 8d)(a_1 + 16d) > (2a_1 + 13d) \cdot 7 + 12$

$(a_1 + 10d)(a_1 + 14d) < (2a_1 + 13d) \cdot 7 + 47$

$(a_1 + 10d)(a_1 + 14d) - 47 > (a_1 + 8d)(a_1 + 16d) - 12$

$a_1^2 + \cancel{24a_1d} + 140d^2 > \cancel{a_1^2} + \cancel{24a_1d} + 128d^2 + 35$

$12d^2 > 35$

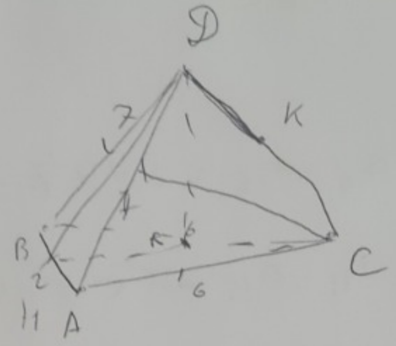
$d^2 > \frac{35}{12}$ $\begin{array}{r|l} 35 & 12 \\ -24 & 2 \\ \hline 11 & \end{array}$

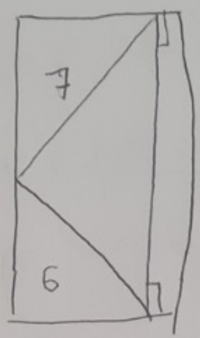
$d \in [-\infty; -3] \cup [3; +\infty)$

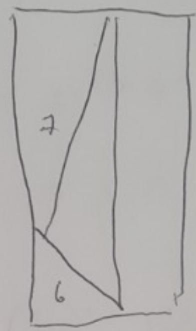
$(a_1 + 24)(a_1 + 48) > (2a_1 + 39) \cdot 7 + 12$

$a_1^2 + 64a_1 + 48 \cdot 24 > 14a_1 + 39 \cdot 7 + 12$

21101112 (U831986 M1297191)







$(x-a)^2+(y-b)^2 \le 25$

$a^2+b^2 \le \min(-8a-6b, 25)$

$a_{11}a_{15}-47 > a_9a_{17}-12$

$a_{11}\cdot a_{15} > a_9\cdot a_{17}+35$

$(a_1+10d)(a_1+14d) > (a_1+8d)(a_1+16d)+35$ $\quad \begin{array}{r} 16 \\ +8 \\ \hline 128 \end{array}$

$\cancel{a_1^2} + \underline{24a_1d} + 140d^2 > \cancel{a_1^2} + \underline{24a_1d} + 128d^2 + 35$

$12d^2 > 35$

$d \geq 3$

21101112 (U831986 M1297191)

Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21101112**

ID профиля: **831986**

Вариант 19

ЧИСТОВИК

№5

1. $\log_{\left(\frac{x}{2}-1\right)^2}\left(\frac{x}{2}-\frac{1}{4}\right)=\frac{1}{2}\log_{\left(\frac{x}{2}-1\right)}\left(\frac{x}{2}-\frac{1}{4}\right)=\frac{a}{2}$, $x>\frac{11}{4}$

2. $\log_{\sqrt{x-\frac{11}{4}}}\left(\frac{x}{2}-1\right)=\dfrac{2}{\log_{\frac{x}{2}-1} x-\frac{11}{4}}=\frac{2}{b}$

3. $\log_{\frac{x}{2}-\frac{1}{4}}\left(x-\frac{11}{4}\right)^2=\dfrac{2\log_{\left(\frac{x}{2}-1\right)}\left(x-\frac{11}{4}\right)}{\log_{\left(\frac{x}{2}-1\right)}\left(\frac{x}{2}-\frac{1}{4}\right)}=\frac{2b}{a}$

$\frac{a}{2}$; $\frac{2}{b}$; $\frac{2b}{a}$

I $\begin{cases}\frac{a}{2}=\frac{2}{b}\\ \frac{a}{2}+1=\frac{2b}{a},\\ \frac{2}{b}+1=\frac{2b}{a},\end{cases}$ $\begin{cases}\frac{2}{b}+1=\frac{b^2}{2} \quad (1)\\ b=\frac{4}{a}\\ \frac{a}{2}+1=\frac{8}{a^2}\end{cases}$

(1) $\cancel{a+b}$ $4+2b=b^3$

$b^3-2b-4=0$

1

21101112 (U831986 M1297192)

чистовик

$b^3-4b+2b-4=0$

$b(b^2-4)+2(b-2)=0$

$(b-2)(b^2+2b+2)=0$

$b=2$ $\quad D<0$

при $b=2$ $\quad a=2$

$\frac{1}{2}\log_{\left(\frac{x}{2}-1\right)}\left(\frac{x}{2}-\frac{1}{4}\right)=1$

$\log_{\left(\frac{x}{2}-1\right)}\left(\frac{x}{2}-\frac{1}{4}\right)=2$

$\frac{x}{2}-\frac{1}{4}=\left(\frac{x}{2}-1\right)^2$

$\frac{x}{2}-\frac{1}{4}=\left(\frac{x}{2}\right)^2-2\frac{x}{2}+1$

$\left(\frac{x}{2}\right)^2-3\left(\frac{x}{2}\right)+\frac{5}{4}=0$

$x^2-6x+5=0$

$x=1$ – не удовл. условию $x>\frac{11}{4}$

$x=5$

Ответ: $x=5$

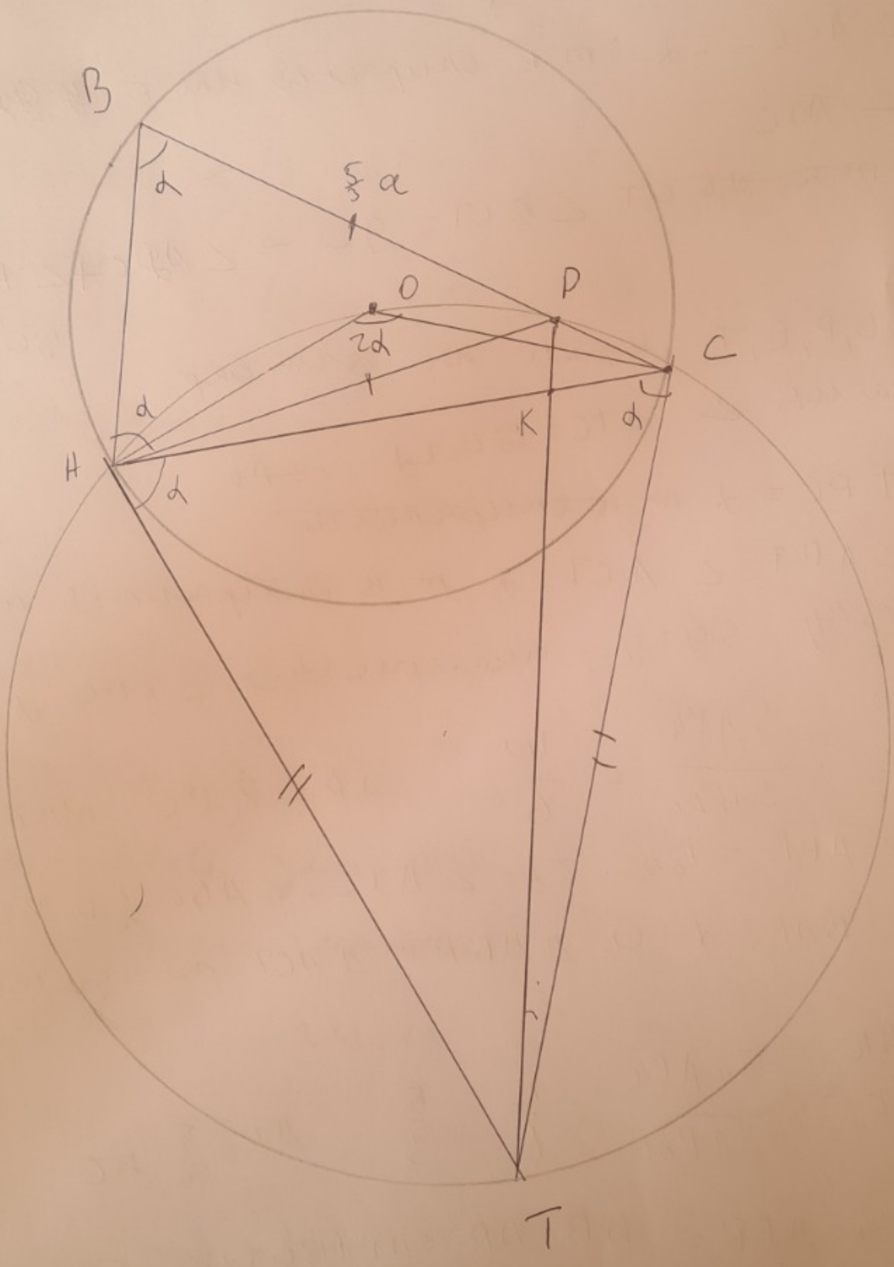
21101112 (U831986 M1297192)

чистовик

~~$S_{\triangle ABC} = \frac{5}{3} S_{\triangle APC} -$~~

чистовик

№6



③

21101112 (U831986 M1297192)

чистовик

1) AT = CT как касат. к окружн.
пусть $\angle TAC = \alpha$

2) $\angle AOC = 2\alpha$ т.к опирается на одну дугу с $\angle TAC$.

3) ~~т.к A, O, C, T~~ $\angle OCT = 90 \Rightarrow \angle AOC + \angle ATC = 180^\circ \rightarrow$

A, O, P, C, T лежат на одной окр.
значит $\angle ATC = 180 - 2\alpha$ ~~, $\angle A$~~

4) ~~$\angle TPC = \alpha$ т.к опирается на~~
$\angle APT = \angle ACT = \alpha$ т.к опираются на одну дугу, аналогично $\angle TPC = \alpha$

$\frac{AP}{PC} = \frac{S_{APK}}{S_{\Delta PKC}} = \frac{10}{6}$, $AP = \frac{5}{3} PC$ пуст $PC = a$

5) $\angle APB = 180^\circ - 2\alpha = \angle ATC$; $\angle ABC = \frac{1}{2} \cup AC = \alpha \Rightarrow$
$\angle BAP = \alpha$ и $\Delta ABP \sim \Delta ACT$ по углам

6) $\frac{AK}{KC} = \frac{S_{\Delta APK}}{S_{\Delta PKC}} = \frac{10}{6} = \frac{5}{3}$ $AK = \frac{5}{3} KC$

7) $S_\Delta ABP = \frac{BP \cdot AP \cdot \sin(180 - 2\alpha)}{2} = \frac{\frac{5}{3} a \cdot \frac{5}{3} a \cdot \sin}{2}$

, $S_{\Delta APC} = \frac{\frac{5}{3} a a \sin 2\alpha}{2} = 10 + 6 = 16 \Rightarrow$ 4 исправлена

~~$S_{\triangle ABP} = \frac{5}{3} S_{\triangle APC} = \frac{16}{3} \cdot 5$~~ $\frac{16 \cdot 3}{2} = 24$

~~тогд $S_{\triangle ABC} = \frac{16}{3} \cdot 5 + 16$~~ $24 + 16 = 40$

~~Ответ: 40~~

$S_{ABP} = \frac{5}{3} S_{\triangle APC} = \frac{16 \cdot 5}{3}$

$S_{\triangle ABC} = 16 + \frac{16 \cdot 5}{3} = \frac{48 + 80}{3} = \frac{128}{3}$

Ответ: $\frac{128}{3}$

№4

НОД $(a; b; c) = 7 \cdot 3$

НОК $(a; b; c) = 3^{17} \cdot 7^{15}$

наимен. значение: $3^1 \cdot 7^1$

наибольш значение: $3^{17} \cdot 7^{15}$

a, b и c - числа: $3^x \cdot 7^y$

$a = 3^{x_1} \cdot 7^{y_1}$

$b = 3^{x_2} \cdot 7^{y_2}$

$c = 3^{x_3} \cdot 7^{y_3}$

5

21101112 (U831986 M1297192)

черновик

вариантов $3^1 \cdot 3^{17} \cdot 3^{r}$: $3! = 6$

т.к $x \in [7; 16]$, то: $15 \cdot 6 = 90$

при $x = 1$ или $x = 17$ - 3 вариант

Итого: $90 + 3 + 3 = 96$

аналогично 7^1; 7^{15}; 7^{2r}:

$13 \cdot 6 + 3 + 3 = 78 + 6 = 84$

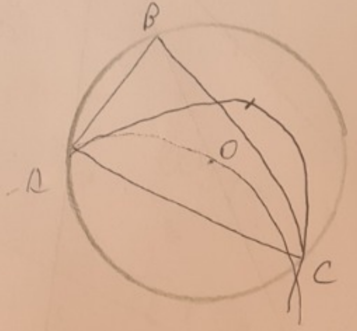
Итог: $96 \cdot 84 = 8064$

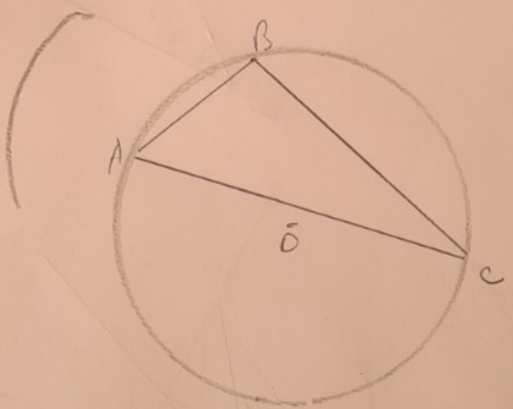
Ответ: 8064

6

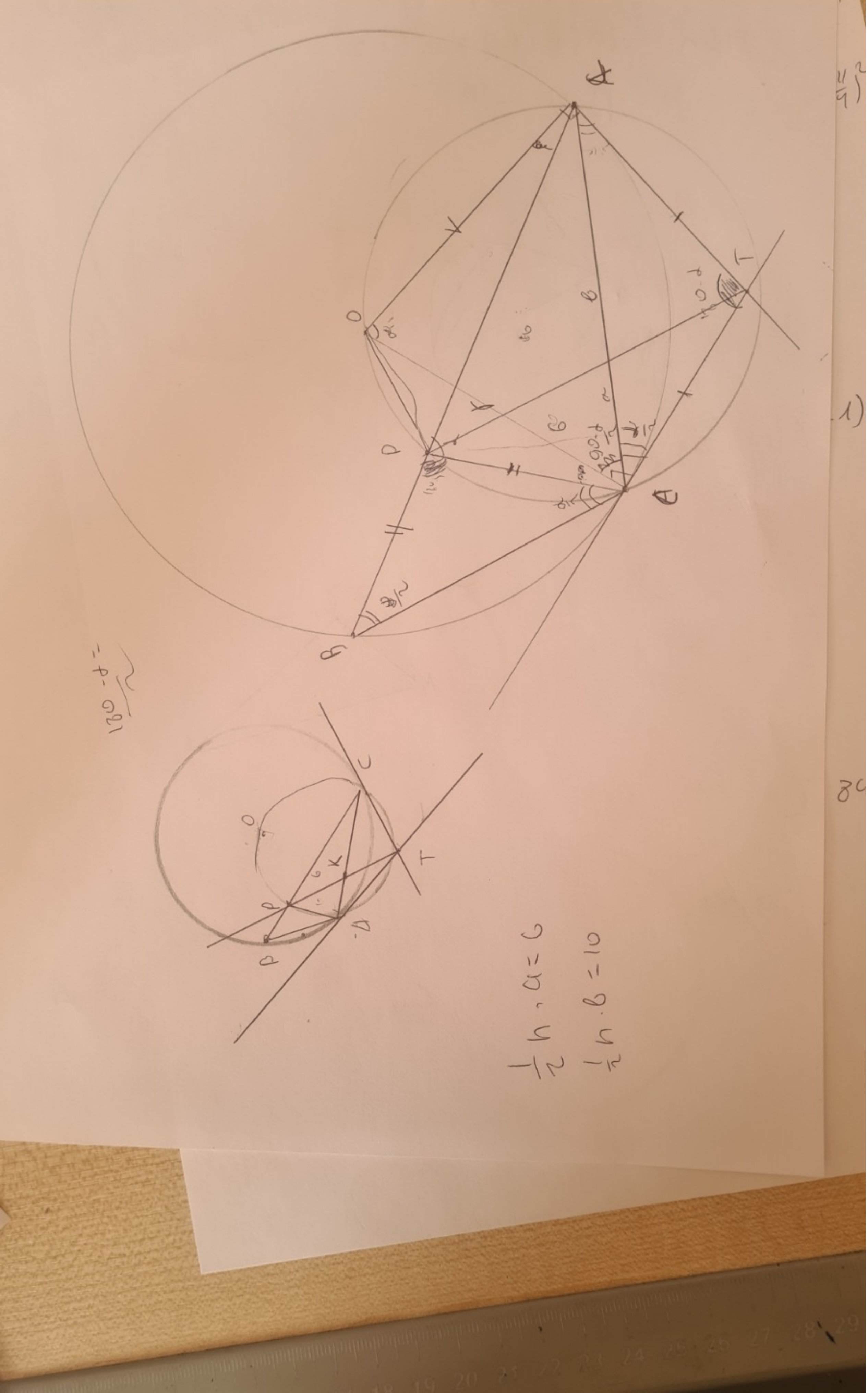
21101112 (U831986 M1297192)

НОД $(a;b;c) = 4$
НОК $(a;b;c) = 3^{17} \cdot 7^{15}$





21101112 (U831986 M1297192)


$\frac{1}{2} h \cdot a = 6$
$\frac{1}{2} h \cdot b = 10$

a b c

1) $\begin{cases} a=b \\ a+b=c+1 \end{cases}$ $a+b+1=c$

$$\begin{cases} \log_{(\frac{x}{2}-1)^2}\left(\frac{x}{2}-\frac{1}{4}\right)=\log_{\sqrt{x-\frac{11}{4}}}\left(\frac{x}{2}-1\right) \\ \log_{(\frac{x}{2}-1)^2}\left(\frac{x}{2}-\frac{1}{4}\right)\cdot 2+1=\log_{\frac{x}{2}-\frac{1}{4}}\left(x-\frac{11}{4}\right)^2 \end{cases}$$

2) $\begin{cases} a=c \\ a+c+1=b \end{cases}$ НОД $(a;b;c)=21$

3) $\begin{cases} c=b \\ c+b+1=a \end{cases}$ НОК $(a;b;c)=3^{17}\cdot 7^{15}$ 8

$$\log_{(\frac{x}{2}-1)^2}\left(\frac{x}{2}-\frac{1}{4}\right)=\log_{(x-\frac{11}{4})^{\frac{1}{2}}}\left(\frac{x}{2}-1\right)$$

$$\frac{1}{2}\log_{(\frac{x}{2}-1)}\left(\frac{x}{2}-\frac{1}{4}\right)=2\log_{(x-\frac{11}{4})}\left(\frac{x}{2}-1\right)$$

$$\frac{1}{2}\log_{(\frac{x}{2}-1)}\left(\frac{x}{2}-\frac{1}{4}\right)=\frac{4}{\log_{(\frac{x}{2}-1)}\left(x-\frac{11}{4}\right)}$$

4 6

42

НОД $=2$

НОК $=24$

21 ; 42 ; 63 ; 84 ; 105

a b c $=21$

НОД

21101112 (U831986 M1297192)

$$
\begin{array}{r}
96 \\
\times\ 84 \\
\hline
384 \\
+\ 768 \\
\hline
8064
\end{array}
$$

21101112 (U831986 M1297192)