Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21101057**

ID профиля: **377919**

Вариант 19

Чистовик
№1

Дано:

$$\begin{cases} a_9 \cdot a_{17} > S+12 \\ a_{11} \cdot a_{15} < S+47 \end{cases}$$

$S = \sum_1^{14} a_n$

a_1 - ?

Решение:

1) по определению $a_n = a_1 + b(n-1)$, подставим в систему

$$\begin{cases} (a_1+8b)(a_1+16b) > S+12 \\ (a_1+10b)(a_1+14b) < S+47 \end{cases} \quad (1)$$

2) Также выразим сумму, через a_1

$$S = \frac{a_1+a_{14}}{2} \cdot 14 = (a_1+a_1+13b)7 = 14a_1+91b \quad (2)$$

3) Подставим (2) в (1)

$$\begin{cases} a_1^2+24a_1b+128b^2 > 14a_1+91b+12 \\ a_1^2+24a_1b+140b^2 < 14a_1+91b+47 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} a_1^2+24a_1b-14a_1-91b > 12-128b^2 \\ a_1^2+24a_1b-14a_1-91b < 47-140b^2 \end{cases}$$

4) Пусть ~~слева~~ слева $f(a_1) = a_1^2+24a_1b-14a_1-91b$; b - какой-то параметр, тогда

$$\begin{cases} f(a_1) > 12-128b^2 \\ f(a_1) < 47-140b^2 \end{cases}$$

5) рассмотрим систему при различных b

при $b=1$

$$\begin{cases} f(a_1) > -116 \\ f(a_1) < -93 \end{cases} \Rightarrow \text{реш. есть}$$

при $b=2$

$$\begin{cases} f(a_1) > -500 \\ f(a_1) < -513 \end{cases} \Rightarrow \text{нет реш.}$$

при $b=3$ и т.д.

$$\begin{cases} f(a_1) > -1140 \\ f(a_1) < -1213 \end{cases} \Rightarrow \text{нет реш}$$

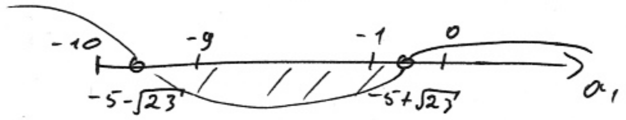
Можно сделать вывод, что при $b>1$ нет решений $\Rightarrow$ $\Rightarrow b=1$

6) Подставим $b=1$ и найдем a_1

$$\begin{cases} a_1^2+24a_1+128 > 14a_1+91+12 \\ a_1^2+24a_1+140 < 14a_1+91+47 \end{cases} \Leftrightarrow$$

Чистовик

$$\Leftrightarrow \begin{cases} a_1^2+10a_1+25>0 \\ a_1^2+10a_1+2<0 \end{cases} \Leftrightarrow \begin{cases} (a_1+5)^2>0 \\ \left(a_1-(-5+\sqrt{23})\right)\left(a_1-(-5-\sqrt{23})\right)<0 \end{cases}$$



$\Rightarrow a_1 \in (-5-\sqrt{23}; -5) \cup (-5; -5+\sqrt{23}$

$-5-\sqrt{23} \vee -10$

$5+\sqrt{23} \vee 10$

$\sqrt{23} \vee \cancel{1}5 \quad {}^{\wedge 2}$

$23<25$

$-5-\sqrt{23} \vee -9$

$5+\sqrt{23} \vee 9$

$\sqrt{23} \vee 4$

$23>16$

$-5+\sqrt{23} \vee 0$

$23<25$

$-5+\sqrt{23} \vee -1$

$23>16$

$\Downarrow$

$a_1 \in \{-9; -8; -7; -6; -4; -3; -2; -1\}$

<u>Ответ:</u> $a_1 \in \{-9; -8; -7; -6; -4; -3; -2; -1\}$

Чистовик

№2

Дано:

$AB=2$

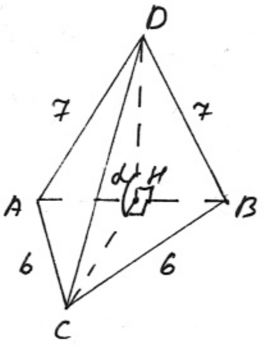
$AC=BC=6$

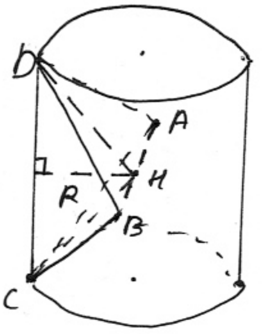
$AD=BD=7$

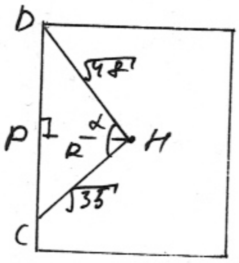
$CD \parallel$ оси

R - min

CD - ?







Решение

1) Две ~~боковые~~ грани тэтраэдра - это равнобедренные треугольники

2) Изменяя длину ребра CD, мы просто меняем величину двугранного угла α: $\alpha = (ABC)\hat{;}(ABD) = DH\hat{;}CH$

3) У цилиндра с вписаным тэтраэдрам будет минимальный радиус $\Leftrightarrow$

$2R = AB$

$R = \frac{AB}{2} = 1$

4) Найдем DH и CH

$DH = \sqrt{BD^2 - \left(\frac{AB}{2}\right)^2} = \sqrt{49-1} = \sqrt{48}$

$CH = \sqrt{CB^2 - \left(\frac{AB}{2}\right)^2} = \sqrt{36-1} = \sqrt{35}$

5) $CD = CP + PD$

$CP = \sqrt{CH^2 - R^2} = \sqrt{35-1} = \sqrt{34}$

$PD = \sqrt{DH^2 - R^2} = \sqrt{48-1} = \sqrt{47}$

$CD = \sqrt{34} + \sqrt{47}$

Ответ: $CD = \sqrt{34} + \sqrt{47}$

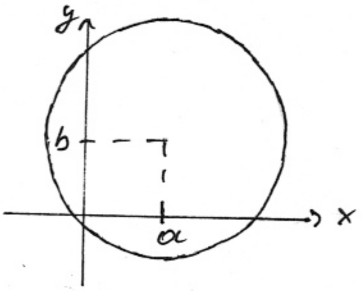
Чистовик

№3

Дано:

$$\begin{cases}(x-a)^2+(y-b)^2\le 25\\ a^2+b^2\le \min(-8a-6b;25)\end{cases}$$

Решение:



1) Первое ур-ие системы задает множество решений, находящееся внутри окружности с радиусом 5 и центром в т. $(a;b)$

2) Если существует вещественная пара чисел $(a;b)$, то множество решений $(x;y)$ не изменится. Просто будет смещаться. ~~центр~~

3) Значит фигура M - окр. с радиусом 5 =>

$=> S=\pi R^2=25\pi$

Ответ: $S=25\pi$

ЧЕРНОВИК

№1

$$a_n \quad \begin{cases} a_9 \cdot a_{17} > S+12 \\ a_{11} \cdot a_{15} < S+47 \end{cases} \Rightarrow \begin{cases} (a_1+b8)\cdot(a_1+b16) > S+12 \\ (a_1+b10)\cdot(a_1+b14) < S+47 \end{cases} \Leftrightarrow$$

$$a_{n+1} = a_n + \cancel{b^{n-1}}\, b(n-1)$$

$$S = \sum_{n=1}^{14} a_n = \frac{a_1+a_{14}}{2}14 = \frac{a_1+a_1+b\cdot 13}{2}14 = \frac{2a_1+13b}{2}14$$

$$\begin{cases} (a_1+8b)(a_1+16b) > S+12 \\ (a_1+10b)(a_1+14b) < S+47 \\ S = (2a_1+13b)7 \end{cases} \Leftrightarrow \begin{cases} a_1^2+24a_1b+8\cdot16b^2 > S+12 \\ a_1^2+24a_1b+140b^2 < S+47 \\ S = 14a_1+91b \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \overbrace{a_1^2+24a_1b}^{k} > S+12-128b^2 \\ a_1^2+24a_1b < S+47-140b^2 \\ S = 14a_1+91b \end{cases} \Rightarrow \quad \cancel{K\Leftrightarrow} \begin{cases} a_1^2+24a_1b > 14a_1+91b+12-128b^2 \\ a_1^2+24a_1b < 14a_1+91b+47-140b^2 \end{cases}$$

$$\begin{cases} a_1^2+24a_1b-14a_1-91b > 12-128b^2 \\ a_1^2+24a_1b\cancel{+}-14a_1-91b < 47-140b^2 \end{cases} \Rightarrow$$

$$k > 12-128b^2 \qquad 12-128b^2 \vee 47-140b^2 \quad {}^{+128b^2}$$

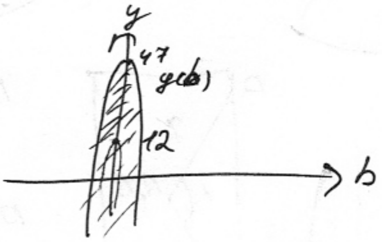
$$k < 47-140b^2 \qquad 12 \vee 47-12b^2$$

$$\exists\ f(b) = 12-128b^2 = 0$$

$$g(b) = 47-140b^2 = 0$$

$$g(x):\ b = \pm\sqrt{\frac{47}{140}} < 1$$

$$f(x):\ b = \pm\sqrt{\frac{12}{128}} < 1$$



$$\text{т.к. } x_1 \text{ и } x_2 < 1 \Rightarrow \exists!\ \ominus$$

$$\begin{array}{r} 13 \\ \times 128 \\ 4 \\ \hline 512 \end{array}$$

$$\begin{cases} a_1^2+(24b-14)a_1+128b^2-91b-12 > 0 \\ a_1^2+(24b-14)a_1+140b^2-91b-47 < 0 \end{cases}$$

что $\forall\ \exists\ a_1 \quad D \geq 0$

$$\begin{cases} (24b-14)^2-4(128b^2-91b-12) \geq 0 \\ (24b-14)^2-4(140b^2-91b-47) \geq 0 \end{cases} \Leftrightarrow$$

$$\begin{cases} 24^2b^2-24\cdot28b+14^2-512b^2+364b+48 \geq 0 \\ 24^2b^2-24\cdot28b+14^2-560b^2+364b+4\cdot47 \geq 0 \end{cases}$$

Черновик

$24^2 b^2 - 24 \cdot 28b + 14^2 - 512b^2 + 364b + 48 \geq 0$

$24^2 b^2 - 24 \cdot 28b + 14^2 - 560b^2$

$$\begin{cases} a_9 \, a_{17} > S + 12 \\ a_{11} \cdot a_{15} < S + 47 \end{cases}$$

$S = \frac{a_1 + a_{14}}{2} 14$

$a_{14} = a_1 + b \cdot 13$

$a_9 = a_1 + 8b$ $\Rightarrow$ $a_{14} = a_9 - 8b + 13b$

$a_{14} = a_9 + 5b$

$a_{17} = a_1 + 16b$

$S = \frac{a_{17} - 16b + a_9 + 5b}{2} 14 = \frac{a_9 + a_7 - 11b}{2} 14$

$a_9 \cdot a_{17} > \left(\frac{a_7 + a_9}{2}\right) 7 - 77b$

№2

Дано:

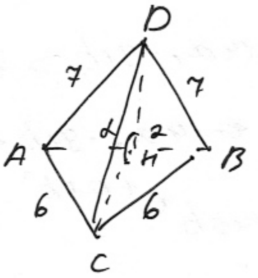
$AB = 2$

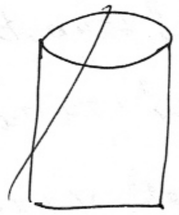
$AC = BC = 6$

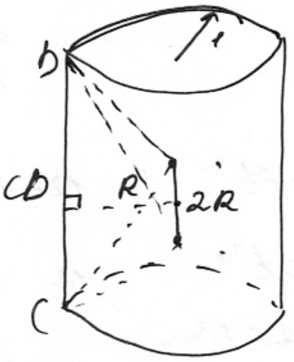
$AD = BD = 7$

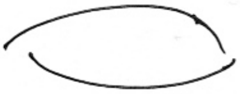
$CD \parallel$ оси

$CD - ?$



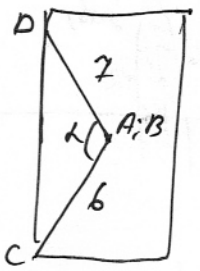


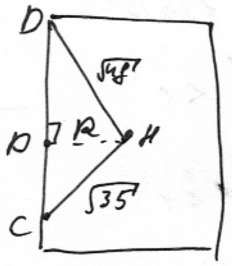




$2R \geq 2$

$R \geq 1$





$S = \overset{-120}{14(-5)} + 91 =$

$= -29$

$(-5+8)(-5+1) > 12 - 29$

$3 \cdot (-4) > -17$

$(-5+10)(-5+14) < 47 - 29$

$5 \cdot 9 < 18$

$CH = \sqrt{36 - 1} = \sqrt{35}$

$DH = \sqrt{49 - 1} = \sqrt{48}$

$CD^2 = PD + CP$

$PD = \sqrt{48 - 1} = \sqrt{47}$

$PC = \sqrt{35 - 1} = \sqrt{34}$

$CD = \sqrt{34} + \sqrt{47}$

$S = \overset{-98}{14 \cdot -7} + 91 = -7$

$(-7+8)(-7+16) > -7 + 12$

$1 \cdot 9 > 5$

$(-7+10)(-7+14) < 47 \overset{-7}{\cancel{7}}$

$3 \cdot 7 < 40$

Черновик

№3

$(x-a)^2 + (y-b)^2 \leq 25$

$a^2+b^2 \leq \min(-8a-6b, 25)$

$] -8a-6b < 25$

$-8a < 25+6b$

$a > -\frac{25}{8} - \frac{6}{8}b$

$a^2+b^2 \leq -8a-6b$

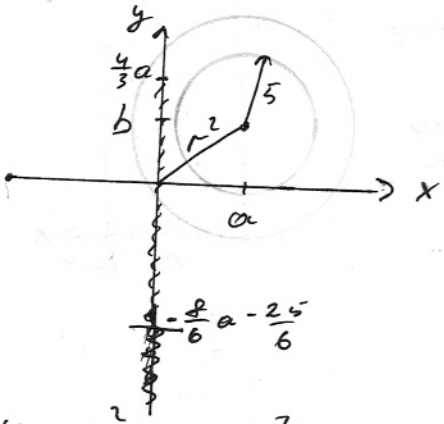
$a^2+8a+b^2+6b \leq 0$

$a^2+8a+16+b^2+6b+9 \leq 0$

$(a+4)^2+(b+3)^2 \leq 5^2$

$] 25 \geq -8a-6b$

$a^2+b^2 \leq 5^2$



$(x-a)^2 + (y-b)^2 \leq 25$

$$\begin{cases}(a+4)^2+(b+3)^2 \leq 25\\ -8a-6b < 25; \quad b > -\frac{8}{6}a - \frac{25}{6}\end{cases}$$

$$\begin{cases}a^2+b^2 \leq 25\\ 25 \geq -8a-6b; \quad b \leq -\frac{4}{3}a - \frac{25}{6}\end{cases}$$

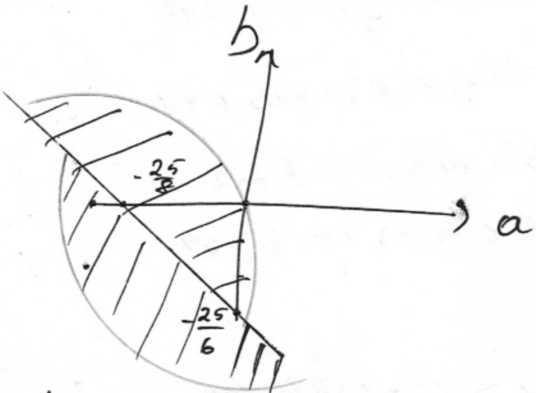
$b = \frac{-8a-25}{6}$

$b = -\frac{4}{3}a - \frac{25}{6}$

$b=0$

$\frac{25}{82} = -\frac{4}{8}a$

$a = -\frac{25}{8}$



$r^2 \leq \min(-8a-6b; 25)$

при $-8a-6b<25; \quad b > -\frac{8}{6}a - \frac{25}{6}$

$r^2 \leq -8a-6b > 0 \quad -6b > 8a$

$b < \frac{4}{3}a$

при $25 \geq -8a-6b$

$r^2 \leq 25$

№1

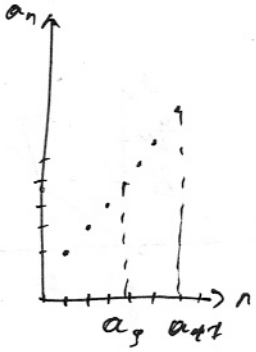
$\frac{a_1+a_2+\ldots a_n}{n} \geq \sqrt{a_1 a_2 a_3 \ldots a_n}$

ЧЕРНОВИК

№1

$$\begin{cases} a_9 \cdot a_{17} > S+12 \\ a_{11} \cdot a_{15} < S+47 \\ S=\frac{a_1+a_{14}}{2}\cdot 14 \end{cases}$$

$a_1 - ?$



$$\begin{cases} \frac{S}{14} \geq \sqrt{a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_{14}} \\ S+12 < a_9 \cdot a_{17} \end{cases}$$

$a_9 = a_1+8b$

$a_{17} = a_1+16b$

$a_{11} = a_1+10b$

$a_{15} = a_1+14b$

$(a_1+8b)(a_1+16b) > S+12$

$(a_1+10b)(a_1+14b) < S+47$

$S=\frac{a_1+a_1+13b}{2}14=\frac{2a_1+13b}{2}14=14a_1+7\cdot 13b$

$\begin{array}{r} {}^{4}16 \\ \times\ 8 \\ \hline 128 \end{array}$ $\begin{array}{r} {}^{2}37 \\ \times\ 3 \\ \hline 111 \end{array}$

$\begin{array}{r} 128 \\ -\ 91 \\ \hline 37 \end{array}$ $\begin{array}{r} 111 \\ -\ 12 \\ \hline 99 \end{array}$

$\begin{array}{r} 37 \\ -\ 12 \\ \hline 25 \end{array}$ $\begin{array}{r} {}^{2}49 \\ \times\ 3 \\ \hline 147 \\ -\ 47 \\ \hline 100 \end{array}$

$\begin{array}{r} 74 \\ -\ 12 \\ \hline 62 \end{array}$

$\begin{array}{r} 98 \\ -\ 47 \\ \hline 51 \end{array}$

$$\begin{cases} a_1^2+24a_1b+8\cdot16b^2 > 14a_1+91b+12 \\ a_1^2+24a_1b+140b^2 < 14a_1+91b+47 \end{cases}$$

$$\begin{cases} a_1^2+24a_1b+128b^2 > 14a_1+91b+12 \\ a_1^2+24a_1b+140b^2 < 14a_1+91b+47 \end{cases}$$

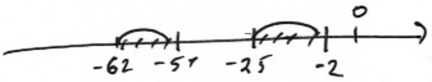
$\exists\ b=const$

$$\begin{cases} a_1^2+24a_1b-14a_1 > 12-\overset{37}{\cancel{128}}b \\ a_1^2+24a_1b-14a_1 < 47-49b \end{cases}$$

$\exists\ f(a_1)=a_1^2+(24b-14)a_1$

$f(a_1) > 12-37b$

$f(a_1) < 47-49b$



$\exists\ b=1$ | $b=2$ | $b=3$

$f(a_1) > -25$ | $f(a_1) > -62$ | $f(a_1) > -99$

$f(a_1) < -2$ | $f(a_2) < -51$ | $f(a_1) < -100$

может быть при $\boxed{b=1 \mid b=2}$

Черновик

$a_1^2 + (24b-14)a_1^2 > 91b - 128b^2 + 12$

$< \cancel{9}\ 47 - 140b^2$

$$\begin{array}{r} {}^{1\,3}\\ \times\ 128 \\ 4 \\ \hline -512 \end{array} \qquad \begin{array}{r} {}^{1}\\ \times\ 140 \\ 4 \\ \hline -560 \\ 47 \\ \hline 513 \end{array}$$

$f(a_1) > 12 - 128b^2$

$f(a_1) < 47 - 140b^2$

$$\begin{array}{r} {}^{\cdot\,9\,10}\\ -\ 140 \\ 47 \\ \hline 93 \end{array}$$

$\longrightarrow$

при $b = 1$

$> 12 - 128 = -116$

$< 47 - 140 = -93$

при $b = 2$

$> 12 - 128 \cdot 4 = -500$

$< 47 - 140 \cdot 4 = -513$

$\Rightarrow \boxed{b = 1}$

$a_1^2 + 24a_1 + 128 > 14a_1 + 91 + 12$

$$\begin{array}{r} {}^{2\,7}\\ \times\ 128 \\ 9 \\ \hline -1152 \\ 12 \\ \hline -1140 \end{array} \qquad \begin{array}{r} {}^{3}\\ \times\ 140 \\ 9 \\ \hline -1260 \\ 47 \\ \hline -1213 \end{array} \qquad \begin{array}{r} -\ 128 \\ 103 \\ \hline 25 \end{array} \qquad \begin{array}{r} -\ 140 \\ 138 \\ \hline 2 \end{array}$$

$100 - 4 \cdot 2 = \cancel{96}$ $\quad \begin{array}{r|l} 92 & 4 \\ \underline{8} & 23 \\ 12 & \end{array}$

$a_1 = \frac{-10 \pm \sqrt{92}}{2} = \frac{-10 \pm 2\sqrt{23}}{2}$

$a_1 = -5 \pm \sqrt{23}$

Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21101057**

ID профиля: **377919**

Вариант 19

Чистовик

№4

Дано:

$$\left[\begin{array}{l} \text{НОД}(a;b;c)=21 \\ \text{НОК}(a;b;c)=3^{17}\cdot7^{15} \end{array}\right.$$

Решение:

1) НОД равносильно

$$\begin{cases} a \vdots 21 \\ b \vdots 21 \\ c \vdots 21 \end{cases} \Rightarrow \begin{cases} a=k21 \\ b=m21 \\ c=n21 \end{cases}, \text{ где } m;n;k \in \mathbb{N}$$

2) НОК равносильно

$$\begin{cases} 3^{17}\cdot7^{15} \vdots a \\ 3^{17}\cdot7^{15} \vdots b \\ 3^{17}\cdot7^{15} \vdots c \end{cases} \Leftrightarrow \begin{cases} 3^{17}\cdot7^{15} \vdots k21 \\ 3^{17}\cdot7^{15} \vdots m21 \\ 3^{17}\cdot7^{15} \vdots n21 \end{cases}, \text{т.к. } 21=7\cdot3 \Rightarrow 3^{16}\cdot7^{14} \vdots (k;m;n)$$

3) Из пункта 2 следует, что

$k;m;n \in \{3;3^2;3^3;\dots;3^{16};7;7^2;7^3;\dots;7^{14};3\cdot7;3\cdot7^2;3 7^3;\dots;3;7^{14};3^2;7;3^2\cdot7^2;\dots\}$

Всего $k;m;n$ могут быть числами

$30+14\cdot16$

4) следовательно a,b,c могут быть также 354 числами

$(354;354;354) \Rightarrow$ ~~всего~~ всего 354^3 троек

Ответ: 354^3

Чистовик

№6

Дано:

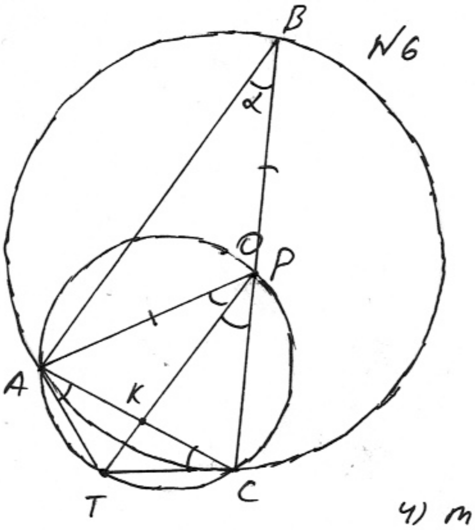
$S_{APK} = 10$

$S_{CPK} = 6$

а) S_{ABC} - ?

б) AC - ?

$\angle ABC = \operatorname{arctg}(2)$



а) 1) Пусть $\angle ABC = \alpha$

т.к. $\alpha = \frac{\overset{\frown}{AC}}{2}$ и $\angle TAK = \angle TCK = \frac{\overset{\frown}{AC}}{2} \Rightarrow$

$\Rightarrow \angle TAK$ и $\angle TCK = \alpha$

2) $\angle APT$ и $\angle ACT$ опираются на $\overset{\frown}{AT} \Rightarrow \angle APT = \alpha$

3) аналогично $\angle CPT = \alpha$

4) т.к. $\angle APK = \angle CPK \Rightarrow KP$ - биссектриса

5) по св-ву биссектрисы

$$\frac{AK}{AP} = \frac{KC}{PC}$$

$$\frac{AK}{KC} = \frac{AP}{PC}$$

6) т.к. $\triangle APK$ и $\triangle KPK$ имеют одинаковую высоту,

то $\frac{AK}{KC} = \frac{S_{AKP}}{S_{CPK}} = \frac{10}{6} = \frac{5}{3} \Rightarrow \frac{AP}{PC} = \frac{5}{3}$

7) $\angle APB = 180 - 2\alpha \Rightarrow \angle BAP = \alpha \Rightarrow$

$\Rightarrow \triangle ABP$ - равнобедренный $\Rightarrow AP = BP \Rightarrow$

$\Rightarrow \frac{BP}{PC} = \frac{5}{3}$

8) $\frac{S_{APB}}{S_{APC}} = \frac{BP}{PC} = \frac{5}{3}$

$$\frac{S_{ABC} - 16}{16} = \frac{5}{3}$$

$$S_{ABC} = \frac{80}{3} + 16$$

$$S_{ABC} = \cancel{\frac{80}{3}} \frac{80+48}{3} = \frac{128}{3}$$

б) 1) $AP = \frac{5}{3}PC$

2) $S_{APC} = \frac{1}{2} AP \cdot PC \cdot \sin 2\alpha$

$$16 = \frac{1}{2} AP \cdot \frac{3}{5} AP \cdot \sin 2\alpha$$

$$16 = \frac{3AP^2 \sin 2\alpha}{10}$$

Чистовик

$AP^2 = \sqrt{\dfrac{160}{3\sin 2\alpha}} = \sqrt{\dfrac{160}{6\sin\alpha\cos\alpha}}$

3)

$\Rightarrow \sin\alpha = \dfrac{2}{\sqrt{5}}$

$\cos\alpha = \dfrac{1}{\sqrt{5}}$

4) $AP = \sqrt{\dfrac{80\sqrt{5}\sqrt{5}}{3\cdot 2\cdot 1}} = \sqrt{\dfrac{80\cdot 5}{6}} = \sqrt{\dfrac{400}{6}} = \dfrac{20}{\sqrt{6}} = \dfrac{10\sqrt{6}}{3}$

~~5) по т. sin для $\triangle ABP$~~

~~$\dfrac{AP}{\sin\alpha} = \dfrac{AB}{\sin(180-2\alpha)}$~~

~~$AB = \dfrac{2AP\cancel{\sin\alpha}\cdot\cos\alpha}{\cancel{\sin\alpha}}$~~

~~$AB = 2\cdot\dfrac{10\sqrt{6}}{3}\cdot\dfrac{1}{\sqrt{5}} = \dfrac{20\sqrt{6}}{3\sqrt{5}} = \dfrac{20\sqrt{30}}{15} = \dfrac{4\sqrt{30}}{3}$~~

~~6) $\dfrac{BP}{\underset{BC-BP}{\overset{\parallel}{PC}}} = \dfrac{5}{3}$~~

~~$3BP = 5BC - 5BP$~~

~~$BC = \dfrac{8}{5}BP = \dfrac{8}{5}AP = \dfrac{80\sqrt{6}}{12} = \dfrac{20\sqrt{6}}{3}$~~

~~7) по т. cos для $\triangle ABC$~~

5) $PC = \dfrac{3}{5}AP = 2\sqrt{6}$

6) по т. cos для $\triangle APC$

$AC^2 = AP^2 + PC^2 - 2AP\cdot PC\cdot\cos 2\alpha$

$AC^2 = \dfrac{100\cdot 6}{9} + 4\cdot 6 - 2\cdot\dfrac{10\sqrt{6}}{3}\cdot 2\sqrt{6}\,(\cos^2\alpha - \sin^2\alpha)$

$AC^2 = \dfrac{600}{9} + 24 - \dfrac{40\cdot 6}{3}\left(\dfrac{1}{5} - \dfrac{4}{5}\right)$

$AC^2 = \dfrac{600+24\cdot 9}{9} - 80\left(-\dfrac{3}{5}\right)$

$AC^2 = \cancel{\dfrac{816}{9}} + \cancel{\dfrac{240}{5}}\quad \dfrac{200}{3} + 24 + \overset{48}{\overset{\parallel}{16\cdot 3}}$

$AC^2 = \dfrac{200}{3} + 72 = \dfrac{200+216}{3}$

$AC = \sqrt{\dfrac{416}{3}} = \sqrt{\dfrac{26\cdot 16}{3}} = 4\sqrt{\dfrac{26}{3}}$

Ответ: а) $S_{ABC} = \frac{128}{3}$ б) $AC = 4\sqrt{\frac{26}{3}}$

ЧЕРНОВИК

$$\log_{\left(\frac{x-2}{2}\right)^2}\left(\frac{x}{2}-\frac{1}{4}\right) = \log_{\frac{(x-2)}{4}}{}^{2}\left(\frac{2x-1}{4}\right) = \frac{\log_2(2x-1)-2}{\log_2(x-2)^2-2}$$

$$\log_{\sqrt{\frac{4x-11}{4}}}\frac{2x-4}{4} = \frac{\log_2(2x-4)-2}{\log_2\sqrt{4x-11}-1}$$

$$\log_{\frac{2x-1}{4}}\left(\frac{4x-11}{4}\right)^2 = \frac{\log_2(4x-11)^2-4}{\log_2(2x-1)-2}$$

$$\frac{\log_2(2x-1)-\log_2 4}{\log_4(x-2)^2-\log_2 4} = \frac{2x-1-4}{(x-2)^2-4} = \frac{2x-5}{x^2-4x}$$

$$\frac{1}{2}\cdot\frac{2x-4-4}{4x-11-4} = \frac{1}{2}\,\frac{2x-8}{4x-15} = \frac{x-4}{4x-15}$$

$$\frac{(4x-11)^2-4}{2x-1-4} = \frac{16x^2-88x+121-4}{2x-5} =$$

$$\begin{array}{r} 72 \\ \times\ 3 \\ \hline 216 \end{array}$$

$$\begin{array}{r|l} 416 & 16 \\ \underline{32} & 26 \\ 96 & \\ 96 & \end{array}$$

$$\frac{\log_2(2x-1)-2}{\log_2(x-2)^2-2} = \frac{\log_2(2x-4)-2}{\log\sqrt{4x-11}-1}$$

$$6\frac{200}{3}+24-\frac{40\cdot6}{3}\left(\frac{1}{5}-\frac{4}{5}\right) = \frac{200}{3}+24+\frac{\overset{8}{\cancel{40}}\cdot6\cdot3}{\cancel{3}\cdot\cancel{5}} =$$

$$= \frac{200}{3}+24+48 = 72 = \frac{200+216}{3} = \frac{416}{3} = \frac{26\cdot16}{1}$$

ЧЕРНОВИК

$$16 = \frac{AP\cdot\frac{5}{3}AP}{2}\cdot\sin 2\alpha$$

$$16 = \frac{5AP^2}{6}\cdot\sin 2\alpha$$

$$AP = \sqrt{\frac{96}{5\cdot\sin 2\alpha}}$$

$$AC = \sqrt{\frac{96}{5\sin 2\alpha}\left(4\cos^2\alpha + \frac{64}{25} - \frac{32}{5}\cos\alpha\right)} =$$

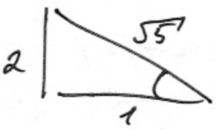
$$= \sqrt{\frac{96\cdot\overset{2}{\cancel{4}}\cos^{\cancel{2}}\alpha}{\underset{5}{\cancel{10}}\sin\alpha\cdot\cancel{\cos\alpha}} + \frac{96\cdot 64}{250\cdot\sin\alpha\cdot\cos\alpha} - \frac{96\cdot\overset{16}{\cancel{32}}\cancel{\cos\alpha}}{\underset{25}{\cancel{50}}\sin\alpha\,\cancel{\cos\alpha}}} =$$

$$= \sqrt{\frac{192}{5}\operatorname{ctg}\alpha + \frac{96\cdot\overset{32}{\cancel{64}}}{125\cdot\sin\alpha\cdot\cos\alpha} - \frac{96-16}{25\cdot\sin\alpha}} =$$

$$= \sqrt{\frac{192}{5}\operatorname{ctg}\alpha + \frac{96\cdot16}{25\sin\alpha}\left(\frac{2}{5\cos\alpha} - 1\right)}$$

$$\frac{192}{10} + \frac{96\cdot16\sqrt{5}}{25\cdot2}\left(\frac{2\sqrt{5}}{5} - 1\right)$$

$$\sqrt{96\left(\frac{1}{5} + \frac{16\sqrt{5}}{50}\left(\frac{2\sqrt{5}-5}{5}\right)\right)}$$



$$=$$

$$AP = \frac{10\sqrt{6}}{3} \qquad PC = \frac{\cancel{3}}{5}\cdot\frac{10\sqrt{6}}{\cancel{3}} = 2\sqrt{6}$$

$$AC = \sqrt{\frac{100\cdot6}{9} + 4\cdot6 - 2\,\frac{10\sqrt{6}}{3}\cdot2\sqrt{6}\cdot2\sin\alpha\cos\alpha} =$$

$$\begin{array}{r|l} \overset{10}{416} & 16 \\ -32 & 26 \\ 96 & \end{array}$$

$$\begin{array}{r|l} 416 & 2 \\ -4 & 208 \;|\; 2 \\ 16 & 104 \;|\; 4 \\ 0 & -8 \;\; 26 \;|\; 4 \\ & 24 \end{array}$$

$72 = 8\cdot9$

$$\begin{array}{r|l} 816 & 9 \\ -81 & 9 \end{array}$$

$$\begin{array}{r|l} 240 & 5 \\ -20 & 48 \\ 40 & \end{array}$$

$$\begin{array}{r} \overset{3}{24} \\ \times\ 9 \\ \hline 216 \\ +\ 600 \\ \hline 816 \end{array}$$

$$\begin{array}{r} 72 \\ \times\ 3 \\ \hline 216 \end{array}$$

$90 + \frac{6}{9} + 48$

$\times\ 138$
9

$138 + \frac{6}{9}$

$$\frac{600 + 8\cdot9^2}{9}$$

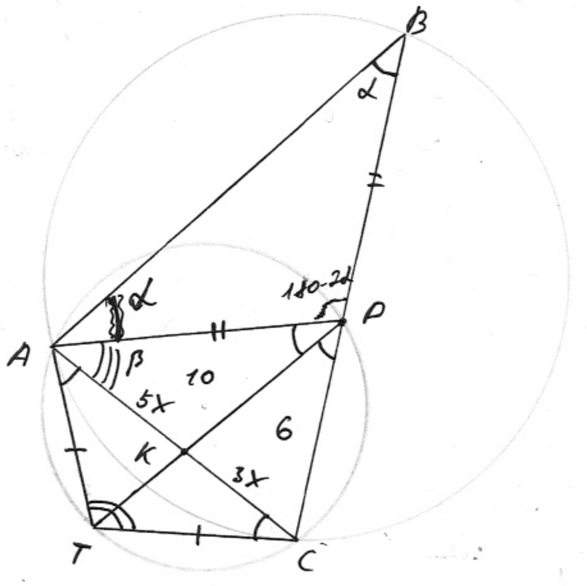
5n

$$\begin{array}{r} \overset{1}{16} \\ \times\ 3 \\ \hline 48 \\ +\ 24 \\ \hline 72 \end{array}$$

$\frac{600}{9} + 24 + 16\cdot3$

$\frac{600}{9} + 72$

$\frac{300}{3} + 72$



$N6$ Черновик

$S_{APK}=10$

$S_{KPC}=6$

S_{ABC} - ?

$\frac{BP}{PC}$ - ?

$$\begin{array}{r|l} -128 & 3 \\ \underline{12} & 42,(6) \\ -8 & \\ \underline{6} & \\ -20 & \end{array}$$

$$\frac{5x}{AP}=\frac{3x}{PC}$$

$$\frac{AP}{PC}=\frac{5}{3} => \frac{BP}{PC}=\frac{5}{3}$$

$$\frac{\overset{S_{ABC}-S_{APC}}{\overset{\shortparallel}{S_{ABC}}}}{S_{APC}}=\frac{5}{3}$$

$$180-(180-2\alpha)-\alpha=2\alpha-\alpha$$

$$\frac{S_{ABC}}{S_{APC}}=\frac{5}{3}+1$$

$$S_{ABC}=\frac{8}{3}S_{APC}=\frac{8}{3}\cdot 16=\frac{8^2\cdot 2}{3}$$

б) 1) $\frac{AP}{\sin\alpha}=\frac{AB}{\overset{\cancel{\cos 2\alpha}}{\sin 2\alpha}}$; $\frac{AP}{\cancel{\sin\alpha}}=\frac{AB}{2\cancel{\sin\alpha}\cdot\cos\alpha}$

$$AB=2AP\cos\alpha$$

2) ~~BC~~= $\frac{BP}{\underset{\overset{\shortparallel}{BC-BP}}{PC}}=\frac{5}{3}$

$$3BP=5BC-5BP$$

$$BC=\frac{8}{5}BP=\frac{8}{5}AP$$

$$AC^2=AB^2+BC^2-2AB\cdot BC\cdot\cos\alpha$$

$$AC^2=4AP^2\cos^2\alpha+\frac{64}{25}AP^2-4\cdot AP^2\cdot\frac{8}{5}\cos\alpha$$

$$AC=AP\sqrt{4\cos^2\alpha+\frac{64}{25}-\cdot\frac{32}{5}\cos\alpha}$$

3) ~~BC~~=

$$16=\frac{AP\cdot PC}{2}\cdot\sin 2\alpha=$$

$$\frac{AP}{PC}=\frac{5}{3}$$

Черновик

№5

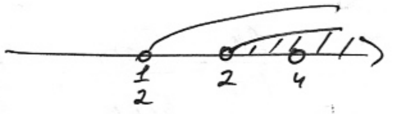
$$\log_{\left(\frac{x}{2}-1\right)^2}\left(\frac{x}{2}-\frac{1}{4}\right) = \frac{1}{2}\log_{\frac{x-2}{2}}\frac{2x-1}{4} = \frac{1}{2}\frac{\ln\frac{2x-1}{4}}{\ln\frac{x-2}{2}} = \frac{1}{2}\frac{\ln(2x-1)-\ln 4}{\ln(x-2)-\ln 2} = \frac{1}{2}\frac{\log_2(2x-1)-2}{\log_2(x-2)-1}$$

$$\log_{\sqrt{x-\frac{11}{4}}}\left(\frac{x}{2}-1\right) = 2\log_{\frac{4x-11}{4}}\frac{x-2}{2} = 2\frac{\log_2(x-2)-1}{\log(4x-11)-2}$$

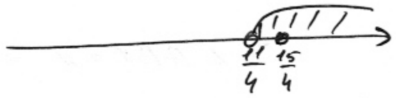
$$\log_{\frac{x}{2}-\frac{1}{4}}\left(x-\frac{11}{4}\right)^2 = 2\log_{\left(\frac{2x-1}{4}\right)}\left|\frac{4x-11}{4}\right| = 2\frac{\log_2|4x-11|-2}{\log_2(2x-1)-2}$$

ОДЗ.

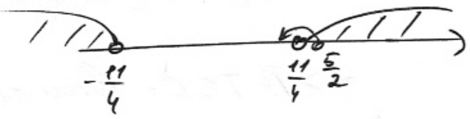
$$\begin{cases}\frac{x}{2}-\frac{1}{4}>0\\ \frac{x}{2}-1>0\\ \frac{x}{2}-1\neq 1\end{cases} \Rightarrow \begin{cases}x>\frac{1}{2}\\ x>2\\ x\neq 4\end{cases}$$



$$\boxed{x\in\left(\frac{11}{4};+\infty\right)\setminus\left\{4;\frac{15}{4}\right\}}$$



$$\begin{cases}x-\frac{11}{4}>0\\ x-\frac{11}{4}\neq 1\end{cases} \Rightarrow \begin{cases}x>\frac{11}{4}\\ x\neq\frac{15}{4}\end{cases}$$



$$\begin{cases}\left(x-\frac{11}{4}\right)^2>0\\ \frac{x}{2}-\frac{1}{4}\neq 1\end{cases} \Rightarrow \begin{cases}|x|>\frac{11}{4}\\ x\neq\frac{5}{2}\end{cases}$$

1) $$\frac{1}{2}\frac{\log_2(2x-1)-2}{\log_2(x-2)-1} = 2\frac{\log_2(x-2)-1}{\log_2(4x-11)-2}$$ 1=2

2=3

$$\left(\log_2(x-2)-1\right)^2 = \frac{1}{4}\left(\log_2(2x-1)-2\right)\left(\log_{(4x-11)}-2\right)$$ 1=3

$$\log_2^2(x-2) - 2\log_2(x-2)$$

2) Метод рационализации

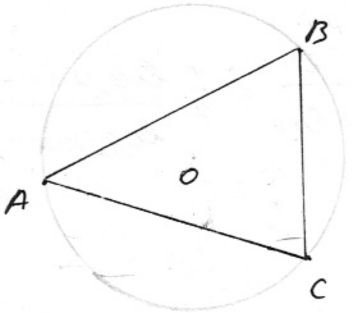
$$\log_{\left(\frac{x}{2}-1\right)^2}\left(\frac{x}{2}-\frac{1}{4}\right) = \frac{1}{2}\frac{\log_2(2x-1)-\log_2 4}{\log_2(x-2)-\log_2 2} \Leftrightarrow \frac{1}{2}\frac{(2-1)(2x-1-4)}{(2-1)(x-2-2)} =$$

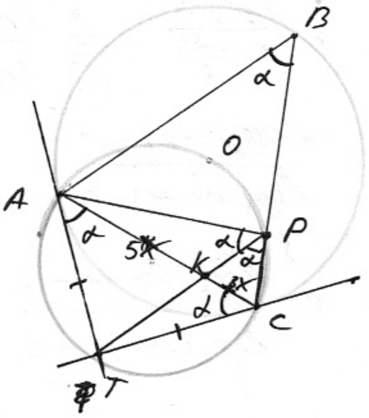
$$= \frac{1}{2}\cdot\frac{2x-5}{x-4}$$

$$\log_{\sqrt{x-\frac{11}{4}}}\left(\frac{x}{2}-1\right) = 2\frac{\log_2|4x-11|-\log_2 4}{\log_2(2x-1)-\log_2 4} \Leftrightarrow 2\frac{(|4x-11|-4)}{(2x-1-4)} = 2\cancel{\frac{|4x-11|-4}{2x-5}}$$

log

№6
Черновик

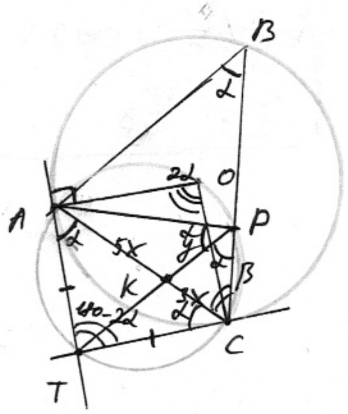




$S_{APK} = \overset{10}{\cancel{S_{CPK}}}$

$S_{CPK} = 6$

S_{ABC} - ?



$\frac{AK}{KC} = \frac{S_{APK}}{S_{CPK}} = \frac{10}{6} = \frac{5}{3}$

$\exists \angle ABC = \alpha$ =>

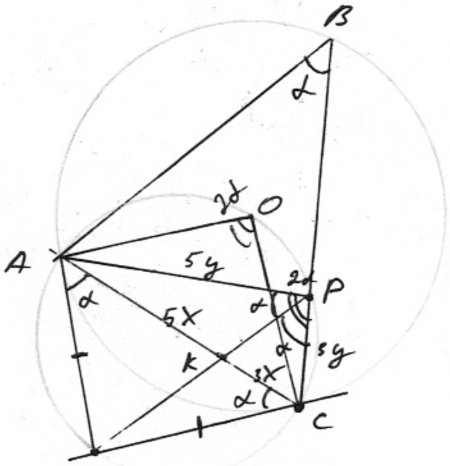
$=> \angle ATC = 180 - 2\alpha =>$

=> ATCO - вписанный 4 уг

$AK \cdot KC = TK \cdot PK$

$15X^2 =$

$S_{APC} = \frac{1}{2} AP \cdot PC \cdot \sin 2\alpha$



PT - биссектриса

$\frac{AK}{KC} = \frac{AP}{PC} = \frac{5}{3}$

$\frac{AC}{\sin\alpha} = 2R$

$\frac{AC}{\sin 2\alpha} = 2r$

$\frac{S_{APK}}{S_{CPK}} = \frac{5y \cdot KP \cdot \sin\alpha}{3y \cdot KP \cdot \sin\alpha}$

$10 = 5y \cdot KP \cdot \sin\alpha$

$6 = 3y \cdot KP \sin\alpha$

$2 = y \, KP \cdot \sin\alpha$

$KP = \frac{2y}{\sin\alpha}$

$3x \cdot 5x = TK \cdot KP$

ЧЕРНОВИК

$$\begin{cases} a \vdots 21 \\ b \vdots 21 \\ c \vdots 21 \\ 3^{17} \cdot 7^{15} \vdots a = 21 \\ 3^{17} \cdot 7^{15} \vdots b = 21 \\ 3^{17} \cdot 7^{15} \vdots c = 21 \end{cases}$$

$3^{\cancel{17}\,16} \cdot 7^{\cancel{15}\,14} \vdots \cancel{3 \cdot 7}$

$a = k21$

$b = m21$

$c = \cancel{d}n21$

$3^{\cancel{17}\,16} \cdot 7^{\cancel{15}\,14} \vdots k\cancel{21}$

$3^{17} \cdot 7^{15} \vdots m\cancel{21}$

$3^{17} \cdot 7^{15} \vdots n21$

$3^{16} \cdot 7^{14} \vdots k$

$k, m, n = \{3; 3^2; 3^3; 3^4; \ldots; 3^{16}; 7^1; 7^2; 7^3; \ldots; 7^{14}\}$

N1

$$\begin{cases} a \vdots 21 \\ b \vdots 21 \\ c \vdots 21 \end{cases} \Leftrightarrow \begin{cases} a = k21 \\ b = m21 \\ c = n21 \end{cases} \quad k; m; n \in \mathbb{N}$$

$(30; 30; 30)$

$3^{17} \cdot 7^{15} \vdots a \vdots b \vdots c$

$3^{\cancel{17}\,16} \cdot 7^{\cancel{15}\,14} \vdots (k, m, n)\, \overset{\cancel{3 \cdot 7}}{21}$

$3^{16} \cdot 7^{14} \vdots (k, m, n) \Rightarrow k; m; n \in \{3; 3^2; 3^3; \ldots; 3^{16}; 7; 7^2; 7^3; \ldots; 7^{14}\}$

кол-во вариантов $(k; m; n) = 30 \Rightarrow$

$\Rightarrow \exists$ 30 вариантов чисел $a, b, c \Rightarrow$

всего ~~30~~ 30^3 троек

$30 + 16 \cdot 14$

$$\begin{array}{r} 2 \\ \times\, 14 \\ 16 \\ \hline 184 \\ 14 \\ \hline +\, 324 \\ 30 \\ \hline 354 \end{array}$$

$$\log_{(\frac{x}{2}-1)^2} \frac{x}{2} - \frac{1}{4} = \log_{x-\frac{11}{4}} \left(\frac{x}{2}-1\right)^2$$

$$\log_{(\frac{x}{2}-1)^2} \frac{x}{2} - \frac{1}{4} = \frac{1}{\log_{(\frac{x}{2}-1)^2} x - \frac{11}{4}}$$

$\frac{x}{2} - \frac{1}{4} =$