

# Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21102796**

ID профиля: **336267**

Вариант 18

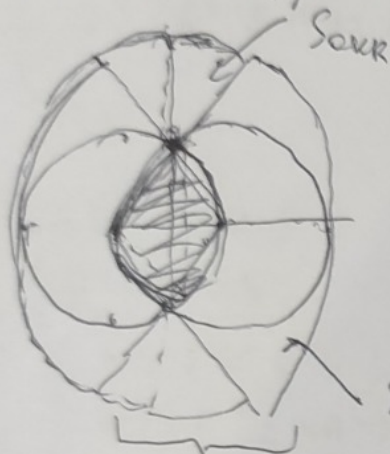
Уровень  
 $\pi_1$

①

5

Уровень ②

Заметим, что линия  $b = 2a - 2,5$  отсекает от обеих окружностей дуги равные  $45^\circ$  и проходит  $2/3$  их точки пересечения. (П.и. расстояние от ок-тей до линии равно  $\frac{\sqrt{5}}{2} = \frac{2}{2}$ )



Заметим, что  $S_{\text{общее}} = 2S_{\text{сектор}} + 4S_{\text{окр}}$ .

$$S_{\text{сектор}} = \frac{\pi \cdot R_{\text{общ}}^2}{4} = \frac{\pi \cdot 4 \cdot 5}{4} = 5\pi$$

$$S_{\text{окр}} = \frac{\pi R_{\text{окр}}^2}{4} = \frac{5\pi}{4}, \text{ Пятикрат!}$$

$$R_{\text{общ.}} = 2R = 2\sqrt{5}$$

$$R_{\text{окр}} = \sqrt{5}$$

$$S_{\text{общее}} = 2 \cdot 5\pi + \frac{5\pi}{4} \cdot 4 = 15\pi$$

Задача 3

$$\begin{cases} (x-a)^2 + (y-b)^2 \leq 5 & (1) \end{cases}$$

$$\begin{cases} a^2 + b^2 \leq \min(4a-2b, 5) & (2) \end{cases}$$

(2):

$$\begin{cases} 4a-2b \geq 5 \\ a^2+b^2 \leq 5 \end{cases}$$

$$\begin{cases} 2b \leq 4a-5 \\ a^2+b^2 \leq 5 \end{cases}$$

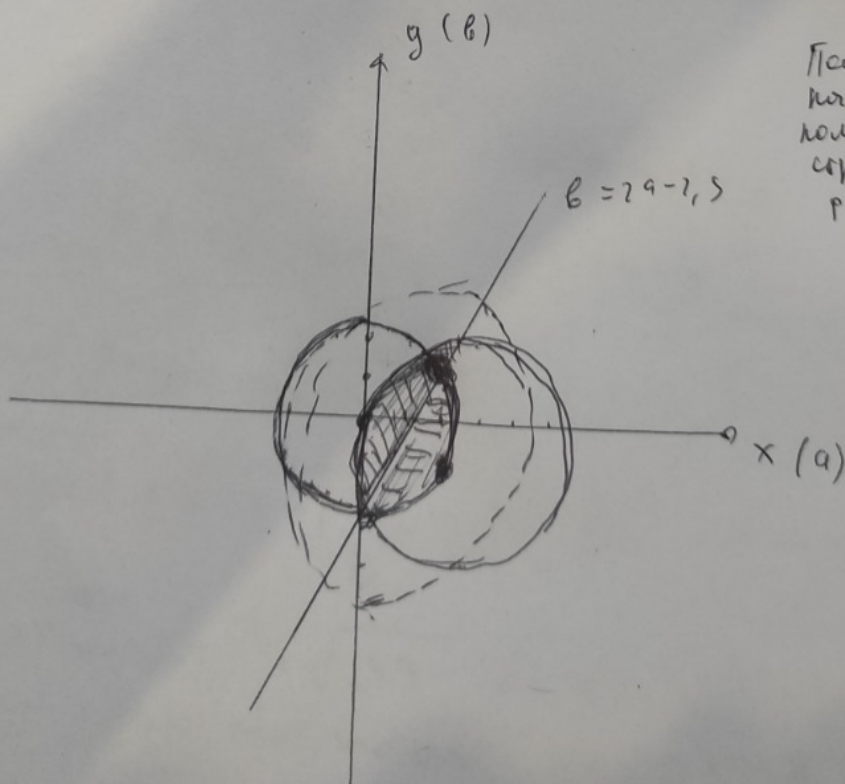
$$\begin{cases} 4a-2b \leq 5 \\ a^2+b^2 \leq 4a-2b \end{cases}$$

$$\begin{cases} 2b \geq 4a-5 \\ (a-2)^2 + (b-1)^2 \leq 5 \end{cases}$$

Получим:

$$\begin{cases} (x-a)^2 + (y-b)^2 \leq 5 \\ a^2 + b^2 \leq 5 \\ b \leq 2a - 2,5 \end{cases}$$

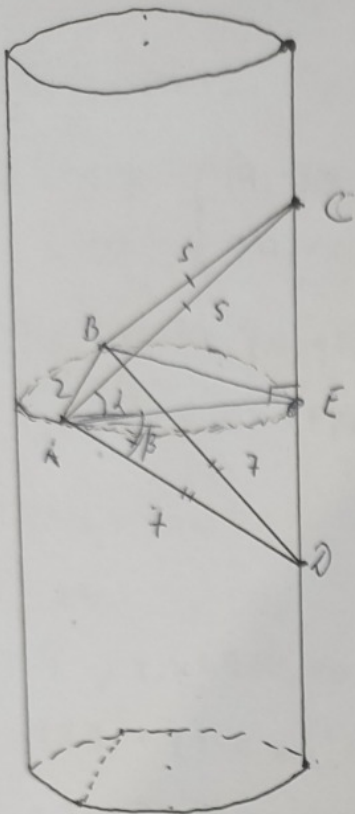
$$\begin{cases} (x-a)^2 + (y-b)^2 \leq 5 \\ (a-2)^2 + (b-1)^2 \leq 5 \\ b \geq 2a - 2,5 \end{cases}$$



Получим, что  
на границе тоже  
конкретно не-сов  
сформулируется  
расстояние  $\sqrt{5}$

Задача ①

$\sqrt{2}$



1)  $AB \parallel$  высоте основания

Очевидно  $AE \perp CD$  ( $BE \perp CD$ )

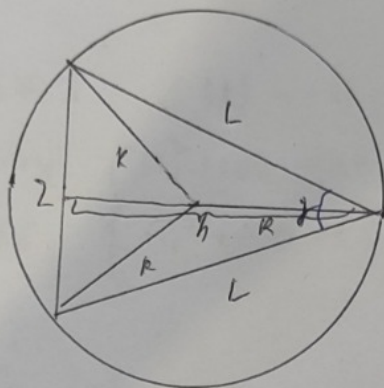
из равенства  $\triangle AEO \cong \triangle BEO$  (3 стороны)  $\Rightarrow$

$\Rightarrow \triangle AEO = \triangle BEO \Rightarrow BE = AE \Rightarrow ABE \perp CD \Rightarrow$   
 $ABE \perp$  высоте осн.

2) Пусть  $AE = L$ ,  $\angle CAE = \alpha$ ,  $\angle EAO = \beta$

$CO = L = CA \cos \alpha = AO \cos \beta = R \cos \beta = 5 \cos \beta$

3) в м-те  $ABE$ :



$$S_{\Delta} = L^2 \cdot \sin \gamma \cdot \frac{1}{2}$$

по теореме синусов:

$$h^2 + 1^2 = L^2$$

$$h = \sqrt{L^2 - 1}$$

$$S_{\Delta} = 2 \cdot h \cdot \frac{1}{2} = \sqrt{L^2 - 1}$$

$$L^2 \cdot \sin \gamma = 2\sqrt{L^2 - 1}$$

$$\sin \gamma = \frac{2\sqrt{L^2 - 1}}{L^2}$$

по теореме синусов:

$$\frac{2}{\sin \gamma} = 2R \Rightarrow R = \frac{L^2}{2\sqrt{L^2 - 1}} \Rightarrow$$

$$\Rightarrow 4R^2(L^2 - 1) = L^2 \Rightarrow L^4 - 4R^2L^2 + 4R^2 = 0$$

$$L^2 = \frac{4R^2 \pm \sqrt{16R^4 - 16R^2}}{2} = \frac{4R^2 \pm 4R^2\sqrt{R^2 - 1}}{2} = 2R^2 \pm 2R^2\sqrt{R^2 - 1}$$

$$CD = 5 \sin \alpha + 7 \sin \beta = 5\sqrt{1 - \cos^2 \alpha} + 7\sqrt{1 + \cos^2 \beta}$$

$$= \sqrt{25 - (5 \cos \alpha)^2} + \sqrt{49 - (7 \cos \beta)^2} = \sqrt{25 - L^2} + \sqrt{49 - L^2} =$$

$$= \sqrt{25 - 2R^2 - 2R^2\sqrt{R^2 - 1}} + \sqrt{49 - 2R^2 - 2R^2\sqrt{R^2 - 1}}$$

Заметим, что минимальное значение равно, что  $\sqrt{R^2 - 1} \geq 0 \Rightarrow R = 1$ . Тогда.

Ответ:  
 (D21102796 ZU33626V M1297091)

Усробику  
 $n_1$

①

$$S = a_1 + a_2 + a_3 + \dots + a_7 = , a_n \in \mathbb{Z}$$

$$a_n = a_1 + k(n-1) \Rightarrow k \in \mathbb{Z}, 2 > 0 \Rightarrow k \in \mathbb{N}$$

$$= a_1 + a_1 + k + a_3 + 2k + \dots + a_1 + 6k = 7a_1 + \frac{6k+0}{2} \cdot 7 = 7a_1 + 21k$$

$$\begin{cases} a_9 \cdot a_{10} < S + 44 \\ a_7 \cdot a_{12} > S + 20 \end{cases} \begin{cases} (a_1 + 8k)(a_1 + 9k) < 7a_1 + 21k + 44 \\ 7a_1 + 21k + 20 < (a_1 + 6k)(a_1 + 11k) \end{cases}$$

$$\begin{cases} a_1^2 + 17ka_1 + 72k^2 < 7a_1 + 21k + 44 & (1) \\ 7a_1 + 21k + 20 < a_1^2 + 17ka_1 + 66k^2 & (2) \end{cases} \text{Сложим обе гр-ы.}$$

$$20 + 72k^2 < 44 + 66k^2$$

$$6k^2 < 24$$

$$k^2 < 4, \text{ т.к. } k \in \mathbb{N}, \text{ то } k = 1$$

$$\begin{cases} a_1^2 + 17a_1 + 72 < 7a_1 + 21 + 44 \\ 7a_1 + 21 + 20 < a_1^2 + 17a_1 + 66 \end{cases} \begin{cases} a_1^2 + 10a_1 + 72 < 65 \\ a_1^2 + 10a_1 + 66 > 41 \end{cases}$$

$$\begin{cases} a_1^2 + 10a_1 + 7 < 0 \\ (a_1 + 5)^2 > 0 \end{cases} \rightarrow \begin{cases} (a_1 + 5)^2 - 25 + 7 < 0 \\ (a_1 + 5)^2 < 18 \Rightarrow \end{cases} \begin{cases} a_1 + 5 = \pm 4 \\ a_1 + 5 = \pm 2 \\ a_1 + 5 = \pm 1 \end{cases}$$

↖  
 берем мин. значение  $a_1$

$$\begin{cases} a_1 = -9 \\ a_1 = -1 \\ a_1 = -7 \\ a_1 = -3 \\ a_1 = -4 \\ a_1 = -6 \end{cases}$$

$$\Rightarrow \text{ответ: } \underline{a = (-9; -7; -6; -4; -3; -1)}$$

$$CD = 5 \sin \alpha + 7 \cos \beta$$

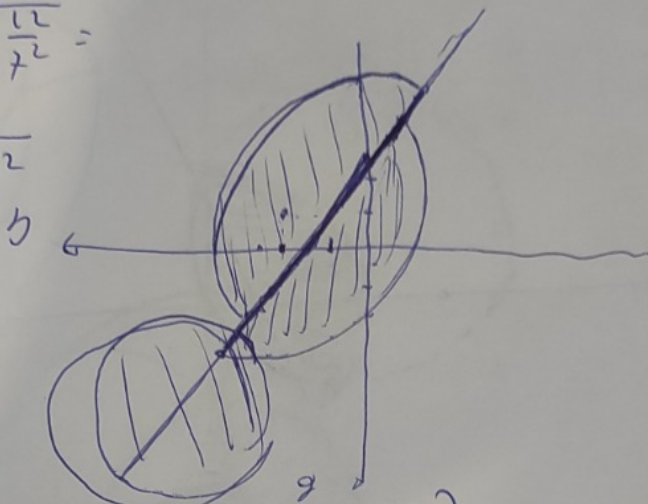
упрощение

$$= 5 \sqrt{1 - \cos^2 \alpha} + 7 \sqrt{1 - \cos^2 \beta}$$

$$\cos \alpha = \frac{L}{5} \quad \cos \beta = \frac{L}{7}$$

$$= 5 \sqrt{1 - \frac{L^2}{25}} + 7 \sqrt{1 - \frac{L^2}{49}} =$$

$$= 5 \sqrt{25 - L^2} + 7 \sqrt{49 - L^2}$$



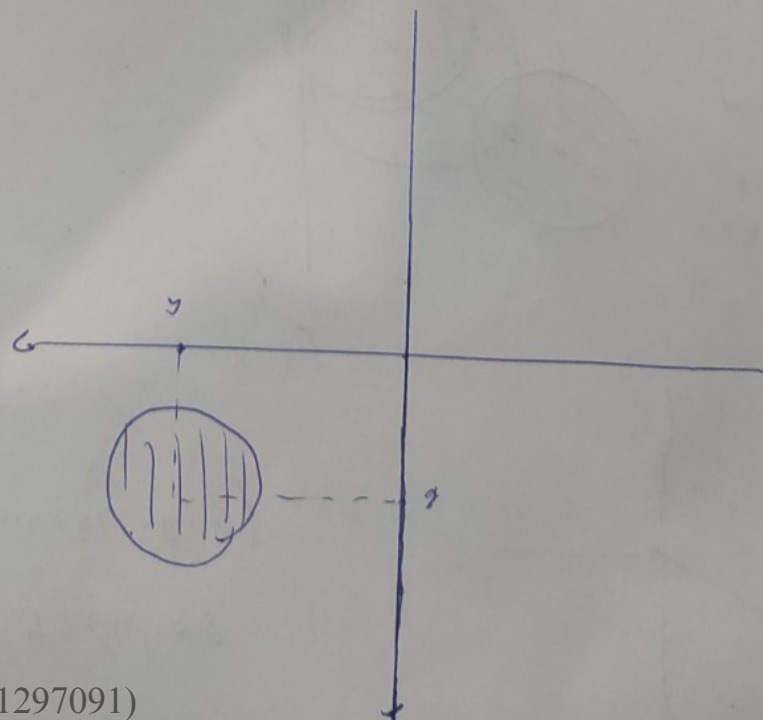
$$\left. \begin{array}{l} 5 \sqrt{1+(1+g)+1(1-b)} \\ 7 - b < g \\ 5 + g > 2b \end{array} \right\}$$

$$\left. \begin{array}{l} 5 \sqrt{2g+2b} \\ 5 + g < 2b \\ 7 - b > g \end{array} \right\}$$

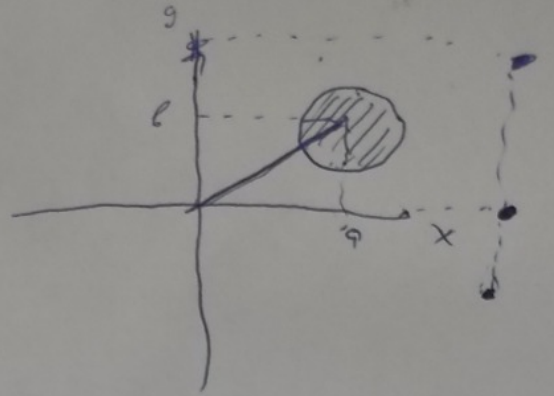
$$\left. \begin{array}{l} 0 \leq 1 - \frac{1}{2}(1+g) + 1 - \frac{1}{2}(2-b) \\ 5 + g > 2b \\ 5 \sqrt{2g+2b} \\ 5 + g < 2b \end{array} \right\}$$

$$\left. \begin{array}{l} g - b \geq 2g + 2b \\ 5 + g < 2b \\ 5 \sqrt{2g+2b} \\ 5 + g < 2b \end{array} \right\}$$

(5 + g - 2b) min ≥ 2g + 2b



$$\begin{cases} (x-a)^2 + (y-b)^2 \leq 5 \\ a^2 + b^2 \leq \min(4a-2b, 5) \end{cases}$$



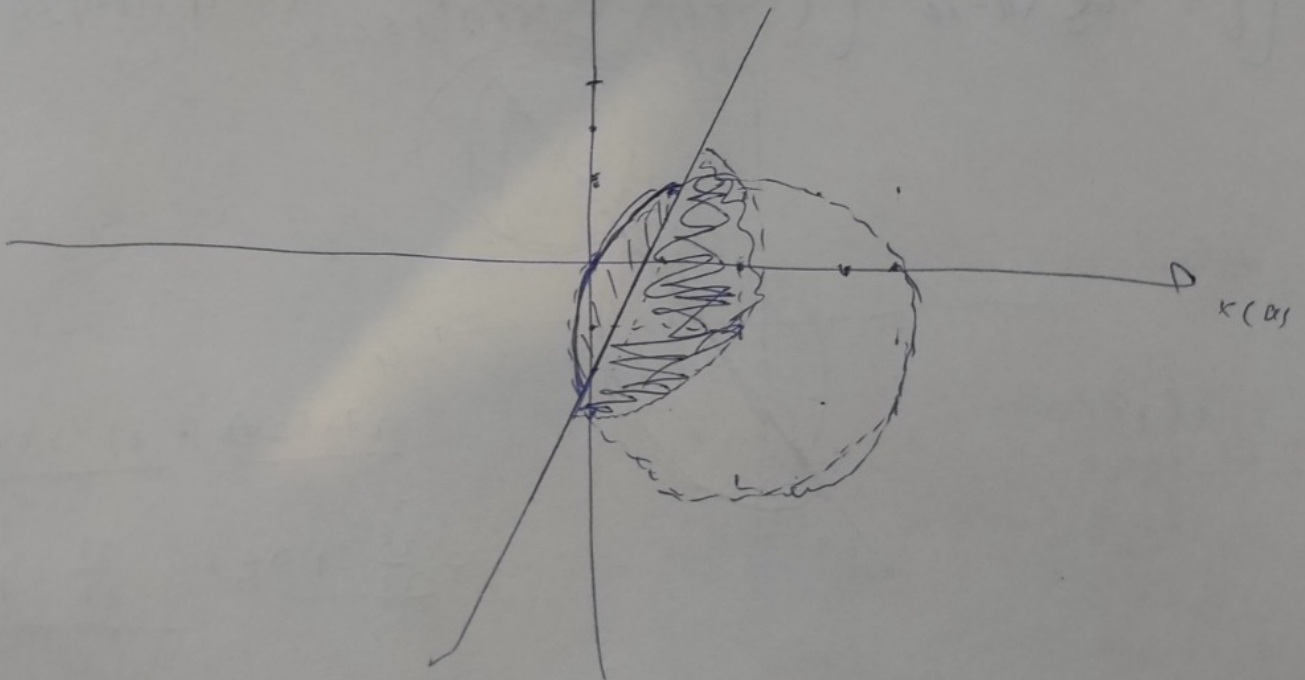
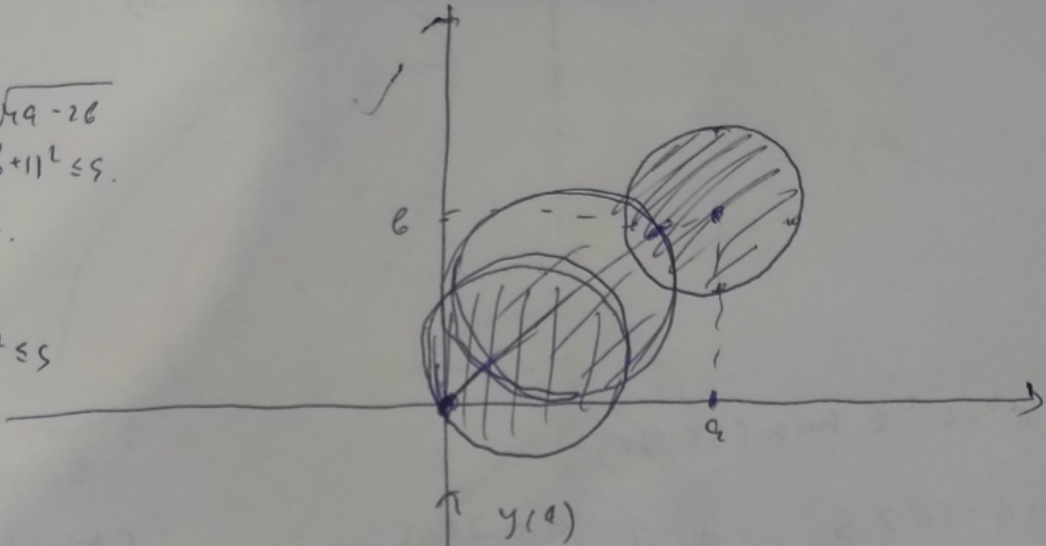
$$\sqrt{2a-b} \leq \sqrt{5}$$

$$4a - 2b < 5$$

$$\begin{cases} \sqrt{a^2+b^2} \leq \sqrt{4a-2b} \\ 4a \leq 5+2b \\ (a-2)^2 + (b+1)^2 \leq 5 \end{cases}$$

$$\begin{cases} a^2 + b^2 \leq 5 \\ 4a > 5 - 2b \end{cases}$$

$$\begin{cases} 2b > 4a - 5 \\ (a-2)^2 + (b+1)^2 \leq 5 \\ 7b < 4a - 5 \\ 9 < a^2 \end{cases}$$



N.L.

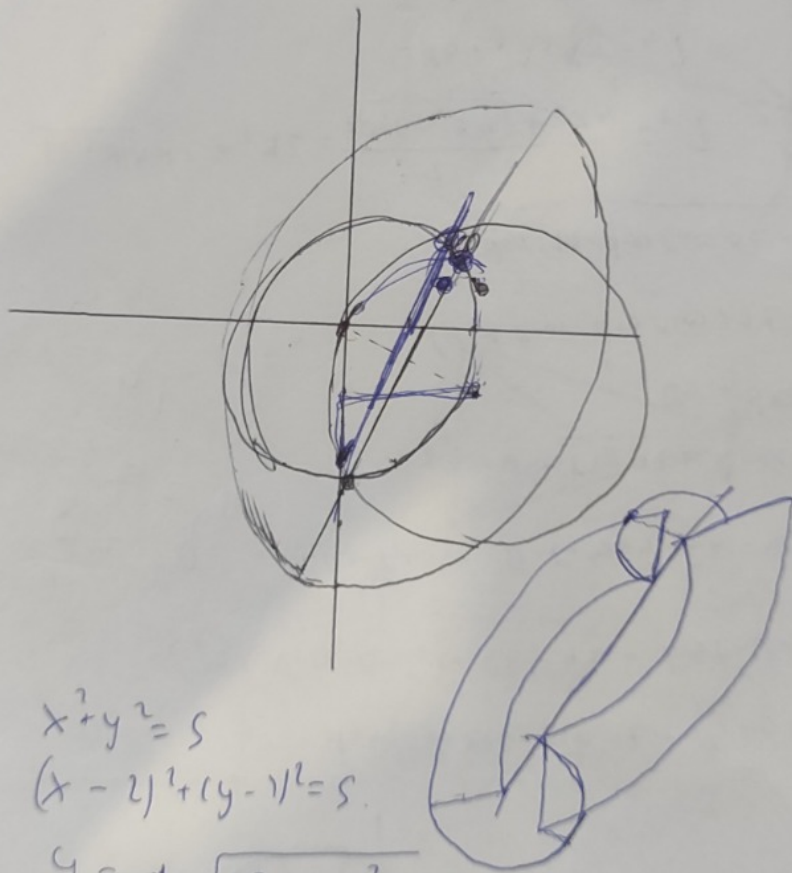
Чиркович

$$\begin{cases} (x-a)^2 + (y-b)^2 \leq S & (1) \\ a^2 + b^2 \leq \min(4a - 2b, S) & (2) \end{cases}$$

Чиркович

$$\begin{cases} \begin{cases} 4a - 2b \geq S \\ a^2 + b^2 \leq S \end{cases} & \begin{cases} 2b \leq 4a - S \\ a^2 + b^2 \leq S \end{cases} \\ \begin{cases} 4a - 2b \leq S \\ a^2 + b^2 \leq 4a - 2b \end{cases} & \begin{cases} 2b \geq 4a - S \\ (a-1)^2 + (b-1)^2 \leq S \end{cases} \end{cases}$$

Точка



$$x^2 + y^2 = S$$

$$(x-2)^2 + (y-1)^2 = S$$

$$y = \pm \sqrt{S - x^2}$$

$$y = \sqrt{S - (x-2)^2} + 1$$

$$\sqrt{S - x^2} = \sqrt{S - (x-2)^2} + 1$$

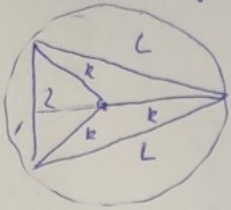


$$CD^2 = 49 + 25 - 70 \cos(\alpha + \beta)$$

$$L = 7 \cos \alpha$$

$$L = 5 \cos \beta$$

непрямой



$$h = \sqrt{L^2 - 1}$$

$$\frac{2}{\sin \alpha} = 2R$$

$$R = \frac{1}{\sin \alpha}, L^2 = \frac{1}{L} \cdot \sin \alpha = S$$

$$\sin \alpha = \frac{25}{L^2}$$

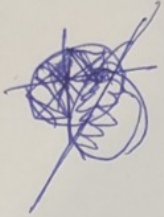
$$R = \frac{L^2}{25} = \frac{L^2}{2\sqrt{L^2-1}}$$

$$4R^2(L^2-1) = 49$$

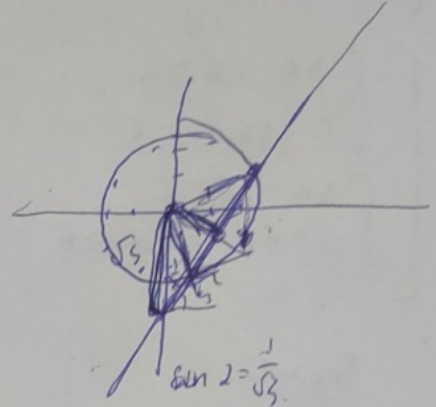
$$L^4 - 4R^2L^2 + 4R^2 = 49$$

$$L^2 = \frac{4R^2 \pm \sqrt{16R^4 - 16R^2}}{2} = 2R^2 \pm 2R\sqrt{R^2-1}$$

$$S = \frac{1}{2} \cdot 2 \cdot \sqrt{L^2-1}$$



$$4a = 28 + 5 \neq 6 \neq 7$$



$$\sin \alpha = \frac{1}{\sqrt{3}}$$

$$CD = \sqrt{74 - 70 \cos \alpha \cos \beta - 7 \sin \alpha \sin \beta}$$

$$CD^2 = 74 - 70(\cos \alpha \cos \beta - \sin \alpha \sin \beta)$$

$$7 \sin \alpha \sin \beta = CD$$

$$49 \sin^2 \alpha + 25 \sin^2 \beta + 70 \sin \alpha \sin \beta = CD^2$$

$$-70 \sin \alpha \sin \beta = 49 - 45 \cos^2 \alpha + 25 - 25 \cos^2 \beta - CD^2$$

$$CD^2 = 74 - 70(\cos \alpha \cos \beta + 74 - 45 \cos^2 \alpha - 25 \cos^2 \beta - CD^2)$$

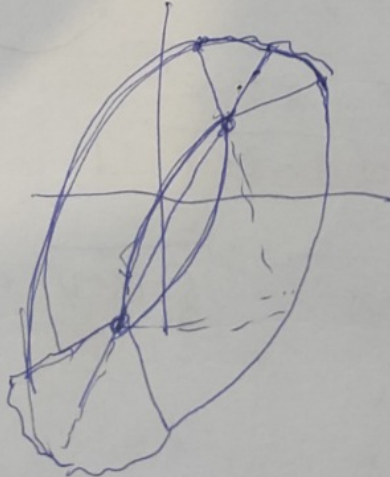
$$CD^2 = 49 \cos^2 \alpha - 70 \cos \alpha \cos \beta + 25 \cos^2 \beta + CD^2$$

$$S = 14$$

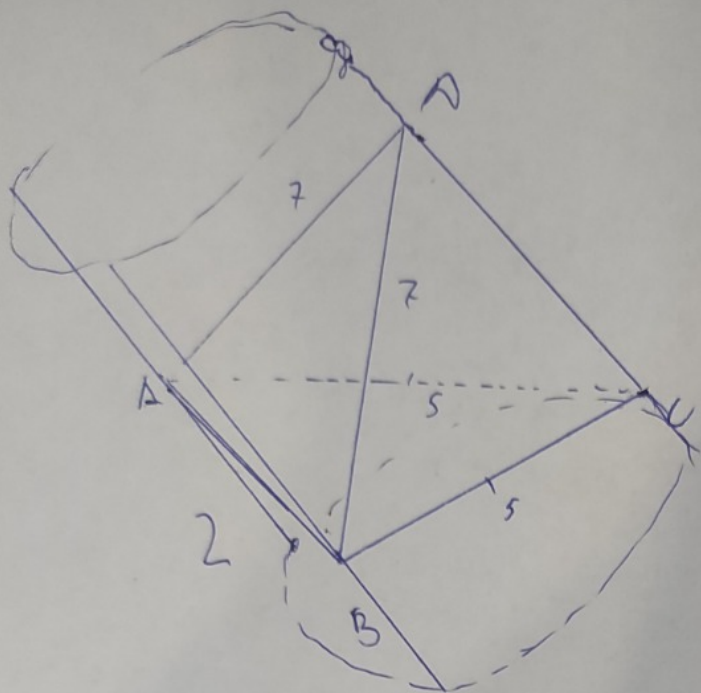
$$S \cdot 10 = 90$$

$$a_9 = 7$$

$$a_{10} = 8$$

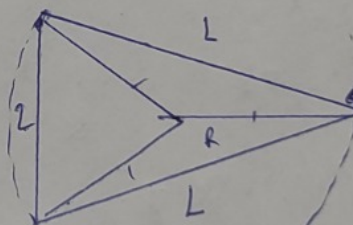
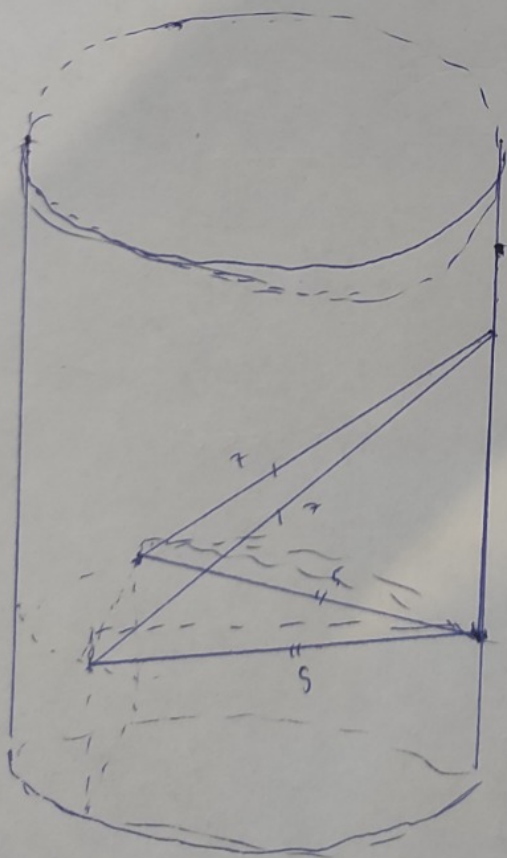
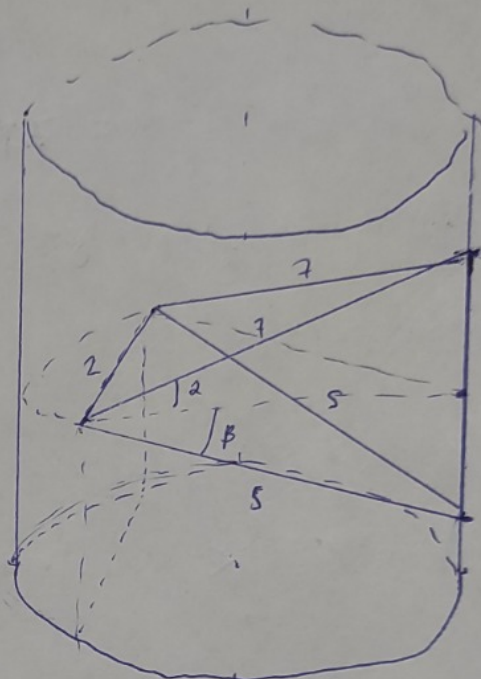


№2. цилиндр

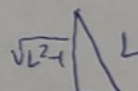


$$CDE(2; 12)$$

$$BC + BD > CD \quad CD < 12$$



$$\frac{968}{45}$$



$$2R = \frac{2}{\sin \alpha}$$

$$L = 7 \cos \alpha \quad CD = 7^2 + 5^2 = 70 \cos(2\alpha)$$

$$L = 5 \cos \beta \quad \cos 2 = \frac{L}{7}$$

$$\cos \beta = \frac{L}{5}$$

$$L^2 \cdot \frac{\sin \alpha}{2} = \sqrt{L^2 - 1} \cdot 2 \cdot \frac{1}{2}$$

$$\sin \alpha = \frac{2\sqrt{L^2 - 1}}{L^2}$$

$$2R = \frac{2}{\sin \alpha} = \frac{L^2}{\sqrt{L^2 - 1}}$$

$$R = \frac{L^2}{2\sqrt{L^2 - 1}}$$

R=

Упробук

$$S = a_1 + a_2 + a_3 + \dots + a_7$$

$$q \in \mathbb{Z}$$

$$\frac{6k+1k}{2} \cdot 6k =$$

$$a_7 \cdot a_{12} > S + 20$$

$$a_1 = ?$$

$$= 21k$$

$$a_9 \cdot a_{10} < S + 44$$

$$\frac{6k+0k}{2} \cdot 7$$

$$a_n = d_1 + kh$$

$$S = 7a_1 + k + 2k + 3k + \dots + 7k = 7a_1 + \frac{7+1}{2} \cdot 7k = 7a_1 + 28k$$

$$\begin{cases} (a_1 + 7k)(a_1 + 12k) > 7a_1 + 28k + 20 \\ (a_1 + 9k)(a_1 + 10k) < 7a_1 + 28k + 44 \end{cases}$$

$$\begin{cases} a_1^2 + 19ka_1 + 84k^2 > 7a_1 + 28k + 20 \\ a_1^2 + 19k + 90k^2 < 7a_1 + 28k + 44 \end{cases} \begin{cases} a_1^2 + 19ka_1 + 84k^2 > 7a_1 + 28k + 20 \\ 7a_1 + 28k + 44 > a_1^2 + 19ka_1 + 90k^2 \end{cases}$$

$$84k^2 + 44 > 20 + 90k^2$$

$$24 > 6k^2$$

$$4 > k^2 \Rightarrow k \in \mathbb{Z} \quad k = 1$$

$$\begin{array}{r} 84 \\ -48 \\ \hline 36 \end{array}$$

$$\begin{cases} a_1^2 + 19a_1 + 84 > 7a_1 + 48 \\ a_1^2 + 19a_1 + 90 < 7a_1 + 76 \end{cases} \begin{cases} (a_1 + 6)^2 > 20, a_1 \in \mathbb{R} \\ a_1^2 + 12a_1 + 36 > 20 \\ a_1^2 + 12a_1 + 14 < 0 \end{cases}$$

$$(a+6)^2 - 36 - 14 < 0$$

$$(a+6)^2 < 36 - 14 = 22 \quad (a+6) \in \mathbb{Z}$$

$$\begin{cases} (a+6)^2 = 16 & a+6 = \pm 4 \\ (a+6)^2 = 4 & a+6 = \pm 2 \\ (a+6)^2 = 1 & a+6 = \pm 1 \end{cases} \begin{cases} a = -10 \\ a = -2 \\ a = -8 \\ a = -4 \\ a = -7 \\ a = -5 \end{cases}$$

# Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21102796**

ID профиля: **336267**

Вариант 18



Задача 2

$\sqrt{4}$ .

III. и  $\text{НОД}(a, b, c) = 15$   $a = 15k_1, b = 15k_2, c = 15k_3$  где  $k_1, k_2, k_3$

$k_1, k_2, k_3$

$3^{15} \cdot 5^{18} : a : b : c$

Заметим:  $abc = 3^3 \cdot 5^3 \cdot k_1 k_2 k_3 = \underbrace{3^3}_{x_1} \cdot \underbrace{5^3}_{x_2} \cdot \underbrace{k_1 k_2 k_3}_{x_3}$

Получим:  $3^{15} \cdot 5^{18} = x_1 x_2 x_3$

III. и  $x_1, x_2, x_3$  взаимно простые, т.е.

$x_1 = 3^{15}$

$x_2 = 5^{18}$

$x_3 = 1$

$$\Rightarrow \begin{cases} k_1 = 3^{15}, 3^{14}, 3^{13}, 3^{12}, 3^{11} & 4 \text{ числа} \\ k_2 = 5^{18}, 5^{17}, 5^{16}, 5^{15} & 4 \text{ числа} \\ k_3 = 1 & 1 \text{ число.} \end{cases}$$

Получим  $\text{НОД}$ -ом  $n = 4^2 \cdot 3!$   $\text{НОД}$ -ом перестановок  $= 16 \cdot 6 = 96$

$\text{НОД}$ -ом разл. комбинаций чисел

$\sqrt{6}$ .



$$3^{15} \cdot 5^{18} = 9^c$$

$$a: 3 \cdot 5$$

$$b: 8 \cdot 3$$

$$c: 13 \cdot 5$$

$$a: 3 \cdot 5$$

$$c: 3 \cdot 5$$

$$b: 3 \cdot 5$$

$$b, c, a: 3 \cdot 5$$

$$a = 15P_1$$

$$b = 15P_2$$

$$c = 15P_3$$

$$3^{12} \cdot 5^{15} = P_1 P_2 P_3$$

$$a = 15 \cdot 5^{15}$$

$$b = 15 \cdot 3^{12}$$

$$c = 15$$

$$P_1 = 5^{15}$$

$$P_2 = 3^{12}$$

$$P_3 = 1$$

$$x > 14$$

$$3^{14} \cdot 5^{17}$$

$$\frac{\log_2(6x-14)^2}{\log_2(\frac{x}{3}+3)}$$

$$\frac{\log_2(x-1)^2}{\log_2(6x-14)}$$

$$\frac{\log_2(\frac{x}{3}+3)}{\log_2(x-1)}$$

$$2 \frac{\log_2(6x-14)}{\log_2(\frac{x}{3}+3)}$$

$$2 \frac{\log_2(x-1)}{\log_2(6x-14)}$$

$$\frac{\log_2(\frac{x}{3}+3)}{\log_2(x-1)}$$

$$\log_2^2(6x-14) = \log_2(x-1) \log_2(\frac{x}{3}+3)$$

$$\log_4 6 - \log_4 e = \log_4 c$$

$$\log_4 \frac{6}{4} = \log_4 c$$

$$\frac{\log_4 \frac{6}{4}}{\log_4 4} = \frac{\log_4 c}{\log_4 e}$$



a c c

$$a = c$$

$$c+1 = a$$

$$\frac{a}{c} = 1$$

$$c + \frac{a}{c} = a$$

$$\frac{c^2 + a}{c} = a \quad c^2 + a = ac$$

$$2 \log_2 (6x-14) = \log_2 \left(\frac{x}{3} + 3\right) \log_2 (x-1)$$

$$\frac{\log_2 \left(\frac{x}{3} + 3\right)}{\log_2 (x-1)} + 1 = \frac{2 \log_2 (6x-14)}{\log_2 \left(\frac{x}{3} + 3\right)}$$

$$2 \log_2 (6x-14) = \log_2 \left(\frac{x}{3} + 3\right) \log_2 (x-1)$$

$$\frac{\log_2 \left(\frac{x}{3} + 3\right) (\log_2 (x-1))}{\log_2 (x-1)} = 2 \log_2 (6x-14)$$

$$2 \log_a b \quad 2 \log_b c \quad \log_c a$$

$$2 \log_a b = 2 \log_b c$$

$$2 \log_c a + 1 = \log_a b$$

$$\ln b^2 = \ln c \cdot \ln a$$

$$\frac{\ln a + \ln c}{\ln c} = \frac{\ln b}{\ln a}$$

$$\begin{cases} (\ln b)^2 = \ln a \ln c \\ (\ln a)^2 + \frac{\ln a \ln c}{(\ln b)^2} = \ln b \ln c \end{cases}$$

$$(\ln a)^2 + (\ln b)^2 = \ln b \ln c$$

$$2 \ln a b = \ln c$$

$$2 \ln a^2 b^2 = 2 \ln_b c^2 + \ln_b b$$

$$\ln b^2 \ln b = (\ln c^2 + \ln b) \ln a$$

$$\frac{\ln b^2}{\ln a} = \frac{\ln c^2}{\ln b} + \frac{\ln b}{\ln b} =$$

$$2 \log_{\left(\frac{x}{3}+3\right)} (6x-14), 2 \log_{6x-14} (x-1), \log_{x+1} \left(\frac{x}{3}+3\right)$$

$$2 \frac{\ln(6x-14)}{\ln\left(\frac{x}{3}+3\right)}, 2 \frac{\ln(x-1)}{\ln(6x-14)}, \frac{\ln\left(\frac{x}{3}+3\right)}{\ln(x-1)} = c$$

$$a \quad b \quad c \quad x+9 = 3x-3$$

$$a \quad b \quad c = 4 \quad x = 6$$

$$a = b$$

$$c+1 = a \quad c = a-1$$

$$a^2 (a-1) = 4$$

$$a^3 - a^2 - 4 = 0$$

$$a = 2 \quad b = 2 \quad c = 1$$

1	-1	0	-4
2	1	0	0

$$x^2 - x + 2 = 0$$

$$(x-2)(x-1)$$

$$x = 1 \pm \sqrt{1-8} \quad \emptyset$$

$$2 \frac{\ln 4}{\ln \frac{5}{3}} \quad 2 \frac{\ln 4}{\ln 4}$$

$$6x-14 = \sqrt{\frac{x}{3}+3}$$

$$b = 2 \frac{\log x-1}{\log \left(\frac{x}{3}+3\right)} \quad c = \frac{\log \left(\frac{x}{3}+3\right)}{\log(x-1)}$$

$$x-1 = \frac{x}{3}+3$$

$$a = 2, b = 2, c = 1$$

$$\frac{x}{3} + 3 = x - 1$$

$$x + 9 = 3x - 3$$

$$12 = 2x \quad \boxed{x=6} \quad \emptyset$$

$$\frac{\ln(x-1)^2}{\ln(6x-14)} = 1$$

$$(x-1)^2 = 6x-14$$

$$x^2 - 2x + 1 = 6x - 14$$

$$x^2 - 8x + 15 = 0$$

$$\begin{matrix} x = 5 \\ x = 3 \end{matrix}$$



