# Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

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Вариант 17

В.17 №1 Чистовик (1)

$$S=\frac{a_1+a_{10}}{2}\cdot 10=(2a_1+9b)\cdot 5$$

$$a_{10}=a_1+9b$$

$$a_6\cdot a_{12}>S+1$$

$$\begin{cases}(a_1+5b)(a_1+11b)>1+10a_1+45b\\(a_1+6b)(a_1+10b)<10a_1+45b+17\end{cases}$$

$$\begin{cases}a_1^2+16ba_1+55b^2>10a_1+45b+1\\a_1^2+16ba_1+60b^2<10a_1+45b+17\end{cases}$$

$$-5b^2>-16$$

$$5b^2<16$$

$b\in\mathbb{Z}$

$$b^2<\frac{16}{5}=3{,}2$$

$-2$ $-\sqrt{3{,}2}$ $\sqrt{3{,}2}$ $2$ $b$

$b=-1, 1$; $b=1$, т.к. прогрессия возр.-я

$b=1$: $$\begin{cases}a_1^2+16a_1+55>10a_1+45+1\\a_1^2+16a_1+60<10a_1+45+17\end{cases}$$

$$\begin{cases}a_1^2+6a_1+9>0\\a_1^2+6a_1-2<0\end{cases}\qquad D=36+4\cdot 2=44$$

$$a_1=\frac{-6\pm\sqrt{44}}{2}=-3\pm\sqrt{11}$$

$$(a_1+3)^2>0$$

$$a_1\neq -3.$$

$-7$ $-3-\sqrt{11}$ $-6$ $0$ $-3+\sqrt{11}$ $1$ $a_1$

$$a_1\in\{-6; -5, \ldots, 0\}.$$

№1. (прод-е)

$a_1=-6$;  $-6,-5,\dots,3$  $S=\frac{-6+3}{2}\cdot 10=-15$

Чистовик 2

$(-1)\cdot 5 > -15+1$

$0\cdot\ldots < -15+17$

$a_1=0$  $0,1,\dots,9$  $S=45$

$5\cdot 11 > 45+1$

$6\cdot 10 < 17+45$.

Ответ: $a_1\in\{-6;-5;\cancel{-4};-3;-2;-1;0\}\cup\{-4\}$.

$a_1=-5$:  $-5,-4,\dots,4$  $S=\frac{-1}{2}\cdot 10=-10$

$0\cdot\ldots > -10$

$1\cdot 5 < 7$

$a_1=-4$:  $-4,-3,\dots,5$  $S=\frac{1}{2}\cdot 10=5$

$1\cdot 7 > 5$ +   $2\cdot 6 < 5+17=22$.

$a_1=-3$  $-3,-2,\dots,6$  $S=\frac{3}{2}\cdot 10=15$

$2\cdot 8 > 15$

$3\cdot 7 < 15+17$ +

$a_1=-2$:  $-2,-1,\dots,7$  $S=\frac{5}{2}\cdot 10=25$

$3\cdot 9 > 26$ +

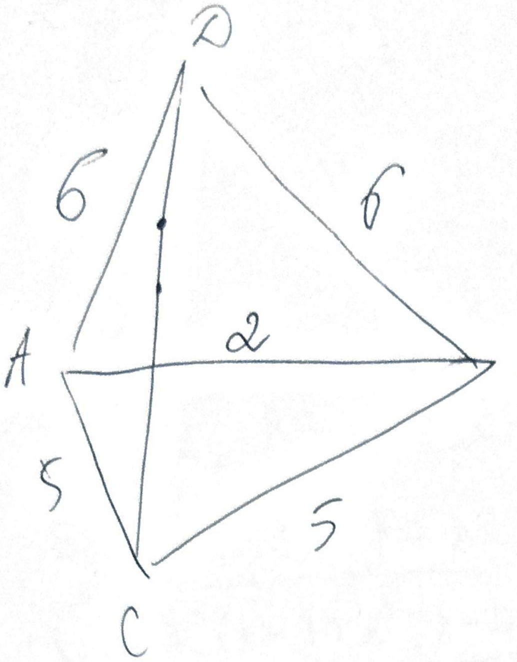
$4\cdot 8 < 25+17$ +

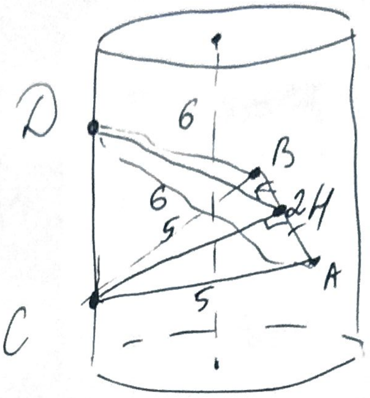
$a_1=-1$:  $-1,0,\dots,8$  $S=\frac{7}{2}\cdot 10=35$

$4\cdot 10 > 35+1$

$5\cdot 9 < 35+17$ +

$-4,-3,-2,-1,0,1,2,3,4,5$  $S=\frac{1}{2}\cdot 10=5$



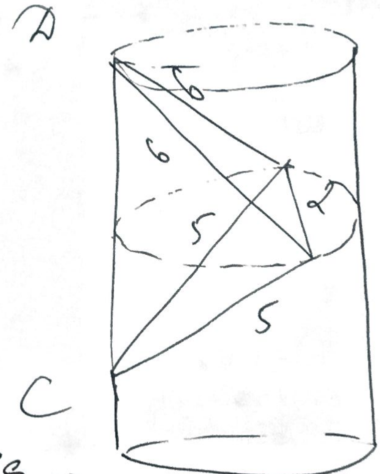

Точка D дви-ся по пересеч-ии сфер $(A;\ r=6)$, $(B;\ r=6)$

$AB \perp DC$

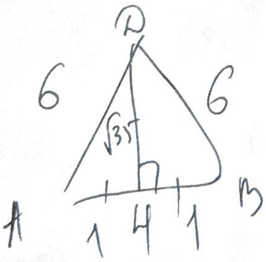
(медиана и выс-та в р/б $\Delta$)

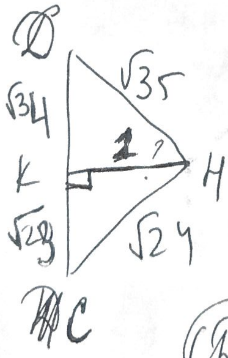
$\Rightarrow AB \perp$ оси $\Rightarrow$ AB лежит на окр-ти,

т.е. в пл-ти, $\parallel$ осн. цил.




Тогда радиус минимален, если AB – диаметр. Иначе AB – хорда $\Rightarrow$ диаметр $> AB = 2$.



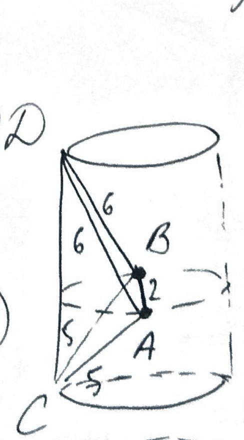




$HK = r = 1$

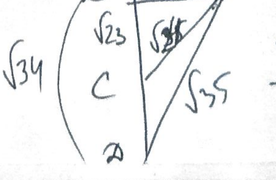
(проекция явл-ся радиусом) радиус $\perp$ оси.

$\Delta CDH$ – остроуг-й.

$CD = \sqrt{34} + \sqrt{23}$








$\Delta CDH$ – тупоуг-й.


✓ $CD = \sqrt{34} - \sqrt{23}$.

Ответ: $CD = \sqrt{34} \pm \sqrt{23}$

Чистовик

4

№3.

круг с $r=\sqrt{2}$

ц. $(x, y)$

$$\begin{cases} (x-a)^2+(y-b)^2 \le 2 \\ a^2+b^2 \le \min(2a+2b,\ 2) \end{cases}$$

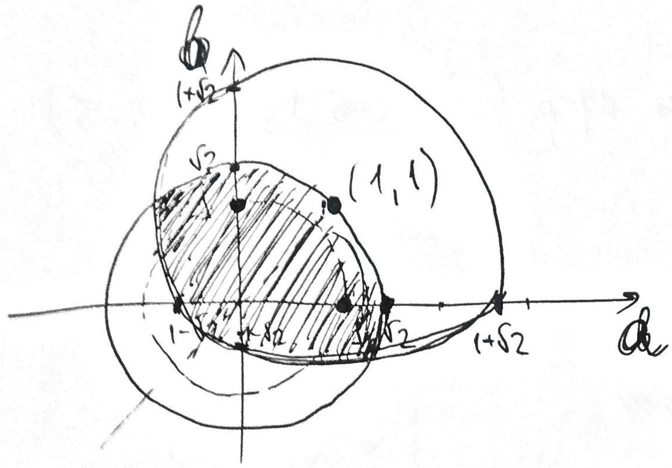
круг $r=\sqrt{2}$

$$\begin{cases} a^2+b^2 \le 2 \\ a^2+b^2 \le 2a+2b \end{cases}$$

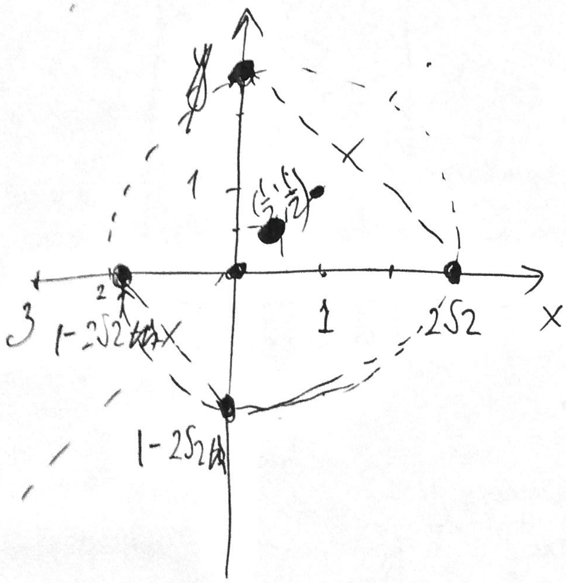
$$a^2-2a+1+b^2-2b+1 \le 2$$

$$(a-1)^2+(b-1)^2 \le 2.$$

круг, $r=\sqrt{2}$.




Круг с $r=\sqrt{2}$, ц. $(x, y)$ должен касаться/пересекать заштрих. обл-ть




крайние точки

$x_{max} = 2\sqrt{2}$, $y=0$

$x_{min} = 1-2\sqrt{2}$, $y=0$

$y_{max} = 2\sqrt{2} \to x=0$

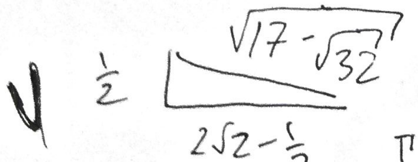
$y_{min} = 1-2\sqrt{2}$, $x=0$

$$\begin{cases} (a-1)^2+(b-1)^2 \le 2 \\ (a-x)^2+(b-y)^2 \le 2 \\ a^2+b^2 \le 2 \end{cases}$$

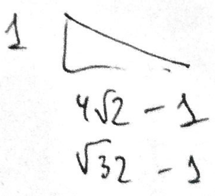
a и b – нек. числа $\Rightarrow$

$\Rightarrow$ бесконечное мн-во кругов.

$r=\sqrt{2}$




п. $(17-\sqrt{32})$



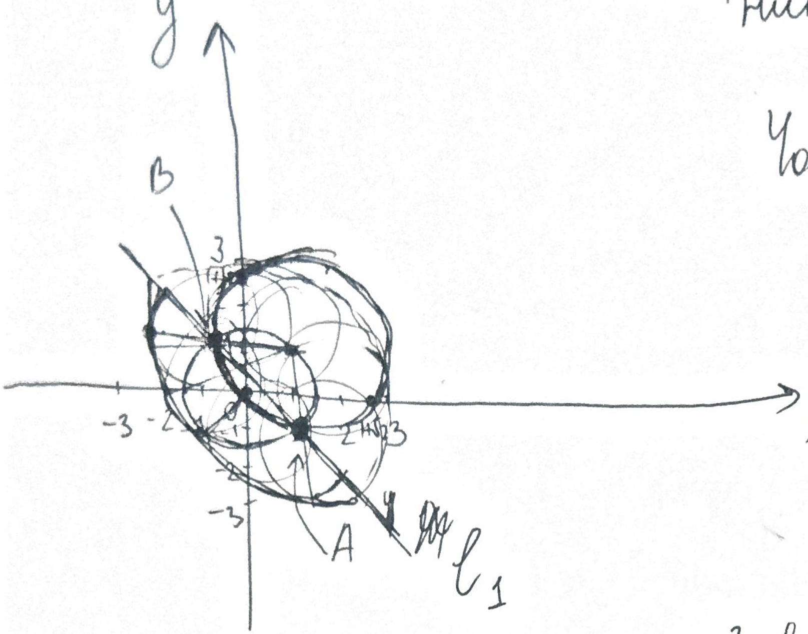

$1+32-2\sqrt{32}+1=$

$=34-2\sqrt{32}$

№3 (продолж-е) Чистовик (5)

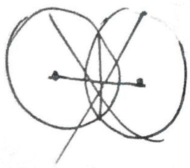
ниже прямой $l_1$:

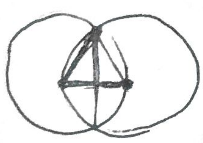
Часть круга $((1;1),\ r=\sqrt{2}+\sqrt{2})$
$+\ 2$ части круга $(A;\sqrt{2})$ $(B;\sqrt{2})$
выше прямой $l_1$.




Часть круга $((0;0);\ r=2\sqrt{2})$
+ часть $(A;\sqrt{2}),\ B(\sqrt{2}))$

$$a^2+b^2=2 \cap (a-1)^2+(b-1)^2=2$$



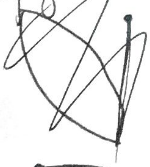


$$a^2+b^2=(a-1)^2+(b-1)^2$$

$$b^2-(b-1)^2=(a-1)^2-a^2$$

$$a=\frac{1}{2},\ b=\frac{1}{2}$$

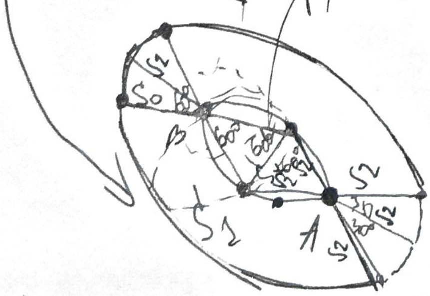
бесконечно много кругов обр-ют с $r=\sqrt{2}+\sqrt{2}=2\sqrt{2}$
часть окружности $l_1$ до точки пересеч.



Далее идет окр-ть с $r=\sqrt{2}$
и у. в точке пересеч.

р/б Δ.

$\frac{3}{2}$




$$0=-2a+1-2b+1$$

$$2=-2a-2b$$

$$1=-a-b$$

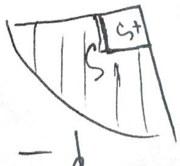
$$b=-a-1.$$

$$(a+1)^2+a^2=2$$

$$2a^2+2a+1-2=0;$$

$$\cancel{2a^2+2a-1=0}$$

$$\cancel{a^2+2a-2=0}$$

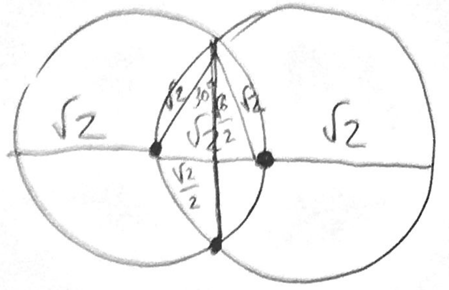


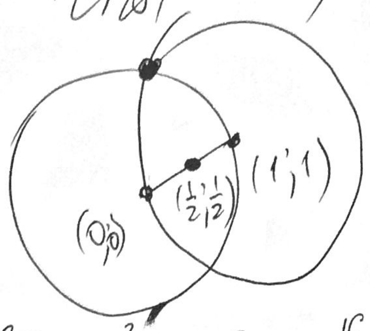

$$S=S_1\cdot 2+4S_0-S_x \qquad S_x=\frac{1}{2}\cdot\sqrt{2}\cdot\sqrt{2}\cdot\frac{\sqrt{3}}{2}\cdot 2=\frac{\sqrt{3}}{2}\cdot 2=\sqrt{3}$$

$$S_1=\pi R^2\cdot\frac{120}{360}=\pi\cdot 4\cdot 2\cdot\frac{1}{3}=\frac{8\pi}{3} \qquad S_0=\pi\cdot 2\cdot\frac{30}{360}=\pi\cdot 2\cdot\frac{1}{12}=\frac{\pi}{6}.$$

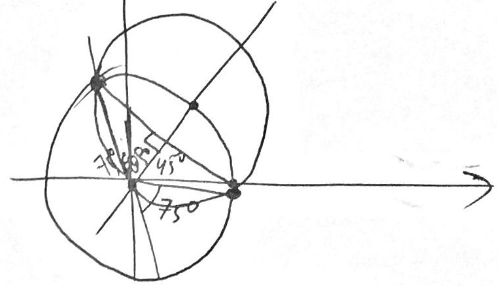
$R=2\sqrt{2}$

$\sqrt{3}$ $(x, y)$ $(m, n)$ Чистовик ⑥




$$S = 2\cdot\frac{8\pi}{3} + 4^2\cdot\frac{\pi}{3} - \sqrt{3} = \frac{16\pi}{3} + \frac{2\pi}{3} - \sqrt{3} = 6\pi - \sqrt{3}$$




$$\begin{cases} m^2 + n^2 = 2 \\ (m-1)^2 + (n-1)^2 = 2 \end{cases}$$

$$\left(m - \frac{1}{2}\right)^2 = \left(n - \frac{1}{2}\right)^2$$

$$\left[\begin{array}{l} m - \frac{1}{2} = n - \frac{1}{2} \Rightarrow m = n \\ m - \frac{1}{2} = \frac{1}{2} - n \Rightarrow m + n = 1 \end{array}\right.$$

$$n = 1 - m$$

$$\begin{cases} m^2 = 1 \\ (m-1)^2 = 1 \end{cases} \quad m = \pm 1 \quad m - 1 = \pm 1$$

$$\begin{cases} (m-1)^2 + m^2 = 2 \\ (m-1)^2 + (\cancel{1} - m - \cancel{1})^2 = 2 \end{cases}$$

$$2m^2 - 2m + 1 = 2$$

$$2m^2 - 2m - 1 = 0$$

$$m^2 - m + \frac{1}{2} = 0;$$

$$D = 4 + 4\cdot 2 = 12$$

$$m = \frac{2 \pm \sqrt{12}}{4} = \frac{1 \pm \sqrt{3}}{2}$$

$$n = 1 - \frac{1 \pm \sqrt{3}}{2} = \frac{1 \mp \sqrt{3}}{2}$$

$$\cancel{S = 2\cdot\frac{2\pi}{3} + \frac{4\pi}{6} - \sqrt{3} = 3\cdot\frac{2\pi}{3} - \sqrt{3} = 2\pi - \sqrt{3}}$$

$$\cancel{S = 2\cdot\frac{2\pi}{3} + \frac{4^2\pi}{6\,3} \cdot \frac{\sqrt{3}}{2} = 3\cdot\frac{2\pi}{3} - \frac{\sqrt{3}}{2} = 2\pi - \frac{\sqrt{3}}{2}}$$

Ответ: $6\pi - \sqrt{3}$

# Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

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Вариант 17

Чистовик. 1

№5

$$\underset{a}{\log_{\sqrt{5x-1}}} \underset{b}{(4x+1)}, \; \log_{4x+1}\left(\frac{x}{2}+2\right)^2, \; \underset{c}{\log_{\left(\frac{x}{2}+2\right)}}(5x-1).$$

$\log_a b, \; 2\log_b c; \; 2\log_c a$

1. $\log_a b = \cancel{2}\log_b c^2 = 2\log_c a + 1.$

$$\frac{\log_2 b}{\log_2 a} = \frac{\log_2 c^2}{\log_2 b},$$

$$\begin{cases} 5x-1>0 \\ 5x-1\neq 1 \\ \frac{x}{2}+2>0 \\ \frac{x}{2}+2\neq 1 \\ 4x+1>0 \\ 4x+1\neq 1 \end{cases}$$

$x > \frac{1}{5}$

$x \neq \frac{2}{5}$

$x > -4$

$x \neq -2$

$$\log_{\sqrt{5x-1}}(4x+1) = \log_{(4x+1)}\left(\frac{x}{2}+2\right)^2;$$

$$x > \frac{1}{5} \Rightarrow \begin{cases} 4x+1 > 1 \\ \frac{x}{2}+2 > 1 \end{cases}$$

$\cancel{4x+1 > \frac{x}{2}+2;}$

$\cancel{8x+2 > x+4}$

$\cancel{7x > 2}$

$\cancel{x > \frac{2}{7}}.$

$$\log_{\sqrt{5x-1}}(4x+1) = \log_{\left(\frac{x}{2}+2\right)}(5x-1)+1$$

$$\log_{\sqrt{5x-1}}(4x+1) = \log_{(4x+1)}\left(\frac{x}{2}+2\right)^2$$

$$\frac{\log_{(4x+1)}(4x+1)}{\log_{(4x+1)}\sqrt{5x-1}} = 2\log_{(4x+1)}\left(\frac{x}{2}+2\right)$$

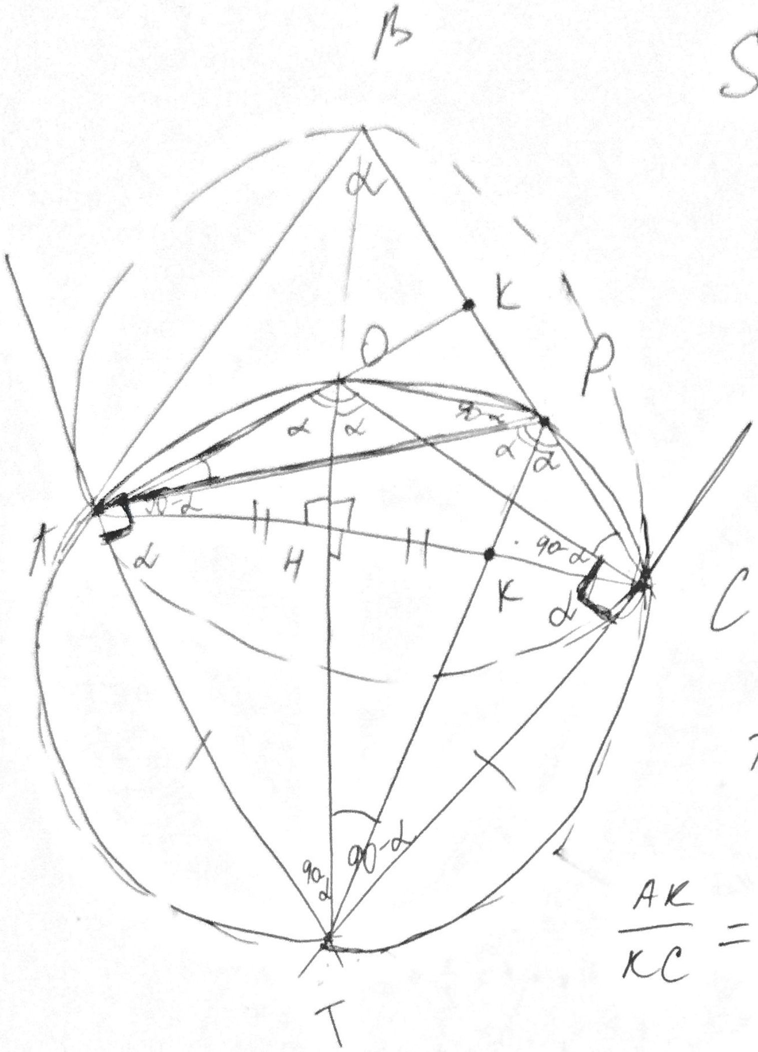
$$\frac{1}{\log_{(4x+1)}\sqrt{5x-1}} = 2\log_{4x+1}\left(\frac{x}{2}+2\right)$$

$$2\log_{(4x+1)}\left(\frac{x}{2}+2\right)\cdot\log_{(4x+1)}\sqrt{5x-1} = 1$$

Чистовик

№6.

(2)




$S_{APK} = 6$, $S_{CPK} = 4$.

$S_{APC} = \frac{1}{2} AP \cdot PC \cdot \sin\angle APC = 10$

$\angle ATO = \angle CTO = 90 - \alpha$

(равные △)

$\angle AOT = \angle COT = \alpha$

$\angle OAC = 90 - \alpha = \angle OTC$

$\Rightarrow T \in$ окр. $AOCP$.

$TK \cdot PK = AK \cdot KC$

$OH \cdot HT = HC^2$

$\frac{AK}{KC} = \frac{S_{APK}}{S_{AKC}} = \frac{3}{2}$.

$AK = \frac{3}{2} KC = \frac{3}{5} AC = \frac{6}{5} AH$.

$S_{ABC} = \frac{abc}{4R}$

$\frac{AP}{PC} = \frac{3}{2}$

$AP \cdot PC \cdot \sin 2\alpha = 20$

$\frac{AC}{\sin 2\alpha} = r$

$\frac{3}{2} \cdot PC^2 \cdot \sin 2\alpha = 20$

$\begin{cases} \frac{3}{2} \cdot PC^2 \cdot \sin 2\alpha = 20 \\ \frac{AC}{\sin 2\alpha} = r \end{cases}$

$AC = CT \cdot 2\cos\alpha$

$AC = OC \cdot 2\sin\alpha$

$OH = AO \cdot \cos\alpha$

$HT = AT \cdot \sin\alpha$

$\underset{R}{AO} \cdot AT \cdot \sin\alpha\cos\alpha = HC^2$

$OT \cdot HC \cdot \frac{\sin 2\alpha}{2} = HC^2$

$\underline{AO = R} = \frac{AC}{2\sin\alpha} = \frac{2\sin 2\alpha \cdot r}{2\sin\alpha} = \underline{2r\cos\alpha}$.

$HC = OT \cdot \frac{\sin 2\alpha}{2}$

$OT$ – диаметр. $AC = x$, $OH \cdot (2r - OH) = \frac{x^2}{4}$.

Числовик ③

$$1+\log_{\left(\frac{x}{2}+2\right)}(5x-1)=\log_{(4x+1)}\left(\frac{x}{2}+2\right)^2;$$

$$\frac{\log_2\left((5x-1)\cdot\left(\frac{x}{2}+2\right)\right)}{\log_2\left(\frac{x}{2}+2\right)}=\frac{\log_2\left(\frac{x}{2}+2\right)}{\log_2(4x+1)}$$

$$\log_{\left(\frac{x}{2}+2\right)}\left((5x-1)\left(\frac{x}{2}+2\right)\right)=\frac{\log_{\left(\frac{x}{2}+2\right)}\left(\frac{x}{2}+2\right)^2}{\log_{\left(\frac{x}{2}+2\right)}(4x+1)}=2;$$

$$\times\begin{cases}\log_{\left(\frac{x}{2}+2\right)}\left((5x-1)\left(\frac{x}{2}+2\right)\right)\cdot\log_{\left(\frac{x}{2}+2\right)}(4x+1)=2\\ 4\log_{(4x+1)}\left(\frac{x}{2}+2\right)\cdot\log_{(4x+1)}\sqrt{5x-1}=2\end{cases}$$

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$$\left(\log_{\frac{x}{2}+2}(5x-1)+1\right)\cdot\cancel{4}\cdot\log_{(4x+1)}\sqrt{5x-1}=\cancel{4}^{\,1}$$

$$\log_{\left(\frac{x}{2}+2\right)}\left((5x-1)\left(\frac{x}{2}+2\right)\right)\cdot\log_{4x+1}\sqrt{5x-1}=1;$$

№4. Чистовик

4

НОД $(a, b, c) = 6 \Rightarrow a \vdots 6,\ b \vdots 6,\ c \vdots 6$

НОК $(a, b, c) = 2^{15} \cdot 3^{16}$

$a, b, c \geq 6.$

Если одно число $c = 6$, то $a = 2^{15} \cdot 3^{16}$, $b = 2^{15} \cdot 3^{16}$

$2^{x} \cdot 3^{y}$

$x \geq 1,\ y \geq 1$

$x \leq 15,\ y \leq 16$

$15 \cdot 16 = 30 \cdot 8 = 240.$

$a = 2^{x_1} \cdot 3^{y_1} \cdot 2^1 \cdot 3^1$

$b = 2^{x_2} \cdot 3^{y_2} \cdot 2^1 \cdot 3^1$ т.к. в НОК нет др. множ.

$c = 2^{x_3} \cdot 3^{y_3} \cdot 2^1 \cdot 3^1$

$\max(x_1, x_2, x_3) = 15 - 1 = 14$

$\max(y_1, y_2, y_3) = 16 - 1 = 15.$

$\min(x_1, x_2, x_3) = 1 - 1 = 0$

$\min(y_1, y_2, y_3) = 1 - 1 = 0$

$x_1 \quad x_2 \quad x_3$ (0-14) $\qquad y_1 \quad y_2 \quad y_3$

$\underline{14} \cdot \underline{0} \cdot \underline{\ } \qquad \underline{15} \cdot \underline{0} \cdot \underline{\ }$

$1 \cdot 1 \cdot 16 = 16.$

$1 \cdot 1 \cdot 15 = 15.$

$1 \cdot 1 \cdot 13 = 13$ - без повт. $\quad 1 \cdot 1 \cdot 14 = 14$ - без повт.

$15 \cdot 16 = 240$ - всего троек,

$13 \cdot 14 = 169 + 13 = 183 - 1 = 182.$

$1092 + 174 =$ $\qquad 182 \cdot 6 = 600 + 240 + 62 \cdot 546 \cdot 2 = 1092$

$= 1274 - 8 =$ $\qquad$ ~~546 + (240 - 182)~~ $(240 - 182) \cdot 3 = 174$

$= 1266.$

6561

№6.

Без повт.: $x_1$ $x_2$ $x_3$ $y_1$ $y_2$ $y_3$ ~~Чистовик~~

5

$\underline{14}$ $\underline{0}$ $\cancel{1}$-13 ($\overline{0-14}$) $\underline{15}\cdot\underline{0}\cdot\underline{1-14}$

$1\cdot 1\cdot 13\cdot 6=78$ $1\cdot 1\cdot 14\cdot 6=84$

порядок важен

$78\cdot 84=(81^2-3^2)=81^2-9.$

$(81^2-9)\cdot 6$

С повт.:

$2\cdot 3=6$ $2\cdot 3=6$

$\cancel{6\cdot 84+6\cdot 78+6\cdot 6}$

$\cancel{15\cdot 16+13\cdot 14=62}$

62.

$6\cdot 84+6\cdot 78+6\cdot 6=6(90+78)=6\cdot 168=$

$=6\cdot 8\cdot 21$

14 0 0,14 15 0 0,15

| | | | | | | |
|---|---|---|---|---|---|---|
| 14 | 0 | 14 | | 15 | 0 | 15 |
| 14 | 14 | 0 | | 0 | 15 | 15 |
| 0 | 14 | 14 | | 15 | 15 | 0 |

$6\cdot 8\cdot 21\cdot 3$ ← тройки чисел в разном порядке

$(81^2-9)\cdot 6+6\cdot 8\cdot 21\cdot 3=6(81^2-9+8\cdot 21\cdot 3)=\cancel{6\cdot 9(27^2-1+8\cdot 7)}=$

$=6(6561-9+504)=6\cdot 7056=42.336$

$81\cdot 81=6561$; $168\cdot 3=504$; $6561+504=7065$; $7056\cdot 6=42336$

Ответ: 42.336

№6.

Чистовик

1. №5

Чистовик ⑥

$$\log_{\sqrt{5x-1}}(4x+1)=\log_{\frac{x}{2}+2}(5x-1)$$

$$\log_{\sqrt{5x-1}}(4x+1)=\frac{2}{\log_{\sqrt{5x-1}}(\frac{x}{2}+2)}$$

$x>\frac{1}{5}$

$x\neq\frac{2}{5}$.

$$\log_{\sqrt{5x-1}}(4x+1)\cdot\log_{\sqrt{5x-1}}(\frac{x}{2}+2)=2$$

$$\log_{5x-1}\underbrace{(4x+1)}_{>1}\log_{5x-1}\underbrace{(\frac{x}{2}+2)}_{>1}=\frac{1}{2}.$$

$$\log_{5x-1}(4x+1)\log_{5x-1}(\frac{x}{2}+2)^2=1.$$

$$\log_{(4x+1)}(\frac{x}{2}+2)^2+1=\log_{\frac{x}{2}+2}(5x-1)$$

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$$\cancel{\log_{5x-1}(4x+1)\log_{(5x-1)}}$$

$$\frac{(\log_{(5x-1)}(4x+1))^2\log_{5x-1}(\frac{x}{2}+2)^2}{\log_{(5x-1)}(\frac{x}{2}+2)^2}=\frac{1}{\log_{(\frac{x}{2}+2)}\left(\frac{(5x-1)}{\frac{x+4}{2}}\right)}$$

$$(\log_{(5x-1)}(4x+1))^2=\frac{}{\log_{(5x-1)}(1-}$$

№6. Чистовик

(7)

$$\frac{AC}{4\sin 2\alpha} = OT \qquad AC \cdot h = 20.$$

$\angle AOC = 2\angle ABC,$ центр. $\qquad PK \parallel AB.$

$AK = KC.$

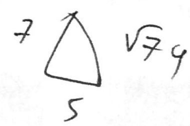
$$\frac{AK}{KC} = \frac{BP}{PC} = \text{(подоб } \triangle)$$

$$\frac{AC}{KC} = \frac{BC}{PC} = \frac{H}{h} = \frac{5}{2}.$$

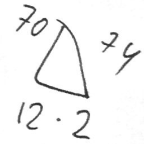
$$H = \frac{5}{2}h. \qquad 20$$

$$S_{ABC} = \frac{1}{2} \cdot AC \cdot H = \frac{1}{2} \cdot AC \cdot \frac{5}{2}h = \frac{5}{4} \cdot 20 = 25$$

$$50 = AB \cdot BC \cdot \frac{7}{\sqrt{74}}.$$

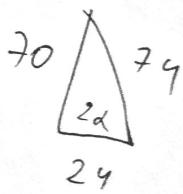



$$AP \cdot PC = \frac{\overset{4}{\cancel{20}} \cdot 37}{\underset{7}{\cancel{35}}} = \frac{3 \cdot 37}{7}.$$

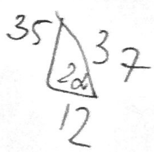



$$PC = \frac{2}{5}BC$$

$$\frac{AC}{2\sin\alpha} = R \qquad \frac{3}{2}PC^2 \cdot \frac{35}{37} = 20$$




$$\frac{AC}{2\sin 2\alpha} = \frac{R}{\cos\alpha}.$$




$$R \cdot \cancel{2}r \cdot \sin\alpha \cdot \frac{\sin 2\alpha}{\cancel{2}} = \left(\frac{AC}{2}\right)^2$$

$$R \cdot r \cdot \sin\alpha \sin 2\alpha \cdot 4 = AC^2 \qquad \frac{OH}{HT} = \frac{2r \cdot \cos\alpha \cdot \cos\alpha}{2r \cdot \sin^2\alpha} =$$

$$AC^2 = 4R \cdot r \sin\alpha \sin 2\alpha$$

$$\frac{49}{25}. \qquad = tg^2\alpha = \left(\frac{7}{5}\right)^2$$

$$\frac{AC^2}{4} = HT^2 \cdot tg^2\alpha$$

$\log_a b \qquad \log_b c^2 \qquad \log_c a^2$

$AC = 2R\sin\alpha = 2r\sin 2\alpha$