

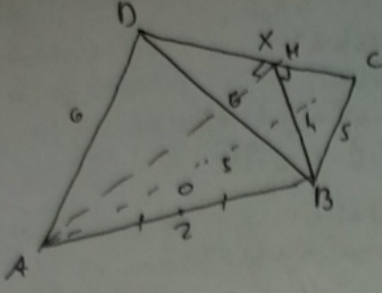
Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21103105**

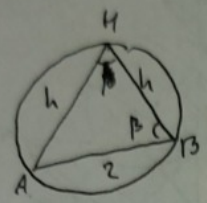
ID профиля: **155255**

Вариант 17



$C, D \in$ площ. $\Rightarrow C, D \in$ площ. \Rightarrow $CD \perp$ основанню

• проведем высоты AH_1 и BH_2 в ΔADC и в ΔBDC соотв.
 A и B симметр. отн. $(DOC) \Rightarrow H_1 = H_2 = H$
 $(AH) \perp$ основанню \Rightarrow окружность отн. оного ΔABC равна основанию



$$2R = \frac{h}{\sin \beta}$$

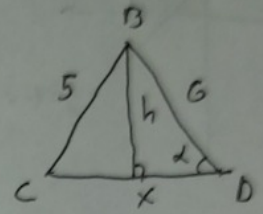
$$\cos \beta = \frac{r^2 + h^2 - r^2}{2 \cdot r \cdot h} = \frac{r}{h}$$

$$\sin \beta = \sqrt{1 - \frac{r^2}{h^2}}$$

$$2R = \frac{h}{\sqrt{1 - \frac{r^2}{h^2}}} = \frac{h^2}{\sqrt{h^2 - r^2}}$$

$$= \frac{36 \left(1 - \left(\frac{x^2 + 11}{12x}\right)^2\right)}{\sqrt{36 - \left(\frac{x^2 + 11}{12x}\right)^2}} \rightarrow \min$$

$$\begin{cases} \frac{dR}{dx} = \left(\frac{f}{\sqrt{f-1}}\right)' = \frac{f' \cdot \sqrt{f-1} - \frac{1}{2\sqrt{f-1}} \cdot f' \cdot f}{f-1} = 0 \\ \frac{d^2R}{dx^2} > 0 \end{cases}$$

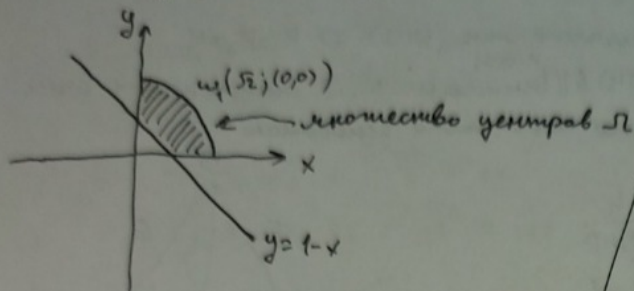


$$\cos \alpha = \frac{6^2 + x^2 - 5^2}{2 \cdot 6 \cdot x} = \frac{x^2 + 11}{12x}$$

$$h = 6 \sin \alpha = 6 \sqrt{1 - \left(\frac{x^2 + 11}{12x}\right)^2}$$

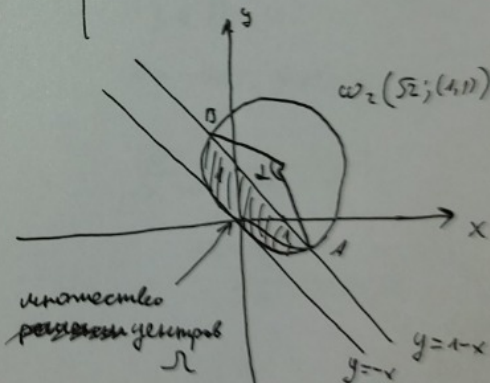
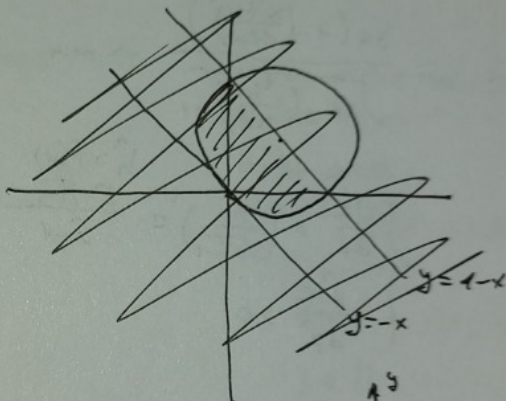
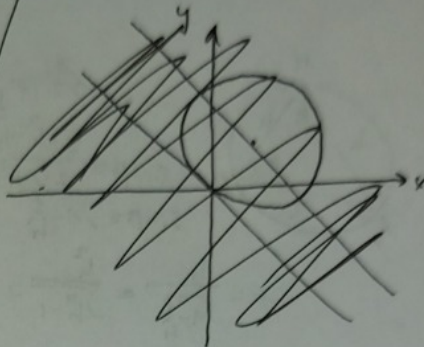
1) $2a+2b \geq 2$

$$\begin{cases} a+b \geq 1 \\ (x-a)^2 + (y-b)^2 \leq 2 - \text{при } \omega_1(\sqrt{2}; (a,b)) \\ a^2 + b^2 \leq 2 - \text{на пръсти } \omega_2 \end{cases}$$

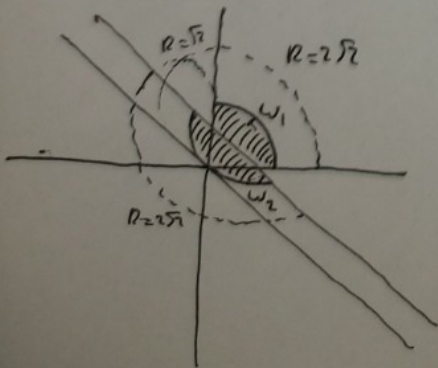


2) $2a+2b \leq 2$

$$\begin{cases} a+b \leq 1 \\ (x-a)^2 + (y-b)^2 \leq 2 \\ 0 \leq a^2 + b^2 \leq 2a+2b \\ \begin{cases} a+b \leq 1 \\ (x-a)^2 + (y-b)^2 \leq 2 \\ (a-1)^2 + (b-1)^2 \leq 2 - \text{при } \omega_2 \end{cases} \end{cases}$$



В итоге, множество центров ω вымодит так:



$$\begin{cases} (x-1)^2 + (y-1)^2 = 2 \\ x+y=1 \end{cases}$$

$$(x-1)^2 + x^2 = 2$$

$$2x^2 - 2x - 1 = 0$$

$$x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}$$

$$y = \frac{1}{2} \mp \frac{\sqrt{3}}{2}$$

$$AB = \sqrt{(\frac{\sqrt{3}}{2})^2 + (\frac{\sqrt{3}}{2})^2} = \sqrt{3}$$

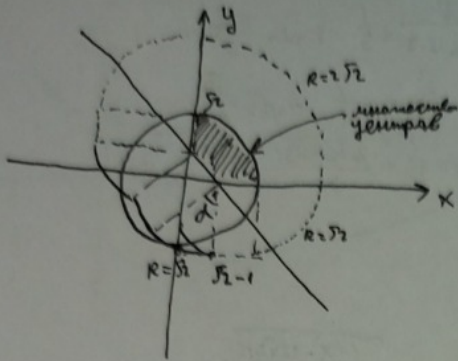
$$\cos \alpha = \frac{2+2-6}{2 \cdot 2 \cdot \frac{1}{2}} = -\frac{1}{2}$$

$$d = \frac{2\sqrt{3}}{3}$$

непроблем

$$2a+2b \geq 2$$

$$\begin{cases} a+b \geq 1 \\ (x-a)^2 + (y-b)^2 \leq 2 \\ a^2 + b^2 \leq 2 \end{cases}$$



$$\alpha = \frac{\pi}{4}$$

$$S = \frac{1}{2} R^2 \alpha$$

$$S = \frac{1}{2} (2\sqrt{2})^2 \cdot \frac{\pi}{4} + 2 \cdot \frac{1}{2} (\sqrt{2})^2 \cdot \frac{\pi}{4} + 2 \cdot (\sqrt{2}-1) \sqrt{2} + 2 \cdot \frac{1}{2} (\sqrt{2})^2 \cdot \frac{\pi}{4} + \sqrt{2} \sqrt{2} = 6 - 2\sqrt{2} + \frac{1}{2}\pi$$

$$4\sqrt{2} + 1 + \frac{1}{2}$$

$$2 \geq 2a+2b$$

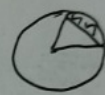
$$\begin{cases} a+b \leq 1 \\ (x-a)^2 + (y-b)^2 \leq 2 \\ 0 \leq a^2 + b^2 \leq 2a+2b \end{cases}$$

архипп

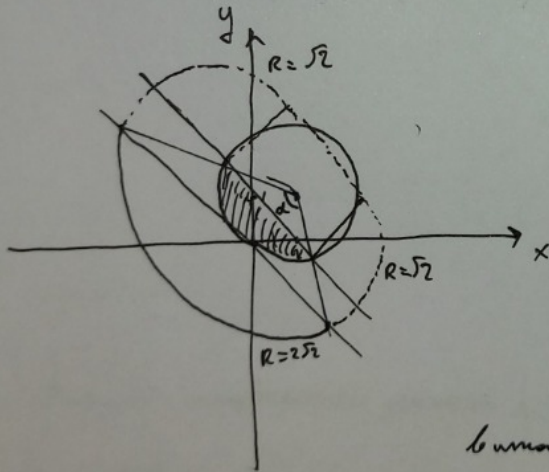
$$0 \leq a+b \leq 1$$

$$a^2 + b^2 \leq 2a+2b$$

$$(a-1)^2 + (b-1)^2 \leq 2$$



$$S = \frac{1}{2} R^2 \alpha - \frac{1}{2} R^2 \sin \alpha$$



$$(x-1)^2 + (y-1)^2 = 2$$

$$x+y=1$$

$$x=1-y$$

$$(x-1)^2 + x^2 = 2$$

$$2x^2 - 2x - 1 = 0$$

$$D = 4 + 4 \cdot 2 = 12$$

$$x = \frac{2 \pm 2\sqrt{3}}{4} = \frac{1 \pm \sqrt{3}}{2}$$

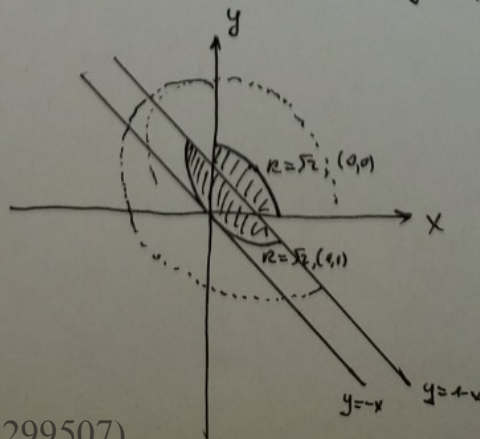
$$a = \sqrt{\left(\frac{1+\sqrt{3}}{2}\right)^2 + \left(\frac{1+\sqrt{3}}{2}\right)^2} = \sqrt{6}$$

$$\cos \alpha = \frac{2+2-6}{2 \cdot \sqrt{2} \cdot \sqrt{2}} = -\frac{1}{2}$$

$$\alpha = \frac{2\pi}{3}$$

$$S = \frac{1}{2} (2\sqrt{2})^2 \cdot \frac{\pi}{3} + \frac{1}{2} (2\sqrt{2})^2 \cdot (\alpha - \sin \alpha) + \dots$$

в шире множество центров:



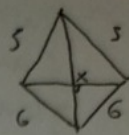
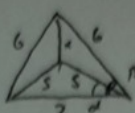
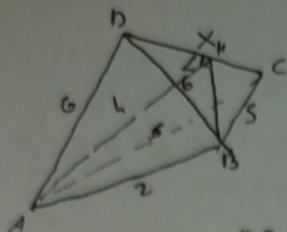
$$f(1, 0, \sqrt{2}), \left(\frac{1-\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}\right)$$

$$f = \frac{1}{4} - \frac{3}{4} + \frac{\sqrt{3}}{2} + \left(\sqrt{2} - \frac{1-\sqrt{3}}{2}\right)^2$$

$$= \frac{1}{4} - \frac{3}{4} + \frac{\sqrt{3}}{2} + 2 + \frac{1}{4} + \frac{3}{4} - \sqrt{2} + \frac{\sqrt{3}}{2}$$

rechnerisch

X_{max}, X_{min}



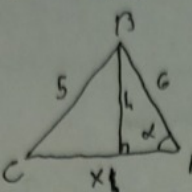
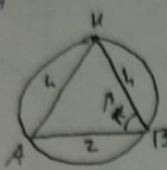
$$x^2 = 5^2 + 6^2 - 2 \cdot 5 \cdot 6 \cdot \cos(\beta - \alpha)$$

$$\cos \alpha = \frac{2^2}{2 \cdot 5 \cdot 2} = \frac{1}{5} \quad \sin \alpha = \frac{2\sqrt{6}}{5}$$

$$\cos \beta = \frac{2^2}{2 \cdot 6 \cdot 2} = \frac{1}{6} \quad \sin \beta = \frac{\sqrt{35}}{6}$$

$$\cos(\beta - \alpha) = \frac{1}{30} + \frac{2\sqrt{6} \cdot \sqrt{35}}{30} = \frac{2\sqrt{210} + 1}{30}$$

DC eodspieg.



$$\cos \beta = \frac{z}{2 \cdot 2 \cdot h} = \frac{1}{h}$$

$$\sin \beta = \sqrt{1 - \frac{1}{h^2}}$$

$$2R = \frac{h}{\sin \beta} = \frac{h}{\sqrt{1 - \frac{1}{h^2}}} = \frac{h^2}{\sqrt{h^2 - 1}}$$

$$\cos \alpha = \frac{6^2 + x^2 - 5^2}{2 \cdot 6 \cdot x} = \frac{x^2 + 11}{12x}$$

$$h = 6 \sin \alpha = 6 \cdot \sqrt{1 - \left(\frac{x^2 + 11}{12x}\right)^2} = 6 \sqrt{\frac{1 - \frac{x^4 - 22x^2 + 121}{144x^2}}{1}}$$

$$h^2 = 36 \cdot \frac{-x^4 - 22x^2 - 120}{144x^2}$$

$$= \frac{36 \cdot \frac{-x^4 - 22x^2 - 120}{144x^2}}{\frac{x^4 + 22x^2 + 120}{144x^2} - 1}$$

$$h^2 = f$$

$$f' = \frac{(-4x^3 - 44x) \cdot 144x^2 - 288x(-x^4 - 22x^2 - 120)}{144^2 x^4}$$

~~$R = \frac{f}{2\sqrt{f-1}}$~~
 ~~$R' = \frac{f' \cdot 2\sqrt{f-1} - \frac{f}{\sqrt{f-1}} \cdot f'}{4\sqrt{f-1}}$~~

$$R = \frac{f}{2\sqrt{f-1}}$$

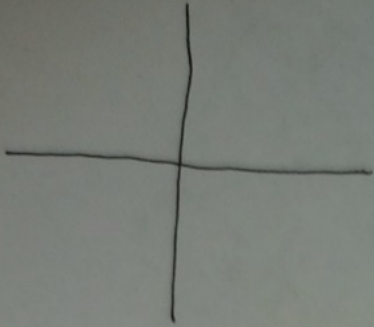
$$R' = \frac{f' \cdot 2\sqrt{f-1} - \frac{f}{\sqrt{f-1}} \cdot f'}{4\sqrt{f-1}} = 0$$

$$f'(2f - 2 - f) = f'(f - 2) = 0$$

representation

$a, b \in \mathbb{R}$

for $1 \leq a, b \leq 2$



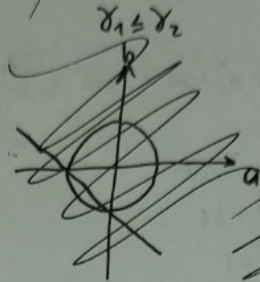
1) $2a + 2b \geq 2$

$a + b \geq 1$

$a^2 + b^2 \leq a + b$

$a^2 + b^2 \leq a + b \iff a^2 - a + b^2 - b \leq 0$

$x_1 \leq x_2$
 $x_1, x_2 \geq 0$
 $\sqrt{x_1} + \sqrt{x_2}$



$2a + 2b \geq 2$
 $a + b \geq 1$

$a^2 + b^2 \leq 2a + 2b$

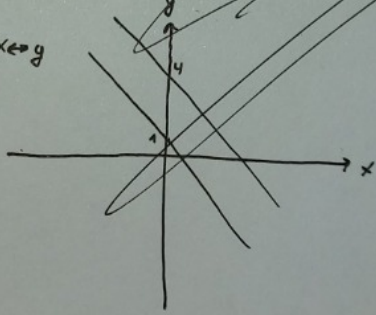
$(a+b)^2 \leq 2a + 2b + 2ab = 2(a+b)$

$= a^2 + b^2 + 2ab$

$\begin{cases} x^2 - 2ax + a^2 + y^2 - 2by + b^2 \leq 2 \\ a + b \geq 1 \\ a^2 + b^2 \leq 2a + 2b \end{cases}$

$\frac{(a+b)^2}{4} \leq \frac{a^2+b^2}{2} \leq a+b$

$1 \leq a+b \leq 4$



$\sqrt{\frac{a^2+b^2}{2}} \geq \frac{a+b}{2}$

$\frac{a^2+b^2}{2} \geq \frac{a^2+2ab+b^2}{4}$

$\frac{a^2-2ab+b^2}{4} \geq 0$

$\frac{(a-b)^2}{4} \geq 0$

$x^2 + y^2 \leq 2x + 2y$

$x^2 - 2x + y^2 - 2y \leq 0$

$0 = 4 - 4y^2 + 8y = -4(y-1)^2 + 8 \geq 0$

$-4(y-1)^2 + 8 \geq 0$

$(y-1)^2 \leq 2$

$-\sqrt{2} \leq y-1 \leq \sqrt{2}$

$-\sqrt{2}+1 \leq y \leq \sqrt{2}+1$

$-\sqrt{2}+1 \leq x \leq \sqrt{2}+1$

$(x-1)^2 + (y-1)^2 \leq 2$

$x = 2 + \sqrt{2} \cos \theta$
 $y = 2 + \sqrt{2} \sin \theta$

a_1 $a_n \in \mathbb{Z}$

неповторяется

$a_2 = a_1 + d$

$a_n = a_1 + (n-1)d$

$$S_n = \frac{2a_1 + (n-1)d}{2} \cdot n$$

$$\begin{cases} (a_1 + 5d)(a_1 + 11d) > \frac{2a_1 + 9d}{2} \cdot 9 \\ (a_1 + 6d)(a_1 + 10d) < \frac{2a_1 + 9d}{2} \cdot 17 \end{cases}$$

$$\begin{cases} a_1^2 + 16a_1d + 55d^2 > \frac{2a_1 + 9d}{2} \cdot 9 + 1 \\ a_1^2 + 16a_1d + 60d^2 < \frac{2a_1 + 9d}{2} \cdot 17 + 1 \end{cases}$$

$$\cancel{a_1^2 + 16a_1d + 55d^2} + (2a_1 + 9d) \cdot 5 + 17 > \cancel{a_1^2 + 16a_1d + 60d^2} + (2a_1 + 9d) \cdot 5 + 1$$

$5d^2 < 16$

$d = \pm 1$

$d = 1$

$$\cancel{(a_1 + 5)} a_1^2 + 16a_1 + 55 > 10a_1 + 45 + 1$$

$a_1^2 + 6a_1 + 9 > 0$

$(a_1 + 3)^2 > 0$

$a_1 \neq -3$

$$a_1^2 + 16a_1 + 60 < 10a_1 + 45 + 17$$

$a_1^2 + 6a_1 - 2 < 0$

$D = 36 + 8 = 44$

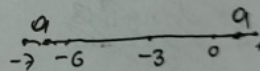
$a_1 = \frac{-6 \pm \sqrt{44}}{2} = -3 \pm \sqrt{11}$

$a_2 = -3 - \sqrt{11}$

$a \in (-3 - \sqrt{11}; -3 + \sqrt{11})$

$3 < \sqrt{11} < 4$

$a = -6; -5; -4; -3; -2; -1; 0$



$$\underline{\underline{a_1 = -6; -5; -4; -2; -1; 0}}$$

арифметическая прогрессия $a_{n+1} = a_n + d$, $a_n = a_1 + (n-1)d$

$$S_{10} = \frac{a_1 + a_{10}}{2} \cdot 10 = (2a_1 + 9d) \cdot 5$$

$$\begin{cases} (a_1 + 5d)(a_1 + 10d) > (20a_1 + 45d) + 1 \\ (a_1 + 6d)(a_1 + 10d) < (20a_1 + 45d) + 17 \end{cases}$$

$$\begin{cases} a_1^2 + 16a_1d + 55d^2 > 20a_1 + 45d + 1 \\ a_1^2 + 16a_1d + 60d^2 < 20a_1 + 45d + 17 \end{cases}$$

$$-5d^2 > -16$$

$$d^2 < \frac{16}{5}$$

$$3 < \frac{16}{5} < 4$$

$$a_n \in \mathbb{Z} \Rightarrow d \in \mathbb{Z} \Rightarrow d = \pm 1; 0$$

$$\text{но по условию } \Rightarrow d = 1$$

$$a_1^2 + 16a_1 + 55 > 20a_1 + 46$$

$$a_1^2 + 6a_1 + 9 > 0$$

$$(a_1 + 3)^2 > 0$$

$$a_1 \neq -3$$

$$a_1^2 + 16a_1 + 60 < 20a_1 + 45 + 17$$

$$a_1^2 + 6a_1 - 2 < 0$$

$$D = 36 + 8 = 44$$

$$a_1 = \frac{-6 \pm \sqrt{44}}{2} = -3 \pm \sqrt{11}$$

$$a \in (-3 - \sqrt{11}; -3 + \sqrt{11})$$

$$3 < \sqrt{11} < 4$$

$$a_1 = -6; -5; -4; -3; -2; -1; 0$$

$$\text{ответ: } a_1 = -6; -5; -4; -2; -1; 0$$

Метр 1

Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21103105**

ID профиля: **155255**

Вариант 17

$$a = 5x-1, b = 4x+1, c = \frac{x}{2}+2$$

$$a, b, c > 0$$

$$a, b, c \neq 1$$

$$\text{данные числа } x = \log_{5a} b = 2 \log_a b$$

$$y = \log_b c^2 = 2 \log_b c$$

$$z = \log_c a$$

заменить, что $xyz = 4$

$$1) \begin{cases} xyz = 4 \\ x = y, z = x-1 \end{cases}$$

$$x \cdot x \cdot (x-1) = 4$$

$$x^3 - x^2 - 4 = 0$$

$$x = 2$$

$$(x-2)(x^2+x+2) = 0$$

$$D < 0$$

$$\begin{cases} x=2 \\ y=2 \\ z=1 \end{cases} \Rightarrow \begin{cases} \log_a b = 1 \\ \log_b c = 1 \\ \log_c a = 1 \end{cases} \Rightarrow a=b=c, \text{ что противно условию}$$

$$5x-1 = 4x+1 = \frac{x}{2}+2$$

\emptyset

$$2) x=z, y=x-1$$

$$\text{аналогично } \begin{cases} x=2 \\ z=2 \\ y=1 \end{cases} \begin{cases} \log_a b = 1 \\ \log_c a = 2 \\ \log_b c = \frac{1}{2} \end{cases} \begin{cases} 5x-1 = 4x+1 \\ 5x-1 = (\frac{x}{2}+2)^2 \\ 4(\frac{x}{2}+2)^2 = 4x+1 \end{cases} \begin{matrix} x=2 \\ 4-1 = (\frac{2}{2}+2)^2 \text{ не } \\ x=2 \end{matrix}$$

$$\underline{\underline{x=2}}$$

$$3) y=z, x=y-1$$

$$\text{аналогично } \begin{cases} y=2 \\ z=2 \\ x=1 \end{cases} \begin{cases} \log_b c = 1 \\ \log_c a = 2 \\ \log_a b = \frac{1}{2} \end{cases} \begin{cases} 4x+1 = \frac{x}{2}+2 \\ 5x-1 = (\frac{x}{2}+2)^2 \\ (4x+1)^2 = 5x-1 \end{cases} \begin{matrix} x = \frac{2}{7} \\ \frac{2}{7} \neq (\frac{15}{7})^2 \\ \emptyset \end{matrix}$$

Ответ: $x=2$

лист 1

a, b, c содержат разное число троек и двоек, ~~но~~ но хотя бы по 1

$$\text{количество} = a \cdot b \cdot c = 2^{16} \cdot 3^{17}$$

пусть a содержит n двоек, тогда b и c могут содержать $1, 2, 3, \dots, 15-n$

$$1 \leq n \leq 14$$

всего вариантов расстановки двоек ~~но~~ \sum_1^{14}

аналогично, если a содержит m троек, то b и c могут содержать $1, 2, 3, \dots, 16-m$

$$1 \leq m \leq 15$$

Черновик

#

$$НОК = 2^{15} \cdot 3^{10}$$

а не сообразил 2

а не сообразил 3

$$AP^2 = \frac{3}{4} PC^2 = \frac{3 \cdot 74}{3 \cdot 7}$$

Упробир

$$AC^2 = AP^2 + PC^2 - 2AP \cdot PC \cdot \cos 2d = \frac{3 \cdot 74}{7} + \frac{4 \cdot 74}{3 \cdot 7} + 2 \cdot \sqrt{\frac{3 \cdot 74}{7} \cdot \frac{4 \cdot 74}{7}} \cdot \frac{24}{74} =$$

$$\cos 2d = \cos^2 d - \sin^2 d = \frac{25}{24} - \frac{49}{24} = -\frac{24}{24}$$

$$= \frac{3 \cdot 74 + 4 \cdot 74 + 2 \cdot 2 \cdot \sqrt{3 \cdot 24 \cdot 3}}{21} =$$

$$= \frac{13 \cdot 74 + 288 \sqrt{3}}{21}$$

$$AC = \sqrt{AC^2}$$

log

$$2 \log_a b = 2 \log_b c$$

$$\log_a a = 2 \log_a b - 1$$

$$\log_a b = \log_b c$$

$$5x-1$$

$$5x-1-4x-1=-x-2$$

$$4x+1$$

$$\frac{x}{2}+2$$

$$x+4$$

$$2 \log_a b = \log_a c$$

$$2 \log_b c = \log_c a - 1$$

$$\frac{2 \ln b}{\ln a} = \frac{\ln a}{\ln c}$$

$$\frac{2 \ln c}{\ln b} = \frac{\ln a - \ln c}{\ln c}$$

$$\log_a b, \log_b c, \log_c a$$

$$2 \log_a b + 2 \log_b c, \log_c a$$

$$2 \log_a b \cdot 2 \log_b c = \frac{4}{\log_c a}$$

$$\begin{cases} xyz=4 \\ x=y, z=x-1 \end{cases}$$

$$x \cdot x \cdot (x-1) = 4$$

$$x^3 - x^2 - 4 = 0$$

$$x=2$$

$$(x-2)(x^2+x+2)=0$$

$$D < 0$$

$$\begin{cases} x=2 \\ y=2 \\ z=1 \end{cases}$$

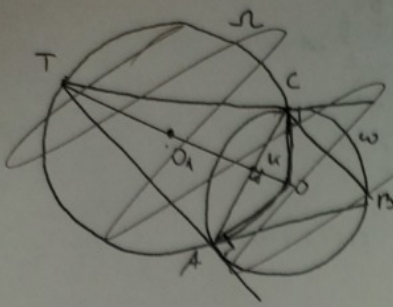
$$2 \log_{(5x-1)} (4x+1) = 2$$

$$2 \log_{(4x+1)} \left(\frac{x}{2}+2\right) = 2$$

$$\log_{\frac{x}{2}+2} (5x-1) = 1$$

$$5x-1 = 4x+1 = \frac{x}{2}+2 \quad \emptyset$$

керновит

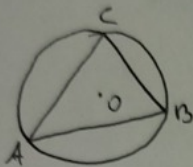


$P=C$
 $S_{APK}=6$
 $S_{CPK}=4$

CT_k - каск \Rightarrow

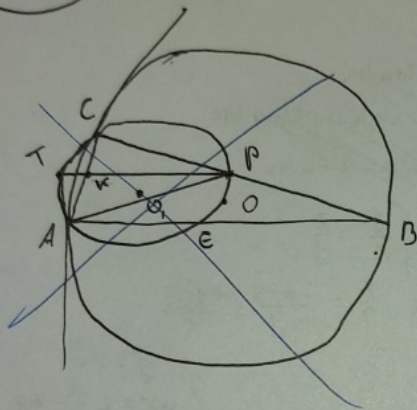
$CT \perp OC$
 $AT \perp AO$
 $\angle AOC = 360^\circ - 90^\circ - 90^\circ = \angle CTA \Rightarrow T \in \Omega$
 T диаметр, противоположн. точке O \Rightarrow
 $\Rightarrow AC \perp TO$

ABC - остроуг. $\Rightarrow O \in ABC$



T диаметр противоположн. O

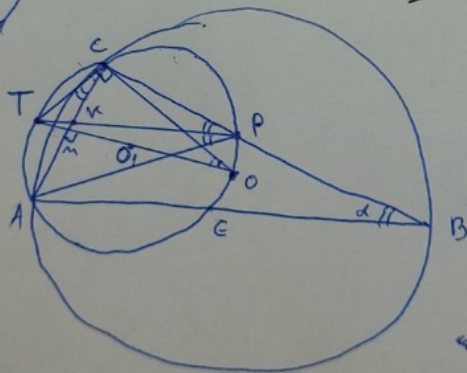
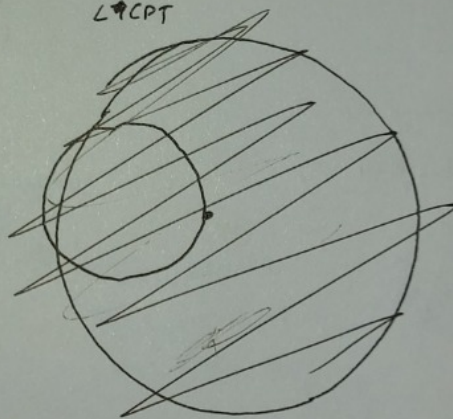
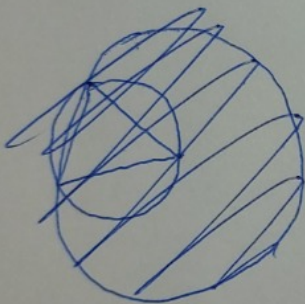
$OT \perp AC$



$S_{APK}=6$
 $S_{CPK}=4$

$\frac{CK}{AK} = \frac{3}{2} \quad h = const$

$\angle \sphericalangle CPT$



$\angle \sphericalangle CPT$

$\angle CPT = \angle COT$ (впис.)

$OT \perp AC \Rightarrow \angle COT = \frac{1}{2} \angle AOC = \angle ABC$ (впис.)

$\angle ABC = \angle CPT \Rightarrow AB \parallel PT \Rightarrow$

$\Rightarrow \triangle ABC \sim \triangle KPC \Rightarrow$
 $\Rightarrow \frac{S_{AKC}}{S_{KPC}} = k^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$S_{ABC} = \frac{25}{4} \cdot 4 = \frac{25}{1} = 25$

Ег $\angle ABC = \frac{7}{5} = \frac{\frac{1}{2} AC}{OM} = \frac{CT}{OE} = \frac{TM}{\frac{1}{2} AC}$
 $CK = 4x$
 $AK = 6x$

$\cos \angle ABC =$

$OT \perp AC \Rightarrow \angle UAT = \angle UTC$

$\angle APT = \angle CPT \quad S = \frac{1}{2} AP \cdot PC \cdot \sin \alpha$

$\frac{PC}{AP} = \frac{2}{3}$

$6 + 4 = \frac{1}{2} PC \cdot AP \cdot \sin \alpha = PC \cdot AP \cdot \sin \alpha \cos \alpha = PC \cdot AP \cdot \frac{35}{24} = 60$

$PC \cdot AP = \frac{240}{35} = \frac{148}{7} = PC \cdot \frac{3}{2} PC \quad PC^2 = \frac{2 \cdot 148}{3}$

$$\log_{\frac{y+1}{5x-1}} \ln$$

$$\log_{\frac{y+1}{5x-1}} (y+1)$$

reprobur

$$5x-1=0$$

$$4x+1=6$$

$$\frac{x}{2} = 12 = c$$

$$\begin{cases} a > 0 \\ a \neq 1 \\ b > 0 \\ b \neq 1 \\ c > 0 \\ c \neq 1 \end{cases} \begin{cases} x > \frac{1}{5} \\ x \neq \frac{1}{5} \\ x > -\frac{1}{4} \\ x \neq 0 \\ x > -1 \\ x \neq -2 \end{cases}$$

+++++

$$\rightarrow \log_a \log_{5a} b = \log_b c^2$$

$$\log_c a = \log_b c^2 - 1$$

$$\begin{cases} \log_a b = \log_b c \\ \log_c a = \log_b \frac{c}{b} \end{cases} \quad \begin{cases} \log_a b = \log_b c \\ \log_c a = 2 \log_a \frac{b}{a} \end{cases}$$

$$\begin{cases} \frac{\ln b}{\ln a} = \frac{\ln c}{\ln b} \\ \frac{\ln a}{\ln c} = \frac{2 \ln c - \ln b}{\ln b} \end{cases} \quad \ln a = \begin{cases} \frac{\ln b}{\ln a} = \frac{\ln c}{\ln b} \\ \frac{\ln a}{\ln c} = \frac{2 \ln b - \ln a}{\ln a} \end{cases}$$

$$\begin{cases} \ln^2 b = \ln a \cdot \ln c \\ \ln a \cdot \ln b = 2 \ln^2 c - \ln b \ln c \end{cases}$$

$$\ln a \ln a \cdot \ln c = 2 \ln^2 c - \ln b \ln c$$

$$\begin{cases} \ln^2 b = \ln a \cdot \ln c \\ \ln^2 a = 2 \ln b \cdot \ln c - \ln a \cdot \ln c \\ \ln^2 b + \ln^2 a = 2 \ln b \ln c \end{cases}$$

$$\begin{cases} \frac{y}{x} = \frac{z}{2} \\ \frac{x}{z} = \frac{2z-y}{2} = \frac{y}{z} \end{cases}$$

$$\begin{cases} x^2 = y^2 \\ y^2 - z^2 = x^2 \end{cases}$$

$$2 \log_a b = \log_b c$$

$$\log_c a = 2 \log_b c - 1$$

$$\begin{matrix} b & c \\ a & b \\ a & c \\ c & b \end{matrix}$$