Часть 1

Олимпиада: **Математика, 11 класс (1 часть)**

Шифр: **21100969**

ID профиля: **852164**

Вариант 17

Чистовик №1 [redacted]

[redacted]

1)

$$S = (a_1 + a_1 + 9d)\cdot\frac{10}{2}$$

$$\begin{cases}(a_1+5d)(a_1+11d) > S+1 \\ (a_1+6d)(a_1+10d) < S+17\end{cases} \Rightarrow \begin{cases}-a_1^2 - 16a_1d - 55d^2 < -S-1 \\ a_1^2 + 16a_1d + 60d^2 < S+17\end{cases}$$

$$\Downarrow$$

$$5d^2 < 16 \Rightarrow \begin{cases} d \in [-\frac{4}{\sqrt{5}}, \frac{4}{\sqrt{5}}] \\ d \in Z\end{cases}$$

т.к. прогрессия ~~…~~ возрастающая

$d = 1$

$$\begin{cases} S = 10a_1 + 45 \\ a_1^2 + 16a_1 + 55 > 10a_1 + 46 \\ a_1^2 + 16a_1 + 60 < 10a_1 + 62 \\ a \in Z\end{cases}$$

$$\begin{cases} a_1 + 6a_1 + 9 > 0 \\ a_1^2 + 6a_1 - 2 < 0 \\ a \in Z\end{cases} \Rightarrow \begin{cases} a \in R \setminus \{-3\} \\ a \in (-3-\sqrt{11}, -3+\sqrt{11}) \\ a \in Z\end{cases}$$

$\Rightarrow$ Ответ:

$a \in \{-6; -5; -4; -2; -1; 0\}$

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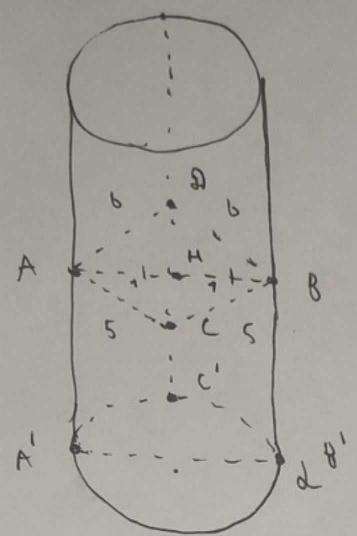
Чистовик №2 [redacted]

[redacted]

2)



$AB = 2$

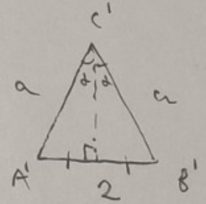
Пусть H — ~~...~~ основание высоты DH в $\triangle ADB$. Тогда $\forall$ тэтраэдр будет симметричен относительно пл-ти CDH.

=> точки A и B находятся на одинаковом расстоянии от пл-ти основ. цилиндра (пл-ть α)

=> при орт. проецировании AB на α $AB \to A'B'$

$A'B' = AB = 2$; $C \to C'$

Получается, что $\triangle A'C'B'$ вписан в основание цилиндра



$$2R = \frac{2}{\sin 2\alpha}$$

$$\Rightarrow R = \frac{1}{2\underbrace{\frac{1}{a}}_{\sin\alpha}\cdot\underbrace{\frac{\sqrt{a^2-1}}{a}}_{\cos\alpha}}$$

$$R = \frac{a^2}{2\sqrt{a^2-1}}\ ; \quad R_{min} \Leftrightarrow R'(a) = 0$$

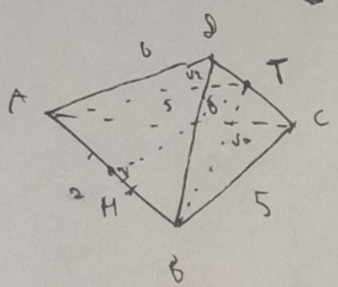
$$R'(a) = \frac{2a\cdot 2(\sqrt{a^2-1})^2 - a^2\cdot 2a}{2\sqrt{a^2-1}}$$

$$= \frac{2a^3 - 4a}{2\sqrt{a^2-1}} = 0$$

$-\sqrt{2}$ 0 $\sqrt{2}$; $R(a)$: $-$ $+$ $-$ $+$

$\Rightarrow a = 0$ или $a = \sqrt{2}$

$\Rightarrow R_{min} = 1$



пл-ть $ABT \perp CD$

$\triangle ABT = \triangle A'C'B'$

$CD = CT + TD$

$CT = \sqrt{CH^2 - HT^2}$.

$TD = \sqrt{DH^2 - HT^2}$

$CH = \sqrt{24}$; $DH = \sqrt{35}$

$HT = \cancel{\sqrt{\ }}\ 1$

ответ:

$\Rightarrow CD = \sqrt{34} + \sqrt{23}$

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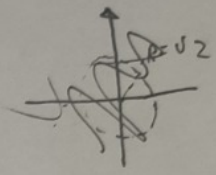
Чистовик №3 [redacted] [redacted]

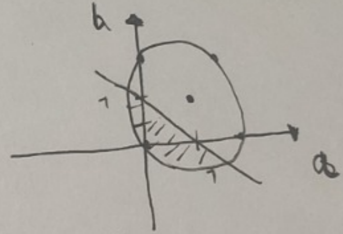
3) $(x-a)^2+(y-b)^2 \le 2$

$a^2+b^2 \le \min(2a+2b,\ 2)$

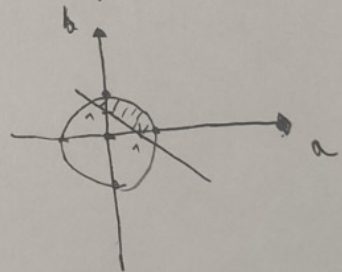
$$\left\{\begin{array}{l} \left[\begin{array}{l} \cancel{(x-a)^2+(y-b)^2} \\ \begin{cases} a^2+b^2 \le 2a+2b \\ 2a+2b \le 2 \end{cases} \quad (1) \Rightarrow \cancel{a^2+b^2 \le 2} \\ \begin{cases} a^2+b^2 \le 2 \\ 2a+2b \ge 2 \end{cases} \quad (2) \end{array}\right. \\ (x-a)^2+(y-b)^2 \le 2 \end{array}\right.$$

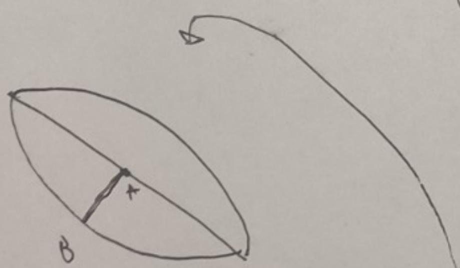


(1) →



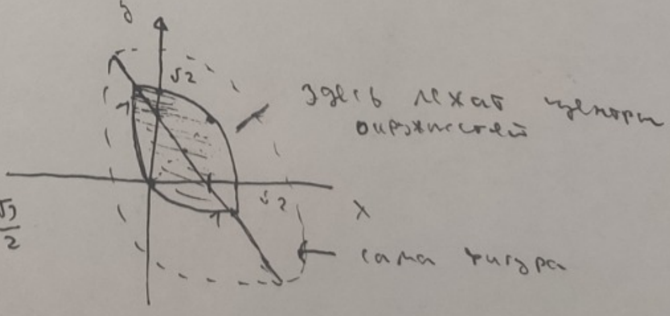
(2) →



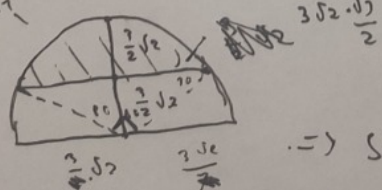


$AB = \frac{3}{2}\sqrt{2}$

$R = 3\sqrt{2}$



$S = S_1$



$\Rightarrow S = 2S_1$

$S_1 = \frac{\pi}{3} 9 \cdot 2 - \frac{1}{2} \cdot 3\sqrt{6} \cdot \frac{3}{2}\sqrt{2} = (6\pi - \frac{9\sqrt{3}}{2}) \Rightarrow S = 12\pi - 18\sqrt{3}$

ответ: $S = 12\pi - 18\sqrt{3}$

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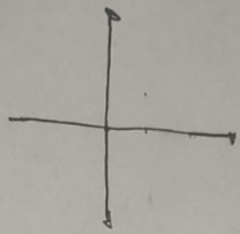
черновик

31 $\begin{cases} (x-a)^2 + (y-b)^2 \leq 2 \\ a^2 + b^2 \leq \min(2a+2b, 2). \end{cases}$

$2(a+b) > 2$

$a^2 - 2a + b^2 - 2b$

$(a-1)^2 + (b-1)^2 \leq 2$



$a^2 + b^2 \leq 2a + 2b$

2π

$x^2 - 2xa + a^2 + y^2 - 2by + b^2 \leq 2$

$2 \cdot \pi$

$x^2 - 2ax + y^2 - 2yb + 2a + 2b \leq 2$

$4 + 8 = 12$

$\begin{cases} (a-x)^2 + (b-y)^2 \leq 2 \\ a^2 + b^2 \leq \min(2a+2b, 2 \end{cases}$

4

$(x-a)^2 + (y-b)^2 \leq 2$

$2a^2 - 2a - 1 \leq 0$

$a^2 + b^2 \leq$

$2a^2 - 2a + 1 \leq 2$

$(a-1)^2 + (b-1)^2 \leq 2$

$\begin{cases} a + b < 1 \\ (a-1)^2 + (b-1)^2 \leq 2 \end{cases}$

$2a + 2b < 1$

$(a-1)^2 + (-a)^2 \leq 2$

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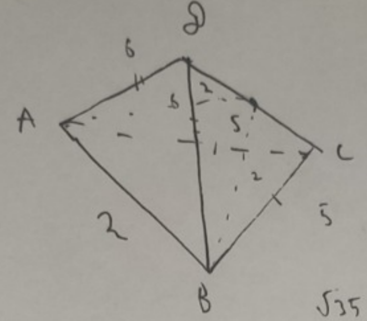
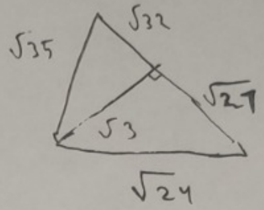
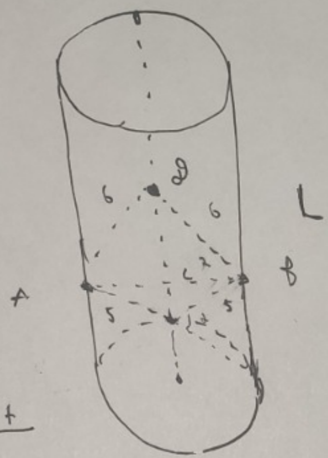
Черновики. [redacted]

2)





$4\sqrt{2}+\sqrt{21}$



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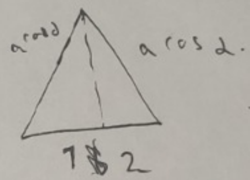
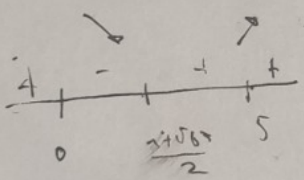


$\frac{1+\sqrt{65}}{2} = R$

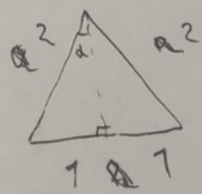
$2\cos\alpha = R$

$\cos\alpha = \frac{R}{2}$



$\frac{1 \pm \sqrt{65}}{2}$

$1 + 4\cdot4\cdot4 = 65$



$2R = \frac{2}{\sin 2\alpha}$

$R = \frac{1}{\sin 2\alpha}$

$2a\cdot 2\sqrt{a^2-1} - \frac{a^2\cdot 2a}{\sqrt{a^2-1}} = 0$

$R = \frac{1}{2\cdot\frac{1}{a}\cdot\frac{\sqrt{a^2-1}}{a}}$

t

$4a(a^2-1)-a^2$ $\quad a<5$ $\quad R = \frac{a^2}{2\sqrt{a^2-1}}$ $\quad \frac{2}{2}=1$

$4a^3-2a^3-4a=0$ $\quad a=\sqrt{2}$

$a=0$ $\quad 4a^2-a-4=0$

$2a^3-4a=0$ $\quad a=0$ $\quad 2a^2=4$

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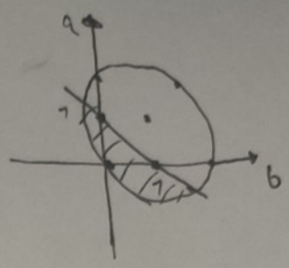
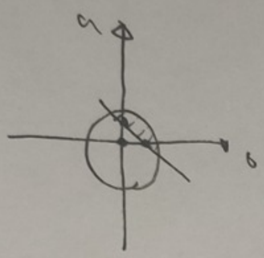
Чорновик.



$$\begin{cases} a^2+b^2 \le 2a+2b \\ a+b \le 1 \end{cases}$$

$(a-1)^2+(b-1)^2 \le 2$



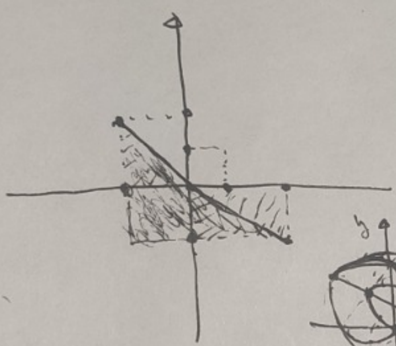
$$\begin{cases} a^2+b^2-2a-2b \le 0 \\ a \le 1-b \end{cases}$$

$a+b \le 1$ $\quad a \le 1-b$

$a \ge 1-b$

$1-2b+b^2+b^2-2+\cancel{2b}-\cancel{2b} \le 0$

$2b^2-2b-1 \le 0$



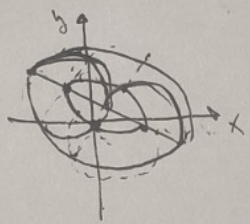
$4+8=12$

$b \in \left[\frac{1-\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}\right]$

$\frac{2 \pm \sqrt{12}}{4}$

$a \in \left[\frac{1-\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}\right]$

$= \frac{1 \pm \sqrt{3}}{2}$



$a \le 1-b$

$a \le \frac{1-\sqrt{3}}{2}$

$a^2+b^2 \le 2$

$\frac{1}{4} - \frac{\sqrt{3}}{2} + \frac{3}{4}$

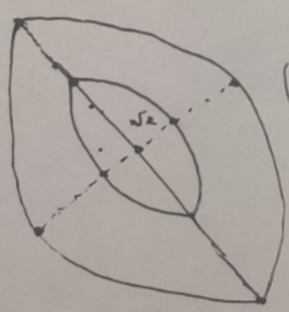
$a+b \ge 1$

$a^2+b^2 \le 2$

$a+b \ge 2$

$a^2+b^2-2a-2b \le 0$

$a+b \ge 2$



$(a-1)^2+(b-1)^2 \le 2 \quad a^2+$ ~~(2a)~~

$a-1 \ge -b$

$a^2-2a+b^2-2b \le -2$

$(-b+1)^2$

$(a-1)^2+(b-1)^2 \le 0$

$(a-1)$

$a=1$
$b=1$.

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Черновик

7)

$a_6 \cdot a_{12} > S+1$

$a_7 \cdot a_{11} < S+17.$

$a_n = a_1 + d(n-1)$

$\frac{(a_1 + a_1 + 9d)\cdot 10}{2} = S$

$(a_1+5d)(a_1+11d) > S+1$

$(a_1+6d)(a_1+10d) < S+17.$

$a_1^2 + 16a_1 d + 55d^2 > S+1$

$a_1^2 + 16a_1 d + 60d^2 < S+17.$

$\frac{4}{\sqrt{5}} \vee 2$

$4 \vee 2\sqrt{5}$

$16 \vee 20.$

$\frac{4}{2...}$

$5d^2 < 16$

$d^2 < \frac{16}{5}$ $\frac{4}{\sqrt{5}}$

$d \in [-1, 1]$

$\Rightarrow d = 1$

$a_1^2 + 6a_1 + 9 > 0$

$(a_1+3)^2 > 0$

$a_1^2 + 16a_1 + 55 - 10a_1 - 46 > 0$

$a_1 \neq -3$

$(a_1+5)(a_1+11) > 10a_1 + 46$

$2a_1 + 9.$

$(a_1+6)(a_1+10) < 10a_1 + 62$

$10a_1 + 45 = S$

$\frac{-6 \pm \sqrt{44}}{2}$

$a_1^2 + 16a_1 d + 60d^2 - 10a_1 - 62 < 0$

$-3 \pm \sqrt{11}$

$36 + 8 = 44$

$a_1^2 + 6a_1 - 2 < 0$

$[-3-3; -3+3]$

$[-6, 0]$

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Часть 2

Олимпиада: **Математика, 11 класс (2 часть)**

Шифр: **21100969**

ID профиля: **852164**

Вариант 17

Черновик

4)

НОД $(a, b, c) = 6$

НОК $(a, b, c) = 2^{15} \cdot 3^{16}$

$a = 2^{a_1} \cdot 3^{b_1}$ 2^{16}

$b = 2^{a_2} \cdot 3^{b_2}$ 2^{15} :

$c = 2^{a_3} \cdot 3^{b_3}$ 1 $2^{15} \cdot 2$

~~НОД~~

$15 \vdots a_1 \vdots a_2 \vdots a_3$

$\frac{abc}{\text{НОД}}$: НОК

$16 \vdots b_1 \vdots b_2 \vdots b_3$

$abc = 2^{16} \cdot 3^{17}$

$2^n \cdot 3^t$ $14 \geq n \geq 1$ $k \geq 1$

$\frac{(1 + 14) 14}{2} \cdot \frac{(1+15) \cdot 15}{2}$

$2^k \; 3^z$ $15 \geq t \geq 1$ $z \geq 1$

$2^{16-n-k} \; 3^{17-t-z}$

1 $\frac{1}{14}$

2 $\frac{1}{13}$

3 $\frac{1}{12}$

14 1 1

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Чистовик №1.

4)

$$\begin{cases} НОД(a,b,c)=6 \\ НОК(a,b,c)=2^{15}\cdot 3^{16} \end{cases}$$

$$\Rightarrow \frac{abc}{НОД(a,b,c)} = НОК(a,b,c)$$

$$\Rightarrow abc = 2^{16}\cdot 3^{17}$$

$$a = 2^{n}\cdot 3^{k}$$

$$b = 2^{t}\cdot 3^{z}$$

$$c = 2^{16-n-t}\cdot 3^{17-k-z}$$

$$14 \geq n \geq 1;\ 14 \geq k \geq 1$$

$$15 \geq t \geq 1;\ 15 \geq z \geq 1$$

n	t	16-n-t
1	1....14	однозначно определено
2	1....13	
⋮		
14	1	

$$\Rightarrow \text{кол-во чисел: } \frac{(1+14)\cdot 14}{2}\cdot\frac{(1+15)\cdot 15}{2}$$

ответ:

$$= 15\cdot 7\cdot 8\cdot 15 = 56\cdot 225 = 12600$$

$$\begin{array}{r} \times\ 56 \\ 225 \\ \hline 280 \\ +112 \\ +112 \\ \hline 12600 \end{array}$$

Чистовик №2

5) $\log_{\sqrt{5x-1}}(4x+1)$; $\log_{4x+1}\left(\frac{x}{2}+2\right)^2$; $\log_{\frac{x}{2}+2}(5x-1)$

Заметим, что произведение чисел $1+x+y+x+1=4$ даёт 4 $\Rightarrow$ одно из чисел должно быть равно $\frac{2}{3}$

$$\Rightarrow \left[\begin{array}{l} \log_{\sqrt{5x-1}}(4x+1)=\frac{2}{3} \quad (1) \\ \log_{4x+1}\left(\frac{x}{2}+2\right)^2=\frac{2}{3} \quad (2) \\ \log_{\frac{x}{2}+2}(5x-1)=\frac{2}{3} \quad (3) \end{array}\right.$$

ОДЗ: $x>\frac{1}{5}$; $x\neq\frac{2}{5}$

(1)

$$5x-1=(4x+1)^3$$

$$5x-1=64x^3+48x^2+12x+1$$

Черновик.

5) $2\log_a b$; $2\log_b c$; $\log_c a$

$4\log_a b \cdot \log_b c \cdot \log_c a$

$4\log_a c \cdot \log_c a = 4$

$\log_{5x-1}(4x+1) = \frac{2}{3}$

$3x+2=4$

$x = \frac{2}{3}$ $\frac{5}{3}$ $\frac{5}{3}$

$(5x-1)^2 = (4x+1)^3$

$\log_{\sqrt{5x-1}} 4x+1 = \frac{2}{3}$

$25x^2 - 10x + \cancel{1} = 64x^3 + 48x + 12x^2 + \cancel{1}$

$x + x + x - 1 = 4$

$7 \cdot 8$

$x = 0$

$x = \frac{5}{3}$

$64x^2 + 58 - 13x = 0$

1 12 80 56

-4 1 8

-7 1 5 45

-4 1 8 48

-2 1 10 60

-1 1 11

6
58
× 58
464
+ 290
3364
− 3328
36

64
× 52
128
+ 320
3328

~~Отв.~~

$32x = \sqrt{x^3}$

-58

$\frac{13 \pm 6}{128}$

$4x+1 = \frac{x^3}{8} + \frac{3}{2}x^2 + 6x + 8$

$\frac{7}{128}$ $\frac{19}{128}$

$4x+1 = (\frac{x}{2}+2)^3$

~~что~~ ~~4x+1~~ =

$1 \cdot 3ux+1$ $\frac{x}{3}+2 = \frac{1}{3}$

$x^3 + 12x^2 + 48x + 64 = \cancel{4x} \; 32x + 8$

$x^3 + 12x^2 + 80x + 56 = 0$

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Черновик

$$\begin{array}{r} 49 \\ +25 \\ \hline 74 \end{array}$$

$4 = \frac{1}{2} \cdot h \cdot 4x$

$25 = \frac{1}{2} AC \cdot H$

$2R \cdot \sin\alpha \cdot \frac{5}{2} h = 25$

$H = \frac{5}{2} h$

$R \cdot h \sin\alpha = 5$

$64x^3 + 48x^2 + 7x + 2 = 0$

$\frac{1}{2} AC \cdot \frac{5}{2} h = 25.$

$\frac{2r \sin\alpha}{\sin\alpha}$

	64	48	7	2
-2	64	80		
-1	64	16		

$PN = h$

$\frac{5x}{\frac{5}{2}h - h}$

$\frac{1}{2} 4x h = 4$

$4x \cdot h \quad xh = 2$

$AC = 2R \sin\alpha$

$2r \cos\alpha = R$

$AC = 2r \sin 2\alpha$

б) $\frac{AC}{\sin 2\alpha} = 2r$

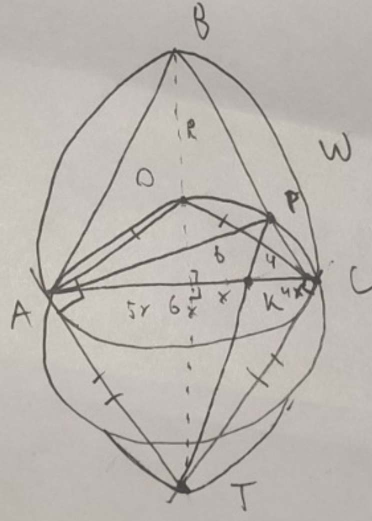
$OT = 2r$

$2r \cos\alpha = R$

$2r = \frac{R}{\cos\alpha}$

$\frac{2r \cos\alpha}{R} = 1$

$25 = \frac{1}{2} AC \cdot h$

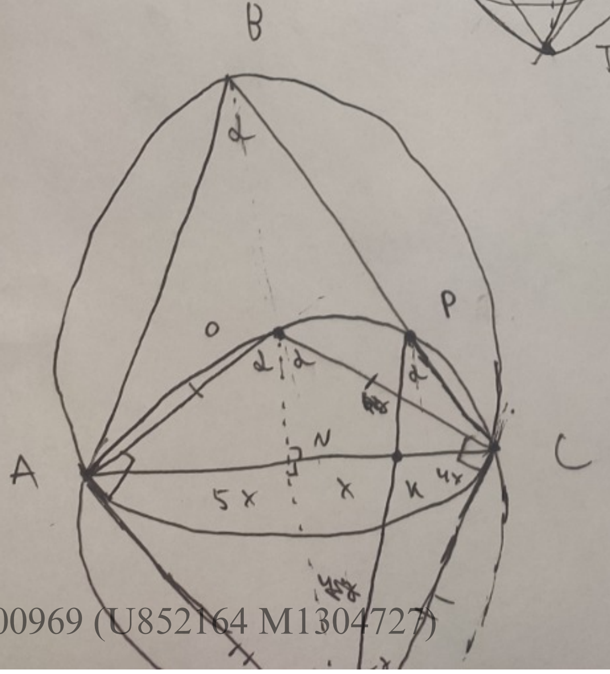


$S_{APC} = 6$

$S_{CPN} = 4.$

$\frac{BP}{CP} - ?$

$BP \cdot BC =$



$\frac{25}{}$

$\frac{4x}{10x}$

$\frac{S_{ABC}}{S_{KPC}} = \left(\frac{10}{4}\right)^2$

$= 0{,}16 \quad \frac{4}{25} \quad \frac{29}{4}$

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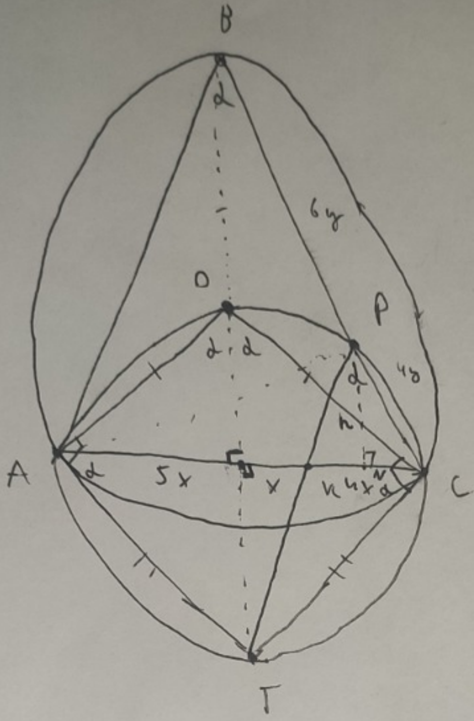
Чистовик № 3

б)



а) 1) ΔCPK и ΔAPK имеют общую высоту

$\Rightarrow AK / KC = \frac{6}{4} = \frac{3}{2}$

2) $AOCT$ - вписан в окр., описанную около ΔAOC, т.к. $\angle AOT = \angle OCT = 90°$ (потому что OC и OA - радиусы, проведенные и касательные)

3) $\angle TPC = \angle TOC = \alpha$

т.к. опираются на дугу TC

$\angle ABC = \frac{\angle AOC}{2} = \alpha$

$\Rightarrow \Delta ABC \sim \Delta KPC$

т.к. $\angle ABC = \angle KPC$ и $\angle ACP$ - общий

$\Rightarrow \frac{S_{ABC}}{S_{KPC}} = \frac{1}{k^2}$; $k = \frac{2}{5}$

$\Rightarrow S_{ABC} = 4 \cdot \frac{25}{4} = 25$

б)

$tg \angle \alpha = \frac{7}{5}$

$\Rightarrow \sin \angle \alpha = \frac{7}{\sqrt{74}}$

$\Rightarrow \cos \angle \alpha = \frac{5}{\sqrt{74}}$

$AC = 2R \sin \angle \alpha$

$25 = \frac{1}{2} H \cdot AC$

$H = \frac{5}{2} PN = \frac{5}{2} h$

$AC = 10x$

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