

Часть 1

Олимпиада: **Математика, 10 класс (1 часть)**

Шифр: **211007041**

ID профиля: **195618**

Вариант 9

задача 3

$$S_{TNC} = S_{TND} (\text{т.к. } \overline{TN} - \text{выс})$$

$$S_{APM} = S_{PMD} (\text{т.к. } PM - \text{выс}) \Rightarrow S_{ABC} = 2 S_{PTNM}$$

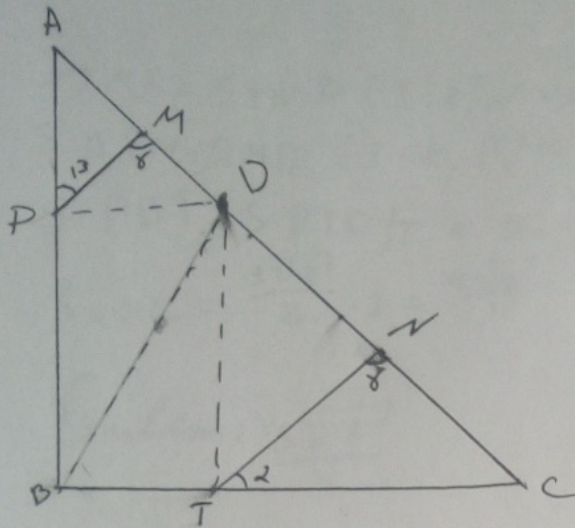
$$S_{PBT} = S_{PTD} (\text{т.к. } PT - \text{выс})$$

$$S_{ABC} = \frac{3\sqrt{7}}{8} \cdot 2 = \frac{3\sqrt{7}}{4}$$

$$\text{Ответ: } \frac{3\sqrt{7}}{4}$$

№ 1

методом (2)



а) Т.к. $MP \parallel NT$ $\angle CNP = \angle CMP = \gamma \Rightarrow \angle TNM = \angle PMA = 180 - \gamma$
 Пусть $\angle NTC = \alpha$, $\angle MPA = \beta \Rightarrow$

2) $\angle BAC = 180 - \beta - (180 - \gamma) = \gamma - \beta$ $\angle BCA = 180 - \alpha - \gamma$

$\angle ABC = 180 - \angle BAC - \angle BCA = \alpha + \gamma - \gamma + \beta = \alpha + \beta$

3) Т.к. BD - медиана ΔBTP - вписан в кр. ур. \Rightarrow

$\Rightarrow \angle BTD = \angle BPD = 90^\circ \Rightarrow \angle NTD = 90 - \alpha$ $\angle MPD = 90 - \beta$

4) $\angle NDT = 180 - (180 - \gamma) - (90 - \alpha) = \alpha + \gamma - 90$ (уг. D T M D)

$\angle PDM = 180 - \gamma - (90 - \beta) = 90 + \beta - \gamma$

$\angle NDT + \angle PDM = 180 - 90 - 90 \Rightarrow \alpha + \gamma - \gamma + \beta + 90 - 90 = 0 \Rightarrow \alpha + \beta = 90$

5) $\angle ABC = \alpha + \beta = 90$ к. м. у.

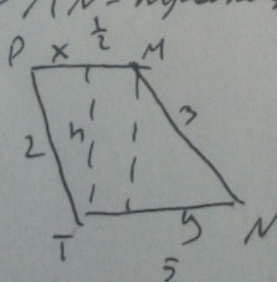
Решение

б) $PM = \frac{1}{2} NT = \frac{5}{2}$ $BD = 2$ т.к. $BTPD$ - прямоугольник $\Rightarrow PT = BD = 2$

Т.к. $DN = NC$ и $\angle DTC = 90^\circ \Rightarrow DN = NC = TN = \frac{5}{2}$

Т.к. $AM = MD$ и $\angle APD = 90^\circ \Rightarrow AM = MD = PM = \frac{1}{2}$

$TPMN$ - трапеция



$\frac{5}{2} - (\frac{1}{2} - x) = y \Rightarrow y - x = 2$

$h^2 + x^2 = 4$

$h^2 + y^2 = 9$

$(y-x)(x+y) = 5$

$x+y = \frac{5}{2} \Rightarrow$

$\begin{cases} x = \frac{1}{4} \\ y = \frac{3}{4} \end{cases} \Rightarrow h = \sqrt{4 - \frac{1}{16}} = \sqrt{\frac{63}{16}}$
 $h = \frac{3\sqrt{7}}{4}$ $S_{TPMN} = \frac{1}{2} \cdot \frac{5\sqrt{7}}{4} = \frac{5\sqrt{7}}{8}$

numerik (1)

N 2

gegeben $a = x + 4$ $b = 6 - x$ u $x \in [-4; 6]$

man $\sqrt{a} + \sqrt{b} = 2\sqrt{ab} - 4 \Rightarrow$

$a + b = x + 4 - x + 6 = 10$

$\Rightarrow (a+b) + 2\sqrt{ab} = 4ab - 16\sqrt{ab} + 16$

$-2\sqrt{ab} = 4ab - 16\sqrt{ab} + 16$

$+\sqrt{ab} = 2ab - 8\sqrt{ab} + 3$

$\Rightarrow 7\sqrt{ab} = 2ab + 3$

$4a^2b^2 + 12ab - 49ab + 9 = 0$

$D = 37^2 - 4^2 \cdot 9 = 1225 = 35^2$

$\Rightarrow (ab)_{1,2} = \frac{7 \pm 35}{8} = \frac{1}{4}, 9$

$24 + 2x - x^2 = \frac{1}{4}, 9$

$x^2 - 2x - 15 = 0$ u $x^2 - 2x - 23\frac{3}{4} = 0$

$D = 4 + 95 = 99 = (3\sqrt{11})^2$

$D = 4 + 60 = 64$

$x_{1,2} = \frac{2 \pm 8}{2} = 5, -3$

$x_{3,4} = \frac{2 \pm \sqrt{99}}{2}$

$(\sqrt{99} < 10 \Rightarrow x_{1,2} \neq \frac{2 \pm 10}{2} = -4, 6 \Rightarrow x_{3,4} \in [-4; 6])$

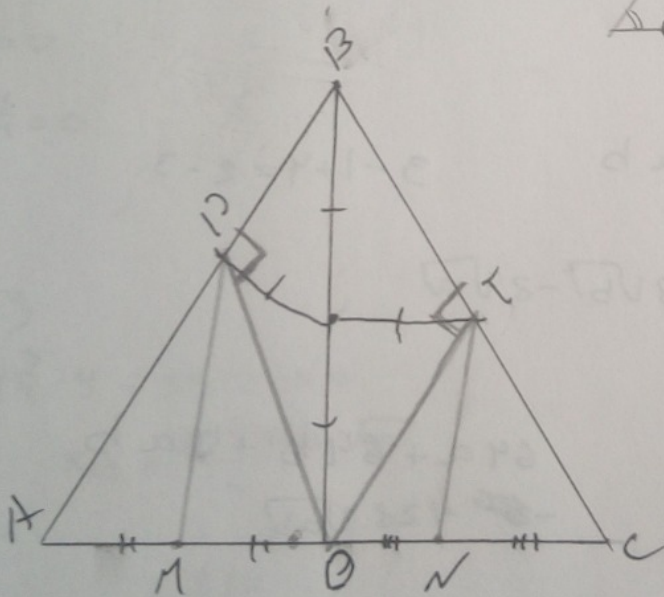
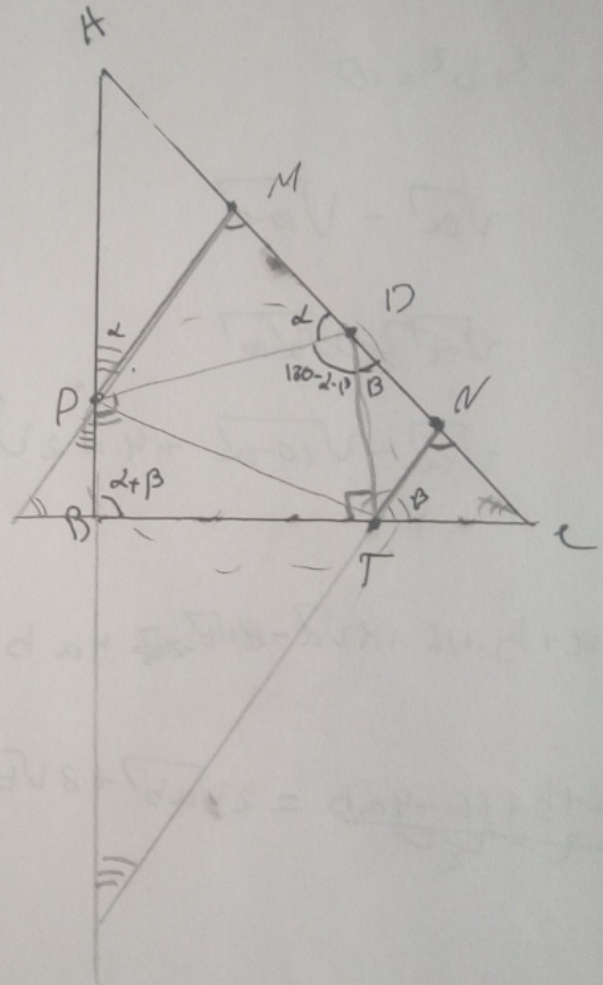
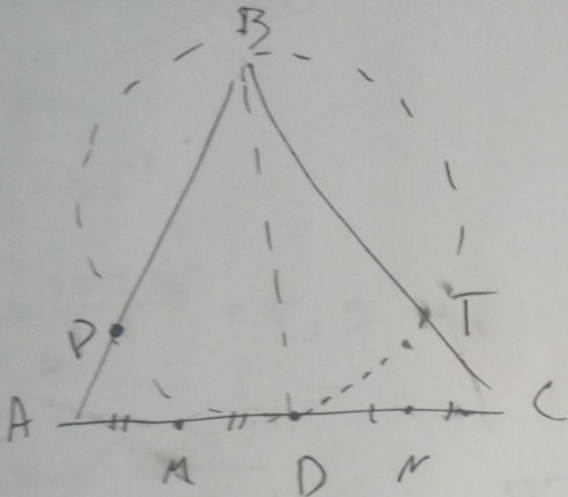
Antwort: $x = -3, 5, \frac{2 + 3\sqrt{11}}{2}, \frac{2 - 3\sqrt{11}}{2}$

чернобыл

$$\sqrt{a} - \sqrt{b} + 4 = 2\sqrt{ab}$$

$$a + b = 2\sqrt{ab}$$

$$2\sqrt{ab} - \sqrt{a} + \sqrt{b} = \sqrt{a}(2\sqrt{b} - 1) = (\sqrt{a} + 1)(\sqrt{b} - \frac{1}{2}) = 3\frac{3}{4}$$



0 → 2

1 + 1 + 1 = 1

уменьш

211007041 (U195618 M1277528)

$$10 - 2\sqrt{ab} = 4ab + 16 - 16\sqrt{ab}$$

$$4ab + 6 = 14\sqrt{ab}$$

$$2ab + 3 = 7\sqrt{ab}$$

$$4a^2b^2 + 6ab + 9 = 49ab$$

$$4x^2 - 37x + 9 = 0$$

$$D = 37^2 - 36 \cdot 4 = 35^2$$

$$\begin{array}{r} 37 \\ \times 37 \\ \hline 259 \\ 111 \\ \hline 1369 \\ - 144 \\ \hline 1225 \\ 35^2 \end{array}$$

$$x_{1,2} = \frac{37 \pm 35}{2} = \frac{1}{4}; 9$$

$$24 + 2x - x^2 = \frac{1}{4}$$

$$x^2 - 2x - 15 = 0$$

$$x^2 - 2x - 23\frac{3}{4} = 0$$

$$D = 4 + 60 = 8^2$$

$$x_{1,2} = \frac{2 \pm 8}{2} = 5; -3$$

$$D = 4 + \bullet \quad 2 \cdot 4\frac{3}{4} \cdot 4 = 99 \approx 100$$

$$x_{1,2} = \frac{2 \pm \sqrt{99}}{2}$$

$$2 \cdot 4 \cdot 4 = 80 + 16 = 96$$

$$\frac{2 \pm 10}{2} = 6; -4$$

$$\sqrt{99} < 10$$

$$0 \rightarrow 2$$

$$1 + 1 + 4 = 1$$

черновик

$$10 - 2\sqrt{ab} = 4ab + 16 - 16\sqrt{ab}$$

$$4ab + 6 = 14\sqrt{ab}$$

$$2ab + 3 = 7\sqrt{ab}$$

$$4a^2b^2 + 6ab + 9 = 49ab$$

$$4x^2 - 37x + 9 = 0$$

$$D = 37^2 - 36 \cdot 4 = 35^2$$

$$x_{1,2} = \frac{37 \pm 35}{2} = \frac{1}{4}; 9$$

$$\begin{array}{r}
 37 \\
 \times 37 \\
 \hline
 259 \\
 111 \\
 \hline
 1369 \\
 -144 \\
 \hline
 1225 \\
 35^2
 \end{array}$$

$$24 + 2x - x^2 = \frac{1}{9}$$

$$x^2 - 2x - 15 = 0$$

$$x^2 - 2x - 23\frac{3}{4} = 0$$

$$D = 4 + 60 = 8^2$$

$$x_{1,2} = \frac{2 \pm 8}{2} = 5; -3$$

$$D = 4 + 2 \cdot 23\frac{3}{4} \cdot 4 = 99 \approx 100$$

$$x_{1,2} = \frac{2 \pm \sqrt{99}}{2}$$

$$24 \cdot 4 = 80 + 16 = 96$$

$$\sqrt{99} < 10$$

$$\frac{2 \pm 10}{2} = 6; -4$$

сепробир

$$a - b + 4 = 2ab$$

$$\sqrt{x+4} = \sqrt{6-x}$$

$$a^2 = x+4$$

$$b^2 = 6-x$$

$$a^2 + b^2 = 10$$

$$\sqrt{a} - \sqrt{10-a}$$

$$\sqrt{a} + \sqrt{a-10}$$

$$\sqrt{a} - \sqrt{10-a} + 4 = 2\sqrt{a(10-a)}$$

$$a+b+16+8\sqrt{a}-8\sqrt{b}-2\sqrt{ab}=4ab$$

$$3-1+4=2-3$$

$$a+b+16-4ab = 2\sqrt{ab} + 8\sqrt{b} - 8\sqrt{a}$$

$$64a + 64b + 4ab \\ - 128\sqrt{ab}$$

$$16 = 2ab + a + b$$

$$4 = 2\sqrt{ab} - \sqrt{a} + \sqrt{b}$$

$$4 + \sqrt{a} = 2\sqrt{ab} + \sqrt{b}$$

$$16 = 2ab + a + b - 4a\sqrt{b}$$

$$16 + a + 8\sqrt{a} = 2ab + b + 4b\sqrt{a}$$

$$16 + a - 2ab - b = 4(b\sqrt{a} - 2\sqrt{a})$$

Часть 2

Олимпиада: **Математика, 10 класс (2 часть)**

Шифр: **211007041**

ID профиля: **195618**

Вариант 9

$$\begin{cases} x^4 + y^4 + 3x^2y^2 = 5 \\ \frac{2}{x^2 + y^2} + x^2y^2 = 2 \end{cases} \quad \sim y \quad \text{мембер}$$

$$\text{Пусть } \begin{cases} x^2 + y^2 = a \\ x^2y^2 = b \end{cases} \Rightarrow \begin{cases} a^2 + b = 5 \\ \frac{2}{a} + b = 2 \end{cases} \Rightarrow a^2 - \frac{2}{a} = 3 \quad a^3 - 2 = 3a$$

$a > 0$ т.к. сумма квадратов

$$a \neq 0 \quad \text{т.к. } \frac{2}{a} \text{ определено} \quad a^3 - 3a - 2 = 0$$

$$(a+1)^2(a-2) = 0 \Rightarrow a = -1; 2 \Rightarrow a = 2 \Rightarrow$$

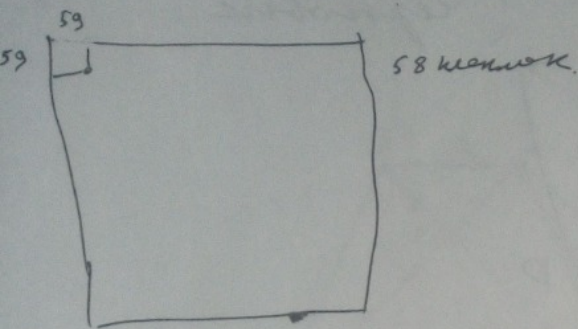
$$\Rightarrow 2^2 + b = 5 \Rightarrow \begin{cases} b = 1 \\ a = 2 \end{cases} \Leftrightarrow \begin{cases} x^2y^2 = 1 \\ x^2 + y^2 = 2 \end{cases} \Leftrightarrow \begin{cases} x^2y^2 = 1 \\ x^2 = 2 - y^2 \end{cases} \Rightarrow \begin{cases} y^4 - 2y^2 + 1 = 0 \\ x^2 + y^2 = 2 \end{cases}$$

$$(y^2 - 1)^2 = 0 \Rightarrow \begin{cases} y = \pm 1 \\ x = \pm 1 \end{cases}$$

①

Ответ: $(1; 1), (1; -1), (-1; 1), (-1; -1)$.

черновик



$$\boxed{58 \cdot 57}$$

$$\boxed{2 \cdot 58 \cdot (58^2 - 58)}$$

$$2 \cdot 58 \cdot (58^2 - 2 \cdot 58 - 56)$$

$$(x^2 + y^2)^2 + x^2 y^2 = 5$$

$$\frac{2}{x^2 + y^2} + x^2 y^2 = 2$$

$$(x^2 + y^2)^3 - 3(x^2 + y^2) + 2 = 0$$

$$a^3 - 3a + 2 = 0$$

$$a=1 \quad (a-1)(a^2+a-2) = (a-1)(a-1)(a+2) = (a-1)^2(a+2)$$

$$x^2 + y^2 = 1 \quad -2$$

$$x^2 + y^2 = 1 \quad x^2 y^2 = 4$$

$$(a+1)(a^2 - a - 2) = (a+1)^2(a-2)$$

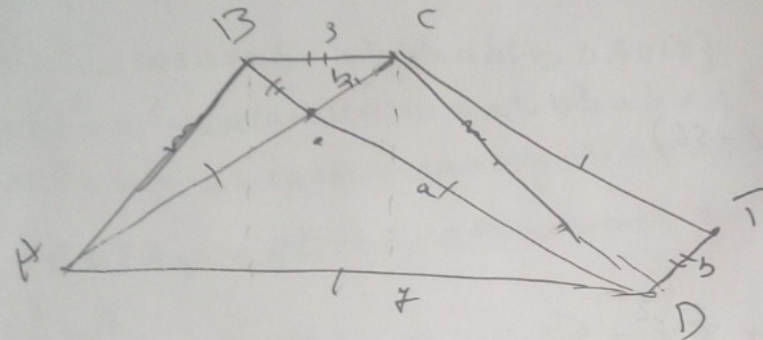
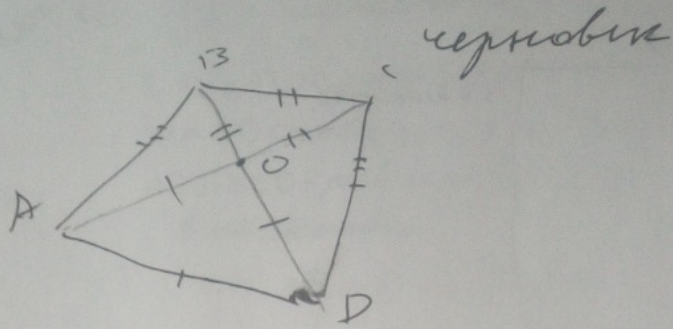
$$x^2 + y^2 = 2 \quad x^2 y^2 = 1$$

$$x^2 = 2 - y^2 \quad 2y^2 - y^4 = 1 \quad y^4 - 2y^2 + 1 = 0$$

$$(y^2 - 1)^2 \quad (y+1)^2 (y-1)^2$$

$$y = \pm 1 \quad x = \pm 1$$

$$\begin{array}{r} 5 \\ \times 58 \\ \hline 406 \\ 290 \\ \times 5306 \\ \hline 113 \\ 9918 \\ 3306 \\ 5306 \\ \hline 383548 \end{array}$$



$$a^2 + b^2 + ab = 2^2$$

~~$$2ab + b^2$$~~

$$(a+b)^2 + b^2 - (a+b)b$$

$$a^2 + 2b^2 + 2ab - ab - b^2 = a^2 + b^2 + ab = 2^2$$

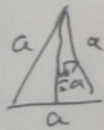
$$h^2 = \sqrt{100 - 25} = 5\sqrt{3}$$

$$9 + 49 + 21$$

$$58 + 21 = 79$$

$$2 = \sqrt{79}$$

$$\frac{105\sqrt{3}}{2}$$



$$\frac{\sqrt{3}}{2} a^2$$

$$\frac{79\sqrt{3}}{2}$$

в квадрате 59×59 - 58^2 узлов ^{и 5 ребер}

1) Пусть узел находится на $y=x$ или $y=59-x$ - I (первый)

\Rightarrow 1 случай I и II узел находится на $y=x$ и $y=59-x \Rightarrow$

\Rightarrow способов - $58 \cdot (58-1) = 58 \cdot 57$

2) случай II не проходит на $y=x$ и $y=59-x \Rightarrow$

кол-во мест для I $58 \cdot 2$ а для II $58^2 - 58 \cdot 2 - 56 =$
($y=x$ и $y=59-x$) (верт
 ось)

$$= 56 \times 57 \times 58 \times 2$$

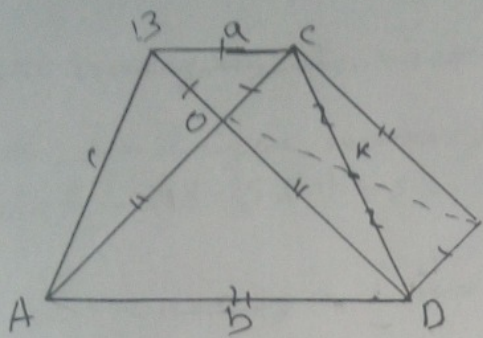
$$\text{всего } 58 \times 57 (112 + 1) = 58 \times 57 \cdot 113$$

(если $x \in \mathbb{N}$ и $x \leq 58$) (при $x=0$ ответ: 0)

2

Ответ: $373 \cdot 578$.

№6 Числовик



Т.к. $\triangle BOC$ и $\triangle AOD$ - равносторонние
 $\Rightarrow \angle OBC = \angle OCB = \angle CAD = \angle BDA = 60^\circ$
 $\Rightarrow ABCD$ - μ -б трапеция
 вписанная

T (т.к. $\triangle COD = \triangle BOA$)

$\triangle CTD = \triangle COD$ и $TD = CO = b$ $CT = OD = a$ и $CD = c$

1) $c^2 = a^2 + b^2 - 2 \cos 120 ab = a^2 + b^2 + ab$ (из $\triangle AOB$)
 $BT^2 = (a+b)^2 + a^2 - 2a(a+b) \cos 60 = a^2 + b^2 + ab = c^2$
 $AT = (a+b)^2 + b^2 - 2b(a+b) \cos 60 = a^2 + b^2 + ab = c^2$
 $\Rightarrow AB = BT = TA \Rightarrow \triangle ABT$ - равносторонний

2) $S_{\triangle ABT} = \frac{\sqrt{3}}{2} c^2$
 $S_{ABCD} = \frac{a+b}{2} \cdot h$ $h = \sqrt{(a+b)^2 - 5^2} = 5\sqrt{3}$
 $S_{ABCD} = 25\sqrt{3}$

$c^2 = 9 + 49 + 21 = 79$
 $S_{\triangle ABT} = \frac{79}{2} \sqrt{3}$

(3)

$\frac{S_{\triangle ABT}}{S_{ABCD}} = \frac{79\sqrt{3}}{50\sqrt{3}} = \frac{79}{50} = 1,58$

Ответ: $\frac{S_{\triangle ABT}}{S_{ABCD}} = 1,58$

① $CK = KD$ $\angle CKO = \angle TKD$ (верт) и $OK = OT$ (из угла) \Rightarrow
 $\triangle COK = \triangle TKD \Rightarrow CO = TD$ и $\angle TDC = \angle OCK$, CD - общ $\Rightarrow \triangle COD = \triangle CTD$