

Часть 1

Олимпиада: **Математика, 10 класс (1 часть)**

Шифр: **211007765**

ID профиля: **870694**

Вариант 12

$$2\sqrt{2,5-\sqrt{6}} = \sqrt{6} - 2$$

Числовые

$$4(2,5-\sqrt{6}) = 6 - 4\sqrt{6} + 4$$

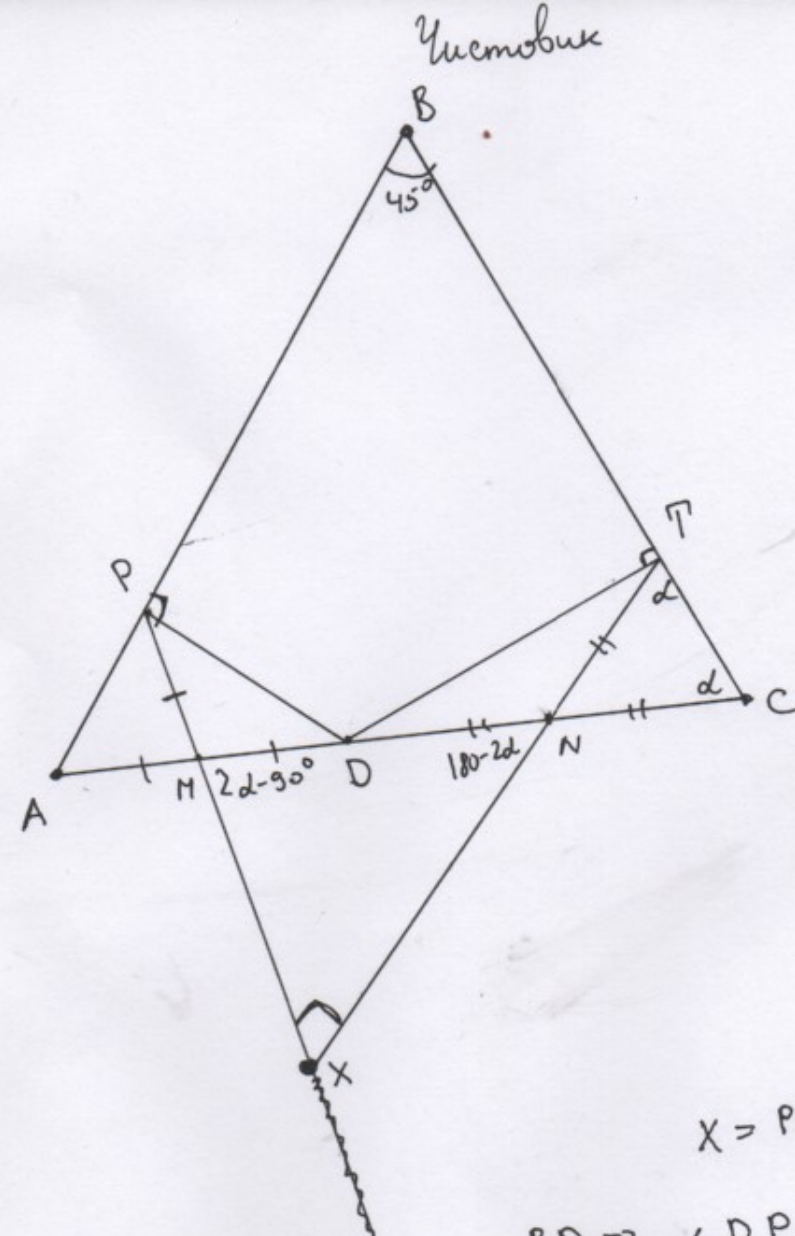
$$10 - 4\sqrt{6} = 10 - 4\sqrt{6}$$

Ч.Т.Р

Во время проверки в левой и в правой части
любого равенства выражение было $\geq 0 \Rightarrow$ если
возводим в квадраты и получаем равенство и
получаем все верно. (Заметим, что $2,5-\sqrt{6} > 0$; $1,5-\sqrt{6} > -1$
т.к. $6,25 > 6$)

Ответ: $3; 1,5-\sqrt{6}$

√1



$X = PM \cap NT ; \angle PXT = 90^\circ$

(2)

DP, BT вискам на оуп. с гуан $BD \Rightarrow \angle DPB = \angle DTB = 90^\circ$
 Потога M - середина вн. пр. пр. $APD \Rightarrow AM = MD = MP$
 Аналогично $TN = ND = NC$

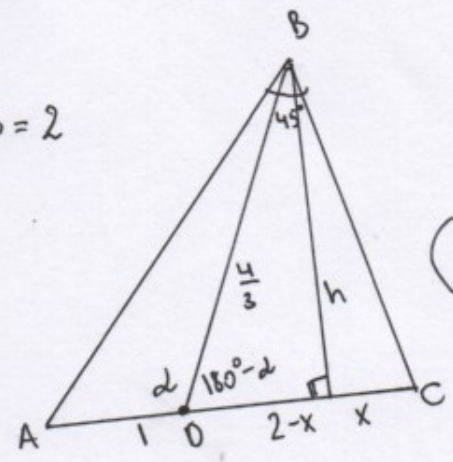
Пучко $\angle C = \alpha \Rightarrow \angle NTC = \alpha$ (м.к. $TNC - \mu(\delta) \Rightarrow \angle MNX = 180 - 2\alpha$
 (как берем. с TNC)

Потога $\angle XMN = 180^\circ - 90^\circ - (180 - 2\alpha) = 2\alpha - 90^\circ \Rightarrow \angle MAP = \angle MPA = \frac{180^\circ - (2\alpha - 90^\circ)}{2} = 135 - \alpha$

$\angle A = 135 - \alpha \Rightarrow \angle B = 45^\circ$
 $\angle C = \alpha$

$1 = TN = DN = NC \Rightarrow CD = 2$

(3) $\frac{1}{2} PM = AM = MD \Rightarrow AD = 1$



(3)

число букв

$$(2-x)^2 + h^2 = \frac{16}{9} \quad (\text{по м. Пифагора})$$

$$x^2 - 4x + 4 + h^2 = \frac{16}{9}$$

$$h^2 = 4x - x^2 - \frac{20}{9}$$

$$BC^2 = h^2 + x^2 = 4x - \frac{20}{9}$$

$$AB^2 = (6-x)^2 + h^2 = 9 + x^2 - 6x + 4x - x^2 - \frac{20}{9} = \frac{61}{9} - 2x$$

$$AC^2 = 2x + \frac{41}{9} - 2 \cdot \frac{\sqrt{2}}{2} \cdot \sqrt{4x - \frac{20}{9}} \cdot \sqrt{\frac{61}{9} - 2x}$$

(по м. кос.) $2x < \frac{61}{9}$
 $x < \frac{61}{18}$
 $x < 3.4$

$$9 \cdot \left(\frac{41}{9} - 2x \right) = -\sqrt{2} \cdot \sqrt{4x - \frac{20}{9}} \cdot \sqrt{\frac{61}{9} - 2x}$$

$$\left(\frac{40}{9} - 2x \right)^2 = 2 \cdot \left(4x - \frac{20}{9} \right) \left(\frac{61}{9} - 2x \right)$$

$$\frac{1600}{81} + 4x^2 - \frac{160}{9}x = -816x^2 + 2 \cdot \left(\frac{4 \cdot 61 + 20 \cdot 2}{9} \right) x - \frac{20 \cdot 61 \cdot 2}{81}$$

$$20x^2 - \frac{160 + 1680 + 80}{9}x + \frac{1600 + 40 \cdot 61}{81} = 0$$

$$20x^2 - \frac{1720}{9}x + \frac{4040}{81} = 0$$

~~$$D = \sqrt{2 \cdot 928704 - 26178200} = 16749504$$~~

$$1620x^2 - 6552x + 4040 = 0$$

$$3405x^2 - 91 \cdot 18x + 1010 = 0$$

$$81 \cdot 5x^2 - 91 \cdot 2 \cdot 3^2 x + 101 \cdot 2 \cdot 5 = 0$$

$$D = 91^2 \cdot 2^2 \cdot 3^4 - 4 \cdot 2 \cdot 5^2 \cdot 3^4 \cdot 101 = 4 \cdot 81 (91^2 - 5050) = 4 \cdot 81 \cdot 8231$$

$$X = \frac{91 \cdot 2 \cdot 3^2 \pm 3^2 \cdot 2 \cdot \sqrt{3231}}{81 \cdot 10} = \frac{91 \pm \sqrt{3231}}{9 \cdot 5}$$

$60 < x < 4$

$$X = \frac{91 - \sqrt{3231}}{9 \cdot 5}$$

$$h^2 = \frac{361 - 2 \cdot 91 \cdot 5 + 3231 - 182 \cdot \sqrt{3231}}{45^2} = \frac{20}{9}$$

(4)

$$S^1 = \frac{\sqrt{\frac{364 - 4\sqrt{3231}}{45}} - \frac{\sqrt{91^2 + 3231 - 182 \cdot \sqrt{3231}} - \frac{20}{9} \cdot 3}{45}}{2} = \frac{\sqrt{368 + 2\sqrt{3231}} \cdot 3}{30} = \frac{\sqrt{368 + 2\sqrt{3231}}}{30}$$

Jawab: $\frac{\sqrt{368 + 2\sqrt{3231}}}{30}$

5

$$\sqrt{x+1} + \sqrt{4-x} + 3 = 2\sqrt{4+3x-x^2}$$

$$1-2+3=2 \cdot 2$$

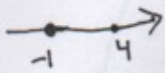
$$0.2.3 \quad 4 \geq x \geq -1$$

$$1-2+3$$

$$-x^2+3x+4 \geq 0$$

$$x^2-3x-4 \leq 0$$

$$(x-4)(x+1) \leq 0$$



$$1,5 + \sqrt{6} < 4$$

$$\sqrt{6} < 2,5$$

$$\sqrt{x+1} = a$$

$$\sqrt{4-x} = b$$

$$-a^2 = -x-1$$

$$1,5 - \sqrt{6} < 1 \quad 5 - a^2 = b^2$$

$$\sqrt{6} > 2,5 \quad b = \sqrt{5-a^2}$$

$$a - b + 3 = 2ab$$

$$-1 < 1,5 - \sqrt{6} < 1$$

$$\sqrt{6} > 2,5$$

$$a - \sqrt{5-a^2} + 3 = 2b\sqrt{5-a^2}$$

$$a+3 = b(2a+1)$$

$$2a-6 = 2b(2a+1)$$

$$(2b-1)(2a+1) = 5$$

$$\sqrt{3} - \sqrt{2+3} = 2\sqrt{6}$$

$$2-1+3=4$$

$$\frac{16x^3 - 96x^2 + 84x + 180}{16x^3 - 48x^2} + \frac{3x+3}{16x^2 - 48x - 60} = \frac{-48x^2 + 84x + 180}{-48x^2 + 144x - 60x + 180}$$

$$384 \overline{) 4}$$

$$384 : 4 = 96$$

$$96 : 4 = 24$$

$$24$$

$$24 : 4 = 6$$

$$\sqrt{x+1} + 3 = 2\sqrt{4+3x-x^2} + \sqrt{4-x}$$

$$x+1+9+6\sqrt{x+1} = 4(4+3x-x^2) + 4-x +$$

$$+ \sqrt{4-x}(\sqrt{x+1})$$

$$x+1+4-x+9+2\sqrt{4+3x-x^2}$$

$$(2\sqrt{x+1}-1)(2\sqrt{4-x}+1) = 5$$

$$\sqrt{x+1} = 16 + 12x - 4x^2 + 4 - x - x - 10$$

$$\sqrt{x+1} = -4x^2 + 10x + 10$$

$$(x+1) = 16x^4 + 100x^2 + 100 - 80x^3 + 200x - 80x^2$$

$$16x^4 + 20x^2 - 80x^3 + 200x + 100 = 0$$

$$16x^4 - 81x^3 + 15x^2 + 192x + 96 = 0$$

$$16x^4 - 96x^3 + 84x^2 + 180x = 0$$

$$x(16x^3 - 96x^2 + 84x + 180) = 0$$

$$x(x-3)(16x^2 - 96x + 180) = 0$$

$$144 + 240 = 384$$

$$\sqrt{6} = x$$

$$2 + 2\sqrt{2.5-x} = x$$

$$2\sqrt{2.5-x} = x-2$$

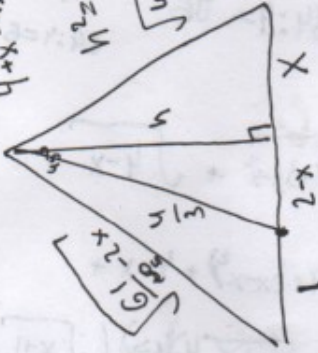
$$4(2.5-x) = x^2 - 4x + 4$$

$$10 - 4x = x^2 + 4 + x^2 - 6x$$

$$(2-x)^2 = \frac{16}{9}$$

$$2-x = \frac{4}{3}$$

$$x = \frac{2}{3}$$



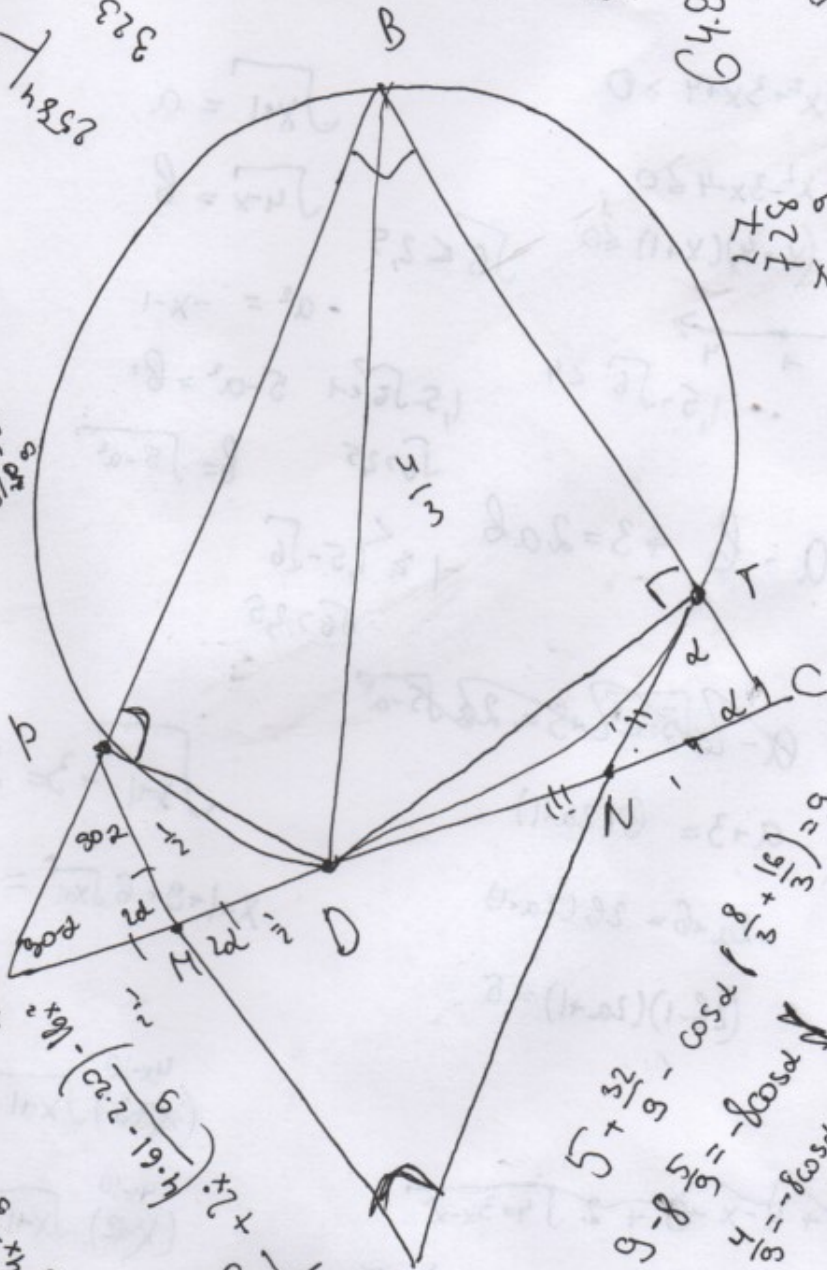
$$\begin{array}{r} 2584 \\ 323 \overline{) 1052} \\ \underline{655} \\ 397 \end{array}$$

$$\begin{array}{r} 2584 \\ 32 \overline{) 852} \\ \underline{64} \\ 212 \\ \underline{184} \\ 28 \end{array}$$

$$\begin{array}{r} 488 \\ + 260 \\ \hline 748 \end{array}$$

$$\begin{array}{r} 4061 \\ 18 \overline{) 576} \\ \underline{36} \\ 216 \\ \underline{180} \\ 36 \end{array}$$

$$50.52 \approx 260$$



$$\begin{array}{r} 2559 \\ 5 \overline{) 877} \\ \underline{255} \\ 622 \\ \underline{622} \\ 0 \end{array}$$

6/1

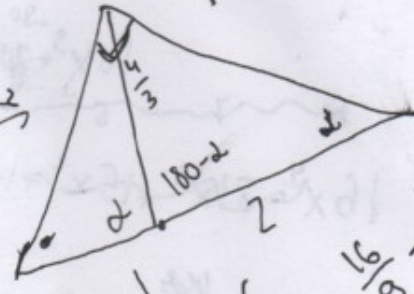
$$\frac{5 \cdot 16}{9} = \frac{80}{9}$$

$$\frac{48}{9}$$

$$2x + \frac{16}{9} = \frac{80}{9}$$

$$2x = \frac{80}{9} - \frac{16}{9} = \frac{64}{9}$$

$$x = \frac{32}{9}$$



$$5 + \frac{32}{9} - \cos \alpha \left(\frac{5}{3} + \frac{16}{9} \right) = 9$$

$$\frac{5}{3} - \cos \alpha = \frac{9 - 5 - \frac{32}{9}}{\frac{5}{3} + \frac{16}{9}}$$

$$\frac{5}{3} - \cos \alpha = \frac{\frac{45 - 32 - 32}{9}}{\frac{5}{3} + \frac{16}{9}}$$

$$\frac{5}{3} - \cos \alpha = \frac{13}{9} \cdot \frac{3}{9} = \frac{13}{27}$$

$$\cos \alpha = \frac{5}{3} - \frac{13}{27} = \frac{35}{27}$$

$$\left(1 + \frac{16}{9} - 2 \cos \alpha \cdot \frac{5}{3} \right) + \left(4 + \frac{16}{9} - 2 \cos \alpha \cdot \frac{8}{3} \right) = 9$$

$$\left(1 + \frac{16}{9} + 1 - \frac{10}{3} \cos \alpha \right) + \left(4 + \frac{16}{9} + 1 - \frac{16}{3} \cos \alpha \right) = 9$$

$$\left(2 + \frac{16}{9} - \frac{10}{3} \cos \alpha \right) + \left(5 + \frac{16}{9} - \frac{16}{3} \cos \alpha \right) = 9$$

$$\left(7 + \frac{32}{9} - \frac{26}{3} \cos \alpha \right) = 9$$

$$\frac{26}{3} \cos \alpha = 7 + \frac{32}{9} - 9 = \frac{63}{9} + \frac{32}{9} - \frac{81}{9} = \frac{14}{9}$$

$$\cos \alpha = \frac{14}{9} \cdot \frac{3}{26} = \frac{14}{42} = \frac{1}{3}$$

$$\frac{101.85 - 1}{10.9} = \frac{100.85}{10.9} = 9.25$$

$$\frac{10.9}{10.9} = 1$$

$$\begin{array}{r} 368 \\ 16012 \\ \hline 11512 \\ 8281 \\ \hline 4500 \end{array}$$

$$\frac{18}{15} = 1.2$$

$$\begin{array}{r} 16380 \\ 1456 \\ \hline 1826 \\ \times 45 \\ \hline 364 \\ 32 \end{array}$$

$$|6380 - 8281 - 3231 - 4500 =$$

=

$$= \frac{364 \cdot 45 - 91^2 - 3231 - 180 \cdot 25 + 2 \sqrt{3231}}{45^2}$$

$$= \frac{364 \cdot 45 - 180 \sqrt{3231} - 91^2 - 3231 + 182 \sqrt{3231} - 20 \cdot 9 \cdot 25}{45^2}$$

$$2a^2 - 2ax$$

3

$$x = \frac{101 \cdot 22 - 1}{18 \cdot 81} = \frac{2222 - 1}{1458} = \frac{2221}{1458}$$

$$\frac{16}{16} \times \frac{81}{81} = \frac{1296}{1296}$$

$$\frac{16}{16} \times \frac{101}{101} = \frac{1616}{1616}$$

$$91^2 \cdot 18^2 - 4 \cdot 3^4 \cdot 5 \cdot 101 \cdot 2 \cdot 5 =$$

$$= 91^2 \cdot 18^2 - 2^3 \cdot 3^4 \cdot 5^2 \cdot 101 =$$

$$= 2^2 \cdot 3^4 (81^2 - 2 \cdot 5^2 \cdot 101)$$

$$\frac{53}{159} = \frac{265}{265}$$

$$\frac{16749504}{26179200} = \frac{42928704}{26179200}$$

$$\frac{51}{51} \times \frac{51}{51} = \frac{261792}{261792}$$

$$\frac{42928704}{39312} = \frac{32760}{32760}$$

3231

$$\frac{8}{20} \times \frac{20}{20} = \frac{16}{160}$$

$$\frac{6552}{6552} = \frac{13104}{13104}$$

$$\frac{89}{89} \times \frac{55}{55} = \frac{4895}{4895}$$

$$\frac{4040}{4040}$$

$$4287374$$

$$\frac{1638}{1638} \times \frac{1638}{1638} = \frac{2683044}{2683044}$$

$$\frac{1638}{81} \times \frac{728}{728} = \frac{1192464}{1192464}$$

$$91^2 = \frac{81}{81} \times \frac{81}{81} = \frac{6561}{6561}$$

$$= \frac{819}{819} - 5050 = 3231$$

$$\frac{182}{182} \times \frac{364}{364} = \frac{66596}{66596}$$

$$\frac{34225}{34225}$$

$$\frac{91 \cdot 8 \cdot 9}{91 \cdot 8 \cdot 9}$$

2

$$g = 684 +$$

$$9 \cdot 56 \cos^2 \alpha + 9 \cdot 76 \cos \alpha - 323 = 0$$

$$9 \cdot 448 \cos^2 \alpha + 9 \cdot 16 \cdot 38 \cos \alpha - 2584 = 0$$

$$448 \frac{9}{8} \cos^2 \alpha + \frac{64+34 \cdot 16}{9} \cos \alpha - \frac{2584}{9} = 0$$

$$\left(64 - \frac{9}{128} \right) \cos^2 \alpha + \left(\frac{64}{9} + \frac{102 \cdot 16}{27} \right) \cos \alpha + \frac{18}{81} - \frac{50 \cdot 52}{81} = 0$$

$$64 \cos^2 \alpha + \frac{9}{64} \cos \alpha + \frac{81}{16} = \frac{50 \cdot 52}{16} \cos \alpha + \frac{81}{16} - \frac{50 \cdot 16}{27} \cos \alpha - \frac{16 \cdot 52}{27} \cos \alpha + \frac{9}{128} \cos^2 \alpha$$

$$\frac{81}{16} + 64 \cos^2 \alpha + \frac{9}{64} \cos \alpha = 2 \cdot \left(\frac{9}{25} - \frac{3}{8} \cos \alpha \right) \left(\frac{9}{52} - \frac{3}{16} \cos \alpha \right)$$

$$\frac{9}{4} + 8 \cos \alpha = - \sqrt{2} \cdot \sqrt{\frac{9}{25} - \frac{3}{8} \cos \alpha} \cdot \sqrt{\frac{9}{52} - \frac{3}{16} \cos \alpha}$$

$$9 = \frac{9}{17} - 8 \cos \alpha - \sqrt{2} \cdot \sqrt{\frac{9}{25} - \frac{3}{8} \cos \alpha} \cdot \sqrt{\frac{9}{52} - \frac{3}{16} \cos \alpha}$$

$$BC^2 = \frac{9}{52} - \frac{3}{16} \cos \alpha$$

$$AB^2 = \frac{9}{25} - \frac{3}{8} \cos \alpha$$

$$AC^2 = AB^2 + BC^2 - 2 \cos 45^\circ \cdot AB \cdot BC$$

$$BC^2 = 4 + \frac{9}{16} + 2 \cos \alpha \cdot \frac{3}{8}$$

$$AB^2 = 1 + \frac{9}{16} - 2 \cos \alpha \cdot \frac{3}{4}$$

10 m. noc.

$$\cdot \cos \alpha - \cos (150^\circ - \alpha) \text{ (korrigieren)}$$

Wiederholen

cos α < 0 (m.k. spiegelbild)

(J)

$$f = 0.81, +$$

$$0 = 2.52 - 2.52 + 2.52 - 2.52$$

$$0 = 2.52 - 2.52 + 2.52 - 2.52$$

$$0 = \frac{2}{2.52} - \frac{2}{2.52} + \frac{2}{2.52} - \frac{2}{2.52}$$

$$ax^2 + 4a^2x - ay + 4a^3 + 2 = 0$$

$$2a^2 - 2ax - 6ay + x^2 + 2xy + 3y^2 = 0$$

$$\frac{18}{10} + \frac{2}{10} + \frac{2}{10} = 5 \cdot \left(\frac{2}{10} - \frac{2}{10} \right) \left(\frac{2}{10} - \frac{2}{10} \right)$$

484

$$368 + 2.52 = 368 + 118 = 486$$

$$0 = \frac{2}{10} - 2 \cos 2\theta - \frac{2}{10} \cdot \frac{2}{10} - \frac{2}{10} \cdot \frac{2}{10}$$

$$BC = \frac{2}{10} - \frac{2}{10} \cos 2\theta$$

$$AB = \frac{2}{10} - \frac{2}{10} \cos 2\theta$$

$$AC = AB + BC - \cos 2\theta \cdot AB \cdot BC$$

$$BC = 1 + \frac{2}{10} + 5 \cos 2\theta \cdot \frac{2}{10}$$

$$AB = 1 + \frac{2}{10} - 5 \cos 2\theta \cdot \frac{2}{10}$$

$$\begin{array}{r} 3481 \\ 2784 \\ \hline 295 \\ 1 \\ \hline 559 \\ \times 59 \\ \hline 659 \\ 275 \\ \hline 28 \end{array}$$

Умножение

√2.

$$\sqrt{x+1} + \sqrt{4-x} + 3 = 2\sqrt{4+3x-x^2} = 2\sqrt{4-x} \cdot \sqrt{x+1} \quad \text{O.D. 3. } -1 \leq x \leq 4$$

$$\sqrt{x+1} + 3 = 2\sqrt{4-x} \cdot \sqrt{x+1} + \sqrt{4-x} \quad \text{Возведем обе части в квадрат.}$$

$$x+1+9+6\sqrt{x+1} = 4 \cdot (4+3x-x^2) + 4-x + 4 \cdot (4-x) \cdot \sqrt{x+1}$$

$$(6 - (16-4x))\sqrt{x+1} = 16+12x-4x^2-x-10+4-x$$

$$(4x-10) \cdot \sqrt{x+1} = -4x^2+10x+10 \quad \text{Возведем обе части в квадрат}$$

$$(16x^2-80x+100)(x+1) = 16x^4+100x^2+100-80x^3-80x^2+200x$$

$$16x^3-80x^2+100x+16x^2-80x+100 = 16x^4-80x^3+20x^2+200x+100$$

$$16x^4-96x^3+84x^2+180x=0$$

Методом подбора найдем корни 0, 3

$$x(x-3)(16x^2-48x-60)=0$$

$$x(x-3)(4x^2-12x-15)=0$$

$$4x^2-12x-15=0$$

$$D = 144 + 240 = 384 = (8\sqrt{6})^2$$

$$x = \frac{12 \pm 8\sqrt{6}}{8} = 1,5 \pm \sqrt{6}$$

$$\begin{array}{r} 16x^3 - 96x^2 + 84x + 180 \quad | \quad x-3 \\ \underline{-16x^3 - 48x^2} \\ -48x^2 + 84x + 180 \\ \underline{-48x^2 + 144x} \\ -60x + 180 \\ \underline{-60x + 180} \\ 0 \end{array}$$

Проверка

$$x=3 \quad 2-1+3=2 \cdot 2 \cdot 1 \quad (6)$$

$$x = 1,5 + \sqrt{6}$$

$$\sqrt{2,5+\sqrt{6}} - \sqrt{2,5-\sqrt{6}} + 3 = 2 \cdot \sqrt{(2,5+\sqrt{6})(2,5-\sqrt{6})} = 2 \cdot \sqrt{6,25-6} = 2 \cdot 0,5 = 1$$

$$\sqrt{2,5+\sqrt{6}} + 2 = \sqrt{2,5-\sqrt{6}} \quad (?!) \quad (\text{т.к. корень справа } < \text{ корень слева, еще + прибавление 2})$$

$$x = 1,5 - \sqrt{6}$$

$$\sqrt{2,5-\sqrt{6}} - \sqrt{2,5+\sqrt{6}} + 3 = 2 \cdot \sqrt{(2,5-\sqrt{6})(2,5+\sqrt{6})} = 2 \cdot \sqrt{6,25-6} = 2 \cdot 0,5 = 1$$

$$\sqrt{2,5-\sqrt{6}} + 2 = \sqrt{2,5+\sqrt{6}}$$

$$2,5-\sqrt{6} + 4 + 4\sqrt{2,5-\sqrt{6}} = 2,5+\sqrt{6}$$

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$$2 + 2\sqrt{2,5-\sqrt{6}} = \sqrt{6}$$

1

Часть 2

Олимпиада: **Математика, 10 класс (2 часть)**

Шифр: **211007765**

ID профиля: **870694**

Вариант 12

Умножение

$\sqrt{4}$

$$x^2 y^2 = a$$

$$a \neq 0$$

$$x^2 y^2 = b$$

$$\begin{cases} \frac{1}{a} + b = \frac{5}{4} \\ 2(x^2 + y^2)^2 + x^2 y^2 = \frac{9}{4} \end{cases} \Rightarrow \begin{cases} \frac{1}{a} - b = \frac{5}{4} \\ 2a^2 + b = \frac{9}{4} \end{cases} \Rightarrow \begin{cases} b = \frac{5}{4} - \frac{1}{a} \\ b = \frac{9}{4} - 2a^2 \end{cases}$$

$$\begin{cases} b = \frac{5}{4} - \frac{1}{a} \\ 2a^2 - \frac{1}{a} = \frac{9}{4} - \frac{5}{4} \end{cases} \Rightarrow \begin{cases} b = \frac{5}{4} - \frac{1}{a} \\ 2a^3 - 1 = a \end{cases} \Rightarrow \begin{cases} b = \frac{5}{4} - \frac{1}{a} \\ (a-1)(2a^2 + 2a - 1) = 0 \\ \Delta = 4 - 8 < 0 \end{cases} \Rightarrow$$

$$\begin{cases} b = \frac{5}{4} - \frac{1}{a} \\ 2a^2 - \frac{1}{a} = \frac{9}{4} - \frac{5}{4} \end{cases} \Rightarrow \begin{cases} b = \frac{5}{4} - \frac{1}{a} \\ 2a^3 - 1 = a \end{cases} \Rightarrow \begin{cases} b = \frac{5}{4} - \frac{1}{a} \\ (a-1)(2a^2 + 2a - 1) = 0 \\ \Delta = 4 - 8 < 0 \end{cases} \Rightarrow$$

$$\begin{cases} b = \frac{5}{4} - \frac{1}{a} \\ 2a^3 - 1 = a \end{cases} \Rightarrow \begin{cases} b = \frac{5}{4} - \frac{1}{a} \\ (a-1)(2a^2 + 2a - 1) = 0 \\ \Delta = 4 - 8 < 0 \end{cases} \Rightarrow$$

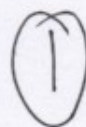
$$\begin{cases} b = \frac{5}{4} - \frac{1}{a} \\ (a-1)(2a^2 + 2a - 1) = 0 \\ \Delta = 4 - 8 < 0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} b = \frac{5}{4} - \frac{1}{a} \\ a = 1 \end{cases} \Rightarrow \begin{cases} b = \frac{1}{4} \\ a = 1 \end{cases}$$

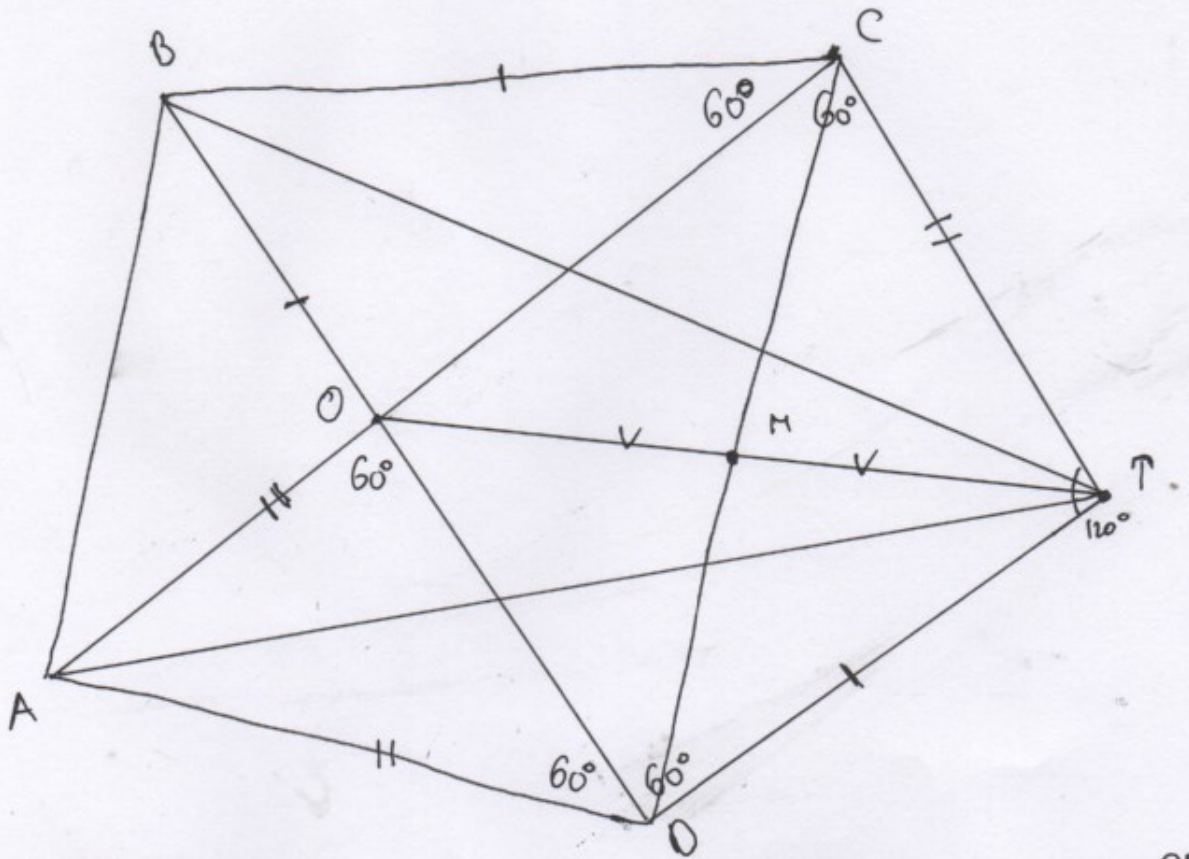
$$\begin{cases} x^2 + y^2 = 1 \\ x^2 y^2 = \frac{1}{4} \end{cases} \Rightarrow \begin{cases} x^2 = 1 - y^2 \\ (1 - y^2)y^2 = \frac{1}{4} \end{cases} \Rightarrow \begin{cases} x^2 = 1 - y^2 \\ y^4 + \frac{1}{4} - y^2 = 0 \end{cases} \Rightarrow \begin{cases} x^2 = 1 - y^2 \\ y^2 = \frac{1}{2} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x^2 = 0,5 \\ y^2 = 0,5 \end{cases}$$

Ответ: $x = \sqrt{0,5}, y = \sqrt{0,5}; x = \sqrt{0,5}, y = -\sqrt{0,5}; x = -\sqrt{0,5}, y = \sqrt{0,5}; x = -\sqrt{0,5}, y = -\sqrt{0,5}$



$\sqrt{6}$



~~CM = MD~~

$\Delta AOD, \Delta BOC$ - рав.

$\Rightarrow \angle OAD = \angle ODA = \angle AOD = \angle BOC = \angle BCO = \angle OBC = 60^\circ$
 $AO = OD = AD; BO = OC = BC$

$OM = MD; OM = MT \Rightarrow BCTD$ - параллелограмм (по двум сторонам и двум углам) $\Rightarrow OD = CT$
 $OC = DT$

$\angle DOC = 180^\circ - 60^\circ = 120^\circ \Rightarrow \angle ODT = \angle OCT = 60^\circ$ (м.к. $OCTD$ - параллелограмм); $\angle BOA = \angle COA = 120^\circ$ (верш.)

Итого $\angle BCT = \angle BTA = \angle TDA = 120^\circ$

~~AD~~
 $CT = OA = AD$
 $BC = BO = TD$

$\Rightarrow \Delta BCT = \Delta TDA$ (по 2 сторонам и углу)
 \Downarrow
 $BA = BT = AT$ ч.т.р.

$BC = 2 \quad 4AD = CT$

По м. косинусов ΔBCT
 $BT^2 = 4 + 16 - 2 \cos 120^\circ \cdot 2 \cdot 4 = 20 - 2 \cdot (-\frac{1}{2}) \cdot 2 \cdot 4 = 28$
 $BT = \sqrt{28}$

$S_{ABT} = \frac{BT \cdot AT \cdot \sin \angle BTA}{2} = \frac{BT^2 \cdot \sqrt{3}}{4} = \frac{28\sqrt{3}}{4} = 7\sqrt{3}$

($\angle BTA = 60^\circ$ м.к. ΔBTA - рав.)

$$S'_{ABCD} = S_{\triangle BOC} + S'_{\triangle AOD} + S'_{\triangle BOA} + S_{\triangle COD}$$

$$S'_{\triangle BOC} = \frac{BO \cdot OC \cdot \sin \angle BOC}{2} = \frac{BO^2 \cdot \sqrt{2}}{4} = \sqrt{2}$$

$$S'_{\triangle AOD} = \frac{AO \cdot OD \cdot \sin \angle AOD}{2} = \frac{AO^2 \cdot \sqrt{2}}{4} = 4\sqrt{2}$$

$$S_{\triangle AOB} = \frac{BO \cdot OA \cdot \sin \angle AOB}{2} = \frac{2 \cdot 4 \cdot \sin 120^\circ}{2} = \frac{8 \cdot \sqrt{2}}{4} = 2\sqrt{2}$$

$$S'_{\triangle COD} = \frac{CO \cdot OD \cdot \sin \angle COD}{2} = \frac{2 \cdot 4 \cdot \sin 120^\circ}{2} = \frac{8\sqrt{2}}{4} = 2\sqrt{2}$$

$$S'_{ABCD} = (4 + 2 + 2 + 1)\sqrt{2} = 9\sqrt{2}$$

$$\frac{S'_{ABT}}{S'_{ABCD}} = \frac{7\sqrt{2}}{9\sqrt{2}} = \frac{7}{9}$$

Answer: $\frac{7}{9}$

Установки

№5.

I случай: на ~~всех~~ на прямых $y=x$ и $y=63-x$ лежит ровно один узел. - узел A

Тогда узлов ~~на~~ ~~прямых~~ не может быть выбраны вторыми
 будет ровно: $61+61-2=120$ узлов - узлы не на выбранных диагоналях,
 но лежащие с A на одной прямой || оси координат
 + $62+62-1=123$ узла - узлы на выбранных диагоналях,
 т.к. по предположению на диаг. лежит ровно
 + 1 выбранный узел
 + 1 место первого узла
 ||
 243 узла.

Всего узлов $62 \times 62 = 3844$

$3844 - 243 = 3601$ - ~~место узла A~~

Позиции где места первого выбранного узла $62+62=124$ и 2 узла при выбранном A.

Тогда в I случае имели $124 \cdot 3600$ вариантов.

II случай: оба выбранных узла лежат на диагоналях.

Тогда ~~где~~ первого ~~узла~~ позиции $62+62=124$, а где второго узла ~~позиции~~ $62+62=124$, а где второго узла

$124 - 1 - 2 = 121$ узлы ~~на~~ ~~прямых~~ с первым узлом ~~прямые~~ || оси координат

$124 \cdot 121 / 2!$ - каждая комбинация повторяется ровно столько раз.

$$\frac{124 \cdot 121}{2} = 62 \cdot 121.$$

$$62 \cdot (121 + 720) = 62 \cdot 841 = 52142$$

211007765 (0870694 M1275089)

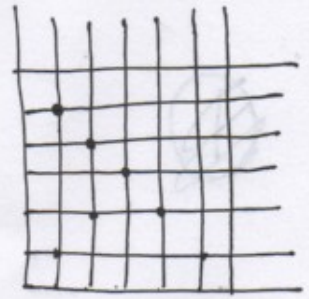
Ответ: ~~453902~~ 52142 способа

(4)

Упробем

$$\begin{cases} \frac{1}{x^2y^2} + x^2y^2 = 5/4 \\ 2x^4 + 2y^4 + 5x^2y^2 = 9/4 \end{cases}$$

$$\begin{cases} x^2y^2 = a \\ x^4y^4 = b \end{cases}$$



$$2(x^2+y^2)^2 + 5x^2y^2 = 9/4$$

$$\begin{aligned} a^2 - a + \frac{1}{4} &= 0 \\ (a - \frac{1}{2})^2 &= 0 \end{aligned}$$

$$\begin{cases} \frac{1}{a} + b = \frac{5}{4} \\ 2a^2 + b = \frac{9}{4} \end{cases}$$

$$b = \frac{5}{4} - \frac{1}{a}$$

$$b = \frac{9}{4} - 2a^2$$

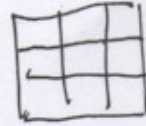
$$\begin{array}{r} 920h5h \\ 8555h \\ \hline 949h1 \\ 29 \\ \hline 523x \end{array}$$

29x79

$$\frac{5}{4} - \frac{1}{a} = \frac{9}{4} - 2a^2$$

$$2a^2 - \frac{1}{a} = 1$$

$$\begin{aligned} a &= 1 \\ b &= \frac{1}{4} \end{aligned}$$

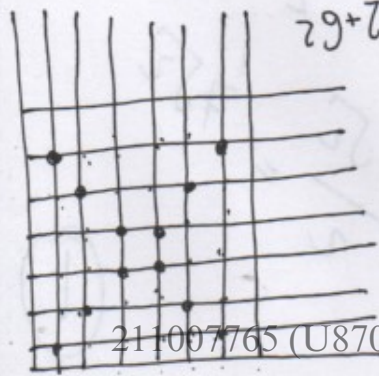


$$2a^3 - 1 = a$$

$$2a^3 - a - 1 = 0$$

$$(a-1)(2a^2+2a+1)$$

$$7+7=12$$



29+79

19

61

5x5=2

6x6=1

11

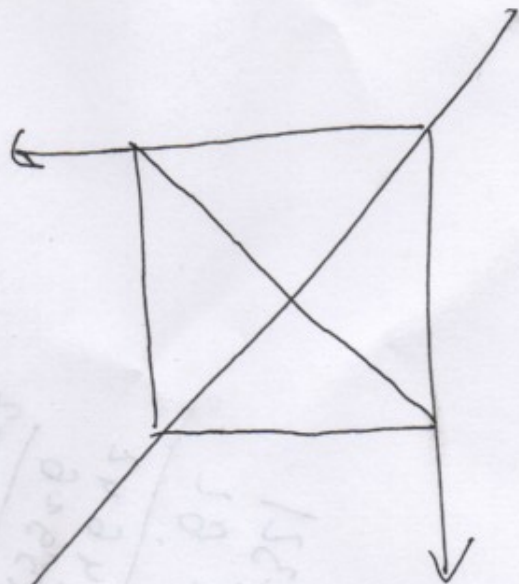
11

$$36 - 11 - 8 - 1 =$$

16

11+8

11



(2)