Часть 1

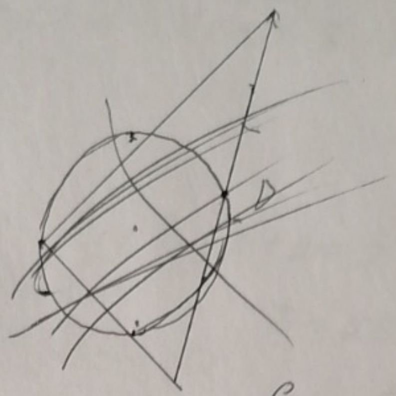
Олимпиада: **Математика, 10 класс (1 часть)**

Шифр: **211007712**

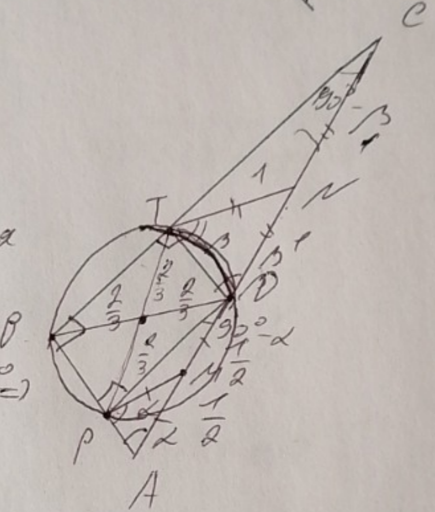
ID профиля: **871254**

Вариант 12

Чистовик №1



1. Дано: $D \in AC$
$(O;R)$; $BD = 2R$
$M, N \in AC$; $CN = DN$
$AM = MD$
$PM \parallel TN$



а) $\angle ABC$ – ?

Решение:

т.к. $\angle BTD$ и $\angle BPD$ опираются ~~опир~~ на диаметр => они равны $90°$ =>
=> $\triangle DPA$ – прямоугольный =>
=> $PM = AM = MD$ (медиана в пр.-/угл. треуг-нике); аналогично в $\triangle CTD$
=> $\angle PAM = \angle MPA$; $\angle NTD = \angle NDT$
Пусть $\angle MAP = \alpha$, а $\angle NDT = \beta$, тогда $\angle PDA = 90° - \alpha$
$NT \parallel PM$
AN – секущая } => $\angle TND = \angle PMA$
$180° - 2\beta = 180° - 2\alpha$ => $\alpha = \beta$ => ~~$\triangle ABC$: $\angle A$, $\angle C = 90° - \alpha$~~
=> $\triangle ABC$: ~~$\angle A$~~ $\angle CAB = \alpha$; $\angle BCA = 90° - \alpha$ => $\angle ABC = 90°$

б) $MP = \frac{1}{2}$; $NT = 1$; $BD = \frac{4}{3}$
S_{ABC} – ?

Решение:

Чистовик №2

Продолжение 1ой задачи.

б) $BD=\frac{4}{3} \Rightarrow PT=\frac{4}{3}$ (т.к. $BTDP$ - прямоугольник)

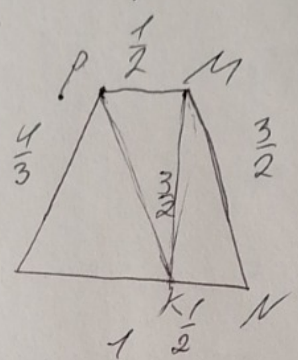
можно заметить, что $MPTN$ - трапеция

Проведем $PK \parallel MN$; $PK=MN$

$PMNK$ - п/г.

$S_{\triangle PKT}=S_{\triangle MNK}=S_{PKM}$

(т.к. основания равны и высоты)



по ф. Герона

$$S_{\triangle PKT}=\sqrt{p\left(p-\frac{4}{3}\right)\left(p-\frac{1}{2}\right)\left(p-\frac{3}{2}\right)}=\sqrt{\frac{5}{3}\left(\frac{1}{3}\right)\cdot\left(\frac{5}{3}-\frac{1}{2}\right)\left(\frac{5}{3}-\frac{3}{2}\right)}=$$

$$p=\frac{1}{2}\left(\frac{4}{3}+\frac{3}{2}+\frac{1}{2}\right)=\frac{5}{3}$$

$$=\sqrt{\frac{5}{3}\cdot\frac{1}{3}\cdot\frac{7}{6}\cdot\frac{1}{6}}=\frac{1}{3}\cdot\frac{1}{6}\sqrt{35}=$$

$$=\frac{\sqrt{35}}{\cancel{18}}$$

$$S_{PMNT}=3S_{\triangle PKT}=\frac{\sqrt{35}}{6}$$

$S_{BTP}=S_{TDP}$; $S_{CTN}=S_{NDT}$

$S_{DPM}=S_{MPA}$ (медианы) $\Big\} \Rightarrow S_{ADC}=2S_{\triangle PKT}=\frac{\sqrt{35}}{3}$

Ответ: а) 90°

б) $\frac{\sqrt{35}}{3}$

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Чистовик № 3

2. $\sqrt{x+1} - \sqrt{4-x} + 3 = 2\sqrt{(x+1)(4-x)} - 3$

$\cancel{x} + 1 + 4 - \cancel{x} - 2\sqrt{(x+1)(4-x)} = 4(x+1)(4-x) - 12\sqrt{(x+1)(4-x)} + 9$

$4(x+1)(4-x) - 10\sqrt{(x+1)(4-x)} + \overset{4}{\cancel{7}} = 0$

$\sqrt{(x+1)(4-x)} = t$

$4t^2 - 10t + \overset{4}{\cancel{7}} = 0$

~~$D = 100 - 80 = 20$~~

~~$t = \frac{10 \pm 2\sqrt{5}}{8} = \frac{5 \pm \sqrt{5}}{4}$~~

$2t^2 - 5t + 2 = 0$

$D = 25 - 16 = 9$

$t = \frac{5 \pm 3}{4} = 2;\ \frac{1}{2}.$

$$\begin{array}{r} 19 \\ \times\ 19 \\ \hline 171 \\ +\ 19 \\ \hline 361 \end{array}$$

$\sqrt{(x+1)(4-x)} = 2;\ \frac{1}{2}.$

$\left[\begin{array}{l} (x+1)(4-x) = 4 \\ (x+1)(4-x) = \frac{1}{4} \end{array}\right.$

$\left[\begin{array}{l} -x^2 + 3x + \cancel{4} = \cancel{0}\,4 \\ -x^2 + 3x + 4 = \frac{1}{4} \end{array}\right.$ $\left[\begin{array}{l} \left[\begin{array}{l} x = 0 \\ x = 3 \end{array}\right. \\ x^2 - 3x - \frac{15}{4} = 0 \end{array}\right.$

$4x^2 - 12x - 15 = 0$

$D = 144 + 240 = 384$

$x = \frac{12 \pm \sqrt{384}}{8} = \frac{12 \pm 4 \cdot \sqrt{24}}{8} = \frac{3 \pm \sqrt{24}}{2}.$

Ответ: $x = 3$

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$\frac{384}{4} = 96 = 4 \cdot \cancel{16}\,24$

Черновик №1

$\sqrt{x+1} - \sqrt{4-x} + 3 = 2\sqrt{(x+1)(4-x)}$

$\sqrt{x+1} - \sqrt{4-x} = 2\sqrt{(x+1)(4-x)} - 3$

$x+1-4+x-2\sqrt{(x+1)(4-x)} = 4(x+1)(4-x)+9 -$

$- 12\sqrt{(x+1)(4-x)}$

$10\sqrt{(x+1)(4-x)} + 2x - 3 = 4(x+1)(4-x)+9$

$10\sqrt{(x+1)(4-x)} = 4(x+1)(4-x) - 2x + 12 = 4(x+1)(4-x) - 2(x-6)$

$100\underbrace{(x+1)(4-x)}_{a} = 16(x+1)^2(4-x)^2 - 16(x+1)(4-x)\underbrace{(x-6)}_{b} + 4(x-6)^2$

$100a = 16a^2 - 16ab + 4b^2$

$-100(x^2-3x-4) = 16(x^2+2x+1)(x^2-8x+16) + 16(x^2-3x-4)(x-6) + 4\cdot$
$\cdot(x^2-12x+36)$

$-100x^2+300x+400 = 16(x^4 - 8x^3 \cancel{+16x^2} + 2x^3 \cancel{-16x^2} + 32x + x^2 - 8x + 16) +$

$+ 16(x^3 - 6x^2 - 3x^2 + 18x - 4x + 24) + 4(x^2 - 12x + 36)$

$-25x^2 + 75x + 100 = 4(x^4 - 6x^3 + x^2 + 24x + 16) + 4(x^3 - 9x^2 + 14x + 24) +$

$+ x^2 - 12x + 36$

$\cancel{21}\ 9 \qquad 60 \qquad -4$

$-25x^2 + \cancel{75x} + \cancel{100} = 4x^4 - 24x^3 + 4x^2 + \cancel{96x} + \cancel{64} + 4x^3 - 36x^2 + 56x + \cancel{96} +$

$+ x^2 \cancel{-12x} + 36$

$4x^4 - 20x^3 - 6x^2 + 65x + 96 = 0.$

$x = 2$

$\frac{96}{4} = \cancel{28}\ 24$

~~$4\cdot 16 - 20\cdot 8 - 6\cdot 4 + 130 + 96 = 16$~~

Черновик №2

$-(x^2-3x-4)=0$

$x=-1$

$x=4$

1) $\sqrt{x+1}-\sqrt{4-x}+3=2\sqrt{4+3x-x^2}$

$\cancel{x+1+9+2\cdot 3\sqrt{x+1}=4(4+3x-x^2)+4-x+4\sqrt{(4-x)(4+3x-x^2)}}$

$\sqrt{x+1}-\sqrt{4-x}+3=2\sqrt{(x+1)(4-x)}$

$\sqrt{x+1}=a\ge 0$

$\sqrt{4-x}=b\ge 0$

$a-b+3=2ab.$

a

$a-2ab-b+3=0$

$\cancel{a-b-ab}$

$\cancel{a-b-3ab+ab+3=0}$

$\cancel{a-b+ab-3(ab-1)=0}$

$a-2ab-b+3=\cancel{(x\ldots)}(2a-m)(-b+n)=$

$=\cancel{2ab-an}-mb+mn$

$n=1$

$m=1$

$mn=3$

$=-2ab+2an+mb-mn$

$n=\frac{1}{2}.$

$m=-1.$

$nm=-\frac{1}{2}$

$(2a+1)(-b+\frac{1}{2})=$

$\cancel{=-2ab+a-b+\frac{1}{2}}$

$=\cancel{2ab-2an-mb+mn}$

$\cancel{b=\frac{1}{2}}$

$\cancel{m=1}$

$n=-\frac{1}{2}$

$m=1.$

$\cancel{(2a-1)(b+\frac{1}{2})}=2ab+a$

Черновик № 2

Черновик № 3

$x=4$

$4^3 \cdot 16 =$

$= 64 \cdot 16$

$\cancel{4x}\ 4^5 - 20 \cdot 64 - 6 \cdot 16 + 260 + 96$

$64 \cdot 16 - 20 \cdot 64 - 6 \cdot 16 + 356 =$

$= -4 \cdot 64 - 6 \cdot 16 + 356 = 16(-4 \cdot 4 - 6) + 356 =$

$= 16 \cdot (-22) + 356 =$

$22 \cdot 16 = 132 + 22 = 352$

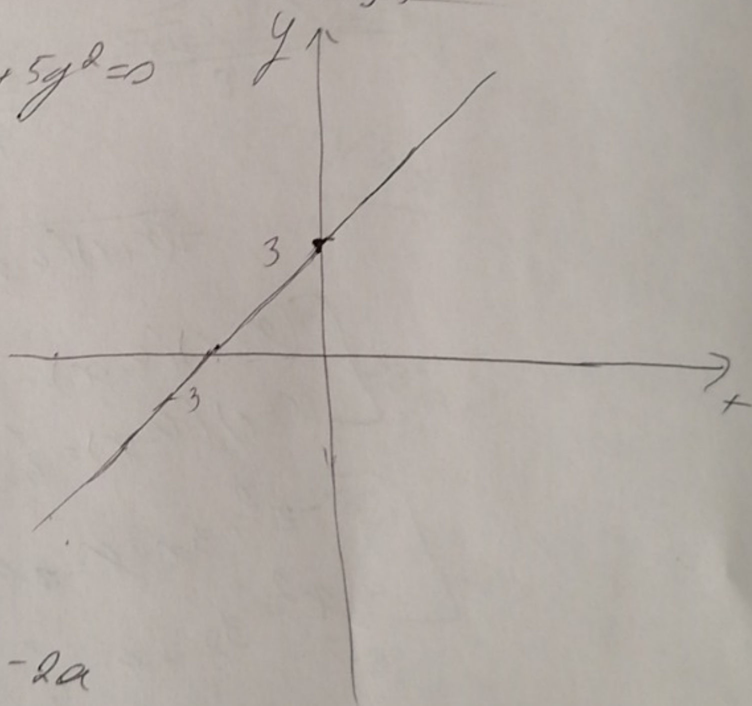
$2a^2 - 2ax - 6ay + x^2 + 2xy + 5y^2 = 0$

A

$ax^2 + 4a^2x - ay + 4a^3 + 2 = 0.$

парабола, В - верш.

$x + y = 3$



$y = x^2 + 4ax + 4a^2 + \frac{2}{a}$

$\cancel{x_B = \frac{-4a}{2} = -2a > -3}$

$a < \frac{3}{2}.$

$x_B = -2a$

$y_B = \cancel{4a^2 - 8a^2 + 4a^2} + \frac{2}{a} = \frac{2}{a}.$

$B\left(-2a; \frac{2}{a}\right)$

$2a^2 - 2ax - 6ay + x^2 + 2xy + 5y^2 = 0$

$(x+y)^2$

Часть 2

Олимпиада: **Математика, 10 класс (2 часть)**

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Вариант 12

Чистовик. N 1

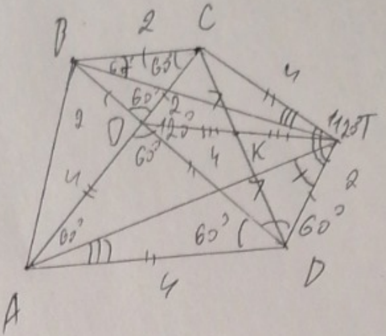
№6.

Дано: ABCD - выпукл. 4х/угольник.

$AC \cap BD = O$.

$\triangle BOC$ и $\triangle AOD$ - правильные

T симметрична т. O отн. середины CD



а) Д-ть: $\triangle ABT$ - правильный

~~Д-во: т. T симметрична т. O => находится~~

Д-во: K - середина CD

т. T симметрична т. O $\Rightarrow OK = KT$

$CTDO: \left.\begin{matrix} OK = KT \\ CK = KD \end{matrix}\right\} \Rightarrow$ по св-ву параллелограмма - CTDO - п/г $\Rightarrow$

$\Rightarrow \begin{matrix} CT = OD \\ CO = TD \end{matrix} \Rightarrow \angle CTD = \angle COD = 120^\circ$

$\angle OCT = \angle ODT = 60^\circ$

Заметим, что $\triangle BCT = \triangle TDA$ (по 2м сторонам и углу между ними) $\Rightarrow$

$\Rightarrow BT = TA$ и $\angle CTB = \angle TAD$, а $\angle CBT = \angle ATD$;

далее $\triangle ATD$: $\angle TAD + \angle ATD = 180^\circ - \underset{120^\circ}{\underset{\parallel}{\angle ADT}} = 60^\circ$; ~~но $\angle ATD$ =~~

но $\angle TAD = \angle BTC \Rightarrow \angle CTD = \angle BTC + \angle BTA + \angle ATD = \angle BTA + \underbrace{\angle TAD + \angle ATD}_{60^\circ} = 120^\circ \Rightarrow \angle BTA = 120^\circ - 60^\circ = 60^\circ$

$\left.\begin{matrix} \angle BTA = 60^\circ \\ BT = AT \end{matrix}\right\} \Rightarrow \triangle BTA$ - правильный

ч.т.д.

Чистовик № 2

Продолжение № 6.

б) $BC = 2$
$AD = 4$.

$\frac{S_{ABT}}{S_{ABCD}}$ - ?

Решение: ~~BC = 2 = OC = TD~~

$AD = OD = CT = 4$.

$\triangle BCT$: по т. косинусов: $BT^2 = BC^2 + CT^2 - 2BC \cdot CT \cos\angle BCT =$

$= 4 + 16 - 2 \cdot 2 \cdot 4 \cdot \cos 120^\circ = 20 + 8 \cdot \cancel{2} \frac{1}{\cancel{2}} = 28$.

$BT = 2\sqrt{7}$

$S_{ABT} = \frac{1}{2} \cdot BT \cdot \overset{= BT}{AT} \cdot \sin 60^\circ = \frac{1}{2} BT^2 \cdot \frac{\sqrt{3}}{2} = \frac{1}{\cancel{2}} \cdot 4 \cdot 7 \cdot \frac{\sqrt{3}}{\cancel{2}} = 7\sqrt{3}$

$S_{ABCD} = \underset{= \frac{1}{2} \cdot 2 \cdot 2 \cdot \sin 60^\circ}{S_{BOC}} + \overset{= \frac{1}{2} \cdot 2 \cdot 4 \cdot \sin 120^\circ}{S_{COD}} + \underset{= \frac{1}{2} \cdot 4 \cdot 4 \cdot \sin 60^\circ}{S_{AOD}} + S_{AOB} = \frac{1}{2} \cdot 2 \cdot 4 \sin 120^\circ$

$= \sin 60^\circ (2 + 4 + 2 \cdot 4 + 4) =$

$= \frac{\sqrt{3}}{2} \cdot 18 = 9\sqrt{3}$

$\frac{S_{ABT}}{S_{ABCD}} = \frac{7\sqrt{3}}{9\sqrt{3}} = \frac{7}{9}$

Ответ: б) $\frac{7}{9}$

Чистовик №3

№4.
$$\begin{cases} \frac{1}{x^2+y^2} + x^2y^2 = \frac{5}{4} \\ 2x^4+2y^4+5x^2y^2 = \frac{9}{4} \end{cases}$$

вычитаем 1ое из 2ого.

$$2x^4+2y^4+4x^2y^2 - \frac{1}{x^2+y^2} = 1.$$

$$2(x^4+2x^2y^2+y^4) - \frac{1}{x^2+y^2} = 1$$

$$2(x^2+y^2)^2 - \frac{1}{x^2+y^2} = 1.$$

Обозначим $x^2+y^2 = t > 0$

$$2t^2 - \frac{1}{t} - 1 = 0$$

$$2t^3 - t - 1 = 0$$

легко угадывается корень $t=1$.; далее по схеме Горнера:

	2	0	-1	-1
1	2	2	1.	0

$$(t-1)(2t^2+2t+1) = 0$$

$$2t^2+2t+1 = 0$$

$$D = 4 - 8 = -4 \Rightarrow t \notin R$$

$t = 1$ – единств. корень.

$$\begin{cases} x^2+y^2 = 1. \\ \frac{1}{x^2+y^2} + x^2y^2 = \frac{5}{4} \end{cases} \Leftrightarrow \begin{cases} x^2+y^2 = 1. \\ 1 + x^2y^2 = \frac{5}{4} \end{cases} \Leftrightarrow \begin{cases} x^2+y^2 = 1 \\ xy = \pm\frac{1}{2} \end{cases}$$

Чистовик № 4.

Продолжение № 4

$$\begin{cases} x^2+y^2=1 \\ xy=\pm\frac{1}{2} \end{cases}$$

1) $\begin{cases} xy=\frac{1}{2} & (1) \\ x^2+y^2=1 & (2) \end{cases}$

домножим (1) на 2 и вычтем из (2):

$x^2-2xy+y^2=0$

$(x-y)^2=0$

$x=y;$

возвращаемся к (1)

$x^2=\frac{1}{2} \Rightarrow x=\pm\frac{\sqrt{2}}{2}=y.$

$\left(\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right),\left(-\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2}\right)$

2) $\begin{cases} xy=-\frac{1}{2} & (1) \\ x^2+y^2=1 & (2) \end{cases}$

домножим (1) на 2 и сложим со (2)

$x^2+2xy+y^2=0$

$(x+y)^2=0$

$x=-y.$

возвращаемся к (1)

$-y^2=-\frac{1}{2} \Rightarrow y=\pm\frac{\sqrt{2}}{2} \Rightarrow x=\mp\frac{\sqrt{2}}{2}$

$\left(\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2}\right),\left(-\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$

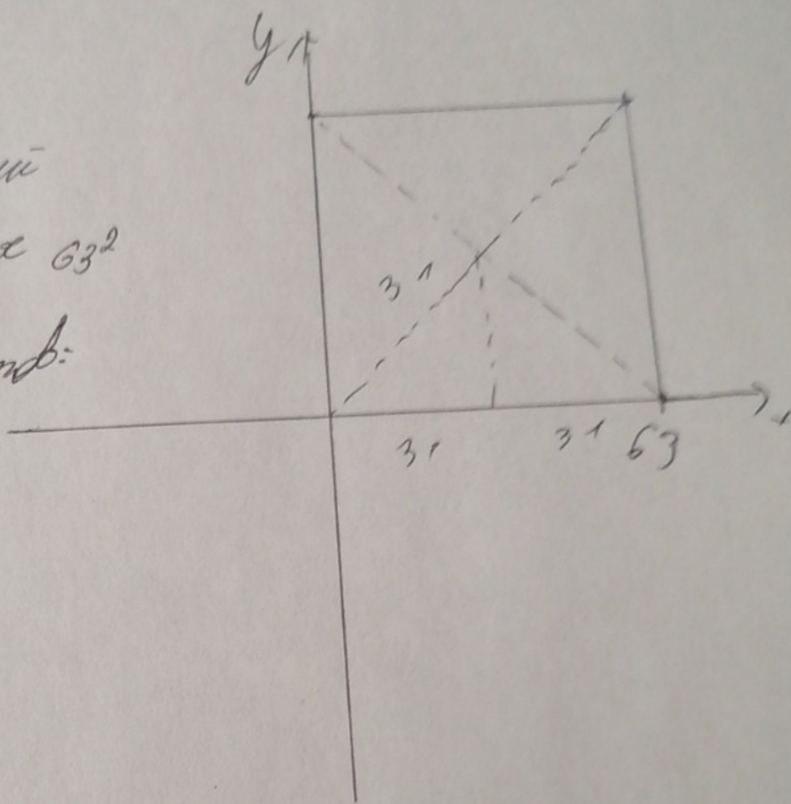
Ответ: $\left(\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right),\left(-\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2}\right),\left(-\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right),\left(\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2}\right)$

Чистовик № 5

~~на каждый~~

на каждый узел, лежащий на прямых, будет приходиться 63^2 других узлов $\Rightarrow$ всего вариантов:

$63^2 \cdot 63 + 63^2 \cdot 62 = 63^2 \cdot 125$



Черновик №1

$$\begin{cases} \frac{1}{x^2+y^2} + x^2y^2 = \frac{5}{4} \\ 2x^4+2y^4+5x^2y^2 = \frac{9}{4} \end{cases}$$

$$2x^4+2y^4+5x^2y^2 - x^2y^2 - \frac{1}{x^2+y^2} = 1.$$

$$2(x^2+y^2)^2 - \frac{1}{x^2+y^2} = 1.$$

$x^2+y^2 = t > 0$

$2t^2 - \frac{1}{t} = 1.$

$2t^3 - t - 1 = 0$

$t = 1.$

	2	0	-1	-1
1	2	2	1	0

$(t-1)(2t^2+2t+1) = 0$

$$\begin{cases} x^2+y^2 = 1. \\ x^2y^2 = \frac{1}{4}. \end{cases}$$

$xy = \pm \frac{1}{2}.$

1) $\begin{cases} xy = \frac{1}{2} \\ x^2+y^2 = 1. \end{cases}$

2) $\begin{cases} xy = -\frac{1}{2}. \\ x^2+y^2 = 1. \ \cancel{xy} \end{cases}$

$(x+y)^2 = 0$

$x = -y \ ; y = -x$

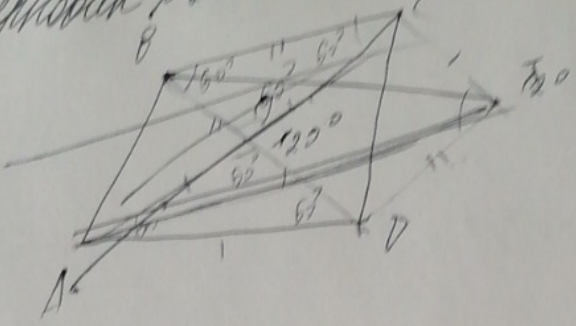
$\cancel{x^2+2xy+y^2}$ $x^2 - 2xy + y^2 = 1 - 1 = 0$

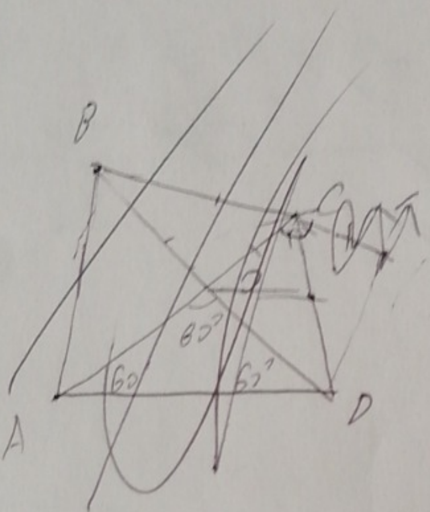
$\cancel{x = y}$ $\left(x = y = \pm \frac{\sqrt{2}}{2}\right)$

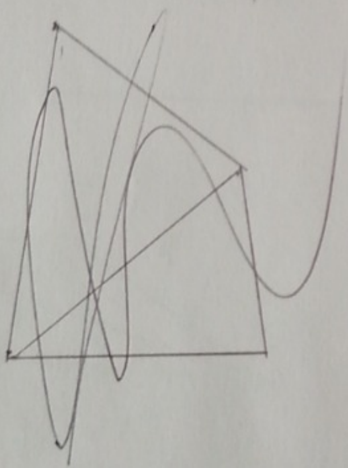
$\left(\begin{cases} x = \frac{\sqrt{2}}{2} \\ y = -\frac{\sqrt{2}}{2} \end{cases}\right) \begin{cases} x = -\frac{\sqrt{2}}{2} \\ y = \frac{\sqrt{2}}{2} \end{cases}$

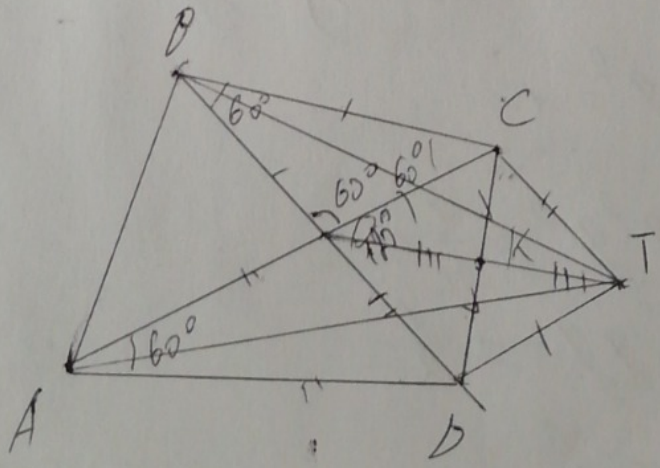
211007712 (U871254 M1275367)

Черновик

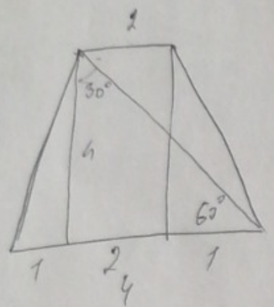






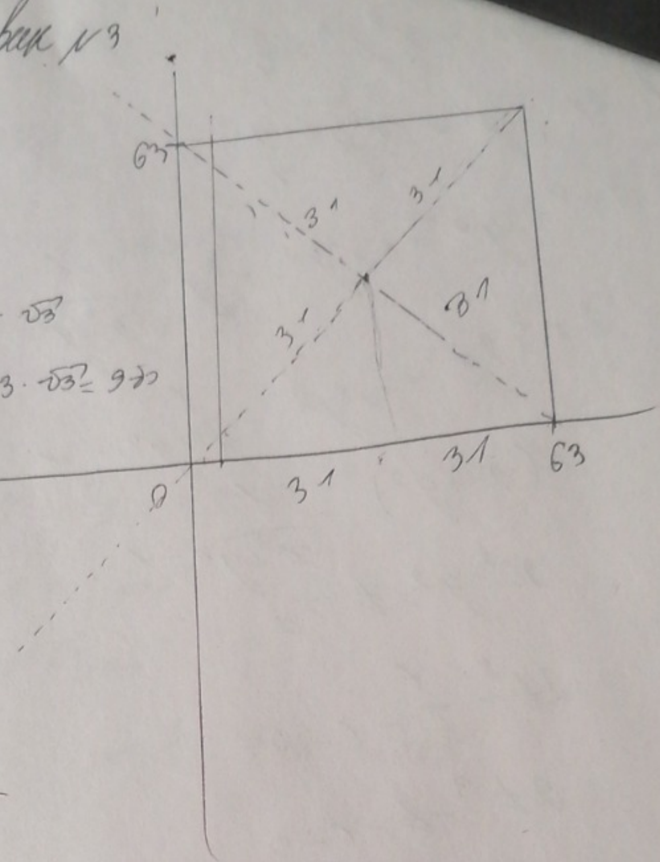


Черновик №3



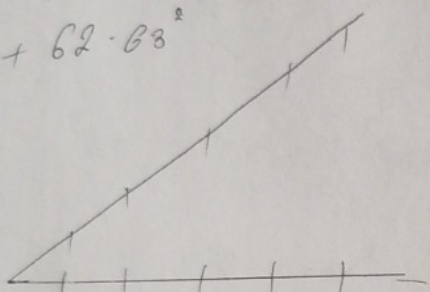
$h = 3 \cdot \sqrt{3}$

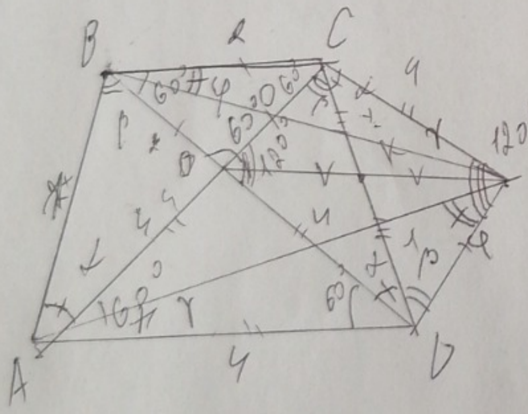
$S = \cancel{4}^{3} \cdot 3 \cdot \sqrt{3} = 9\sqrt{3}$



~~63~~ $63^2 \cdot 63 +$

$+ 62 \cdot 63^2$





Д-ть $AO = OT = AT$

$\angle BAD = 180^\circ - \angle ABC$

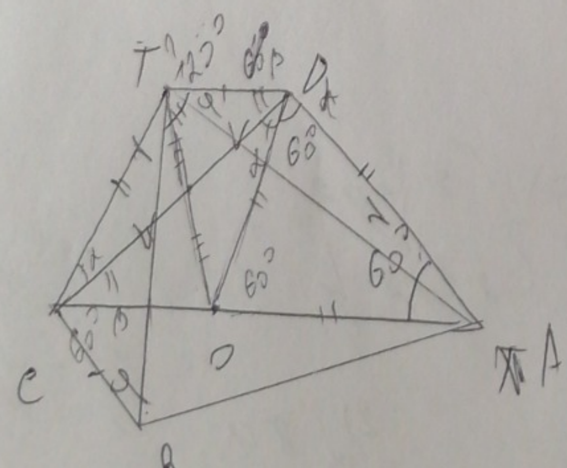
$\alpha + \beta = 60^\circ$

$\varphi + \gamma$ – ?

$\varphi + \gamma + \angle OTA = 120^\circ$

$\angle OTC = \angle TAD$

~~$\angle O\ TC + C$~~



$\angle TAD + \angle ATD = 60^\circ \Rightarrow$

$\Rightarrow \angle OTA = 60^\circ$

a)

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Черновик № 5

Черновик № 4

$$\begin{cases} \frac{1}{x^2y^2} + x^2y^2 = \frac{5}{4} \\ 2x^4+2y^4+5x^2y^2 = \frac{9}{4} \end{cases}$$

$$2x^4+2y^4+5x^2y^2-x^2y^2-\frac{1}{x^2+y^2}=1$$

$$2x^4+2y^4+4x^2y^2-\frac{1}{x^2+y^2}=1$$

$$2(x^2+y^2)^2-\frac{1}{x^2+y^2}=1$$

$$x^2+y^2=t>0$$

$$2t^2-\frac{1}{t}-1=0$$

$$2t^3-t-1=0$$

$$t=1.$$

	2	0	−1	−1
1	2	2	1	0

$$(t-1)\underbrace{(2t^2+2t+1)}_{D<0}=0$$

$$t=1.$$

$$\begin{cases} x^2+y^2=1. \\ \frac{1}{x^2+y^2}+x^2y^2=\frac{5}{4} \end{cases} \Leftrightarrow \begin{cases} x^2+y^2=1 \\ x^2y^2=\frac{1}{4} \end{cases} \Leftrightarrow \begin{cases} x^2+y^2=1 \\ xy=\pm\frac{1}{2} \end{cases}$$

1) $\begin{cases} xy=\frac{1}{2}. \\ x^2+y^2=1. \end{cases}$

$(x-y)^2=0$

$y=\pm\frac{\sqrt{2}}{2}$

2) $\begin{cases} xy=-\frac{1}{2} \\ x^2+y^2=1 \end{cases}$

$(x+y)^2=0 \quad x=-y.$

Черновик № 5

б) $S_{ABT} = \frac{1}{2} \cdot BT^2 \cdot \sin 60^\circ$

$BT^2 = BC^2 + CT^2 - 2BC \cdot CT \cdot \cos 120^\circ$

$BT^2 = 4 + 16 + \cancel{2} \cdot 2 \cdot 4 \cdot \frac{1}{\cancel{2}} = 28$

$BT = 2\sqrt{7}$

$S_{ABT} = \frac{1}{\cancel{2}} \cdot 4 \cdot 7 \cdot \frac{\sqrt{3}}{\cancel{2}} = 7\sqrt{3}$

$$S_{ABCD} = \underbrace{S_{BOC}}_{\frac{1}{2} \cdot 2 \cdot 2 \cdot \sin 60^\circ} + \overbrace{S_{AOD}}^{\frac{1}{2} \cdot 4 \cdot 4 \cdot \sin 60^\circ} + 2\underbrace{S_{ABO}}_{\cancel{2} \cdot \frac{1}{\cancel{2}} \cdot 2 \cdot 4 \cdot \sin 120^\circ} =$$

$= \sin 60^\circ (2 + 2 \cdot 4 + 2 \cdot 4) = \frac{\sqrt{3}}{2} \cdot 18 = 9\sqrt{3}$

$\frac{S_{ABT}}{S_{ABCD}} = \frac{7}{9}$