Часть 1

Олимпиада: **Математика, 10 класс (1 часть)**

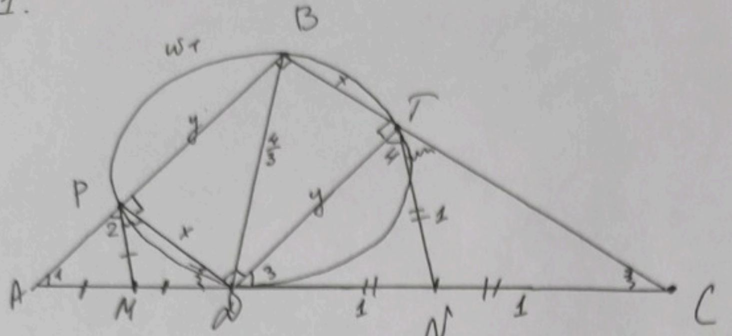
Шифр: **211007006**

ID профиля: **195539**

Вариант 12

Чистовик

№1.



Дано: $\triangle ABC$, $\omega_1 \cap AB = P$, $\omega_1 \cap BC = T$, BD - диаметр ω_1. $M \in AD$, $AM = MD$, $BD \cap AC = D$, $N \in DC$, $DN = NC$, $PM \parallel TN$.

а) $\angle ABC$ - ?

1) BD-диаметр $\Rightarrow \angle BPD = \angle BTD = 90° \Rightarrow \begin{cases} \angle APD = 90° \\ \angle DTC = 90° \end{cases}$

2) PM- медиана прямоугольного $\triangle APD \Rightarrow AM = PM = MD \Rightarrow \angle 1 = \angle 2 = x$, тогда $\angle DPM = 90 - x = \angle PDA$ ($\triangle MPD$ - равнобедренный)

TN- медиана прямоугольного $\triangle DTC \Rightarrow TN = DN = NC \Rightarrow$ $\Rightarrow \triangle DTN$ - ~~равнобедр~~ равнобедренный $\Rightarrow \angle 3 = \angle 4$, $\triangle NTC$ - равнобедренный $\Rightarrow$ $\Rightarrow \angle NTC = \angle TCN$.

3) $PM \parallel TN \Rightarrow \angle PMA = \angle TND \Rightarrow \angle 1 = \angle 2 = \angle 3 = \angle 4 = x$

$\begin{cases} \angle PDA + \angle PDT + \angle TDN = 180° \\ \angle PDA = \angle APD = 90 - x \\ \angle TDN = \angle 3 = x \end{cases} \Leftrightarrow 90 - x + \angle PDT + x = 180° \Rightarrow \angle PDT = 90°.$

4) ~~прямоугольник~~ четырёхугольник $PBTD$ вписан в $\omega_1 \Rightarrow \angle BPD + \angle BTD = \angle PBT + \angle PDT \Rightarrow$

$\Rightarrow \angle PBT = \angle ABC = 180° - 90° = 90°$.

б) $S_{\triangle ABC}$ - ?

Дано: $MP = \frac{1}{2}$, $NT = 1$, $BD = \frac{4}{3}$.

1) $PD = BT = x$, $PB = DT = y$

2) $\angle 1 = \angle 2 = \angle 3 = \angle 4 \Rightarrow \triangle APM \sim \triangle DTN \Rightarrow \frac{AP}{PD} = \frac{PM}{NT} \Rightarrow AP = \frac{1}{2} y$

3) $\begin{cases} \triangle APD: \frac{1}{4}y^2 + x^2 = 1 \\ \triangle BPD: x^2 + y^2 = \frac{16}{9} \end{cases}$ $\begin{cases} x^2 = 1 - \frac{1}{4}y^2 \\ y^2 - \frac{1}{4}y^2 = \frac{16}{9} - 1 \end{cases}$ $\begin{cases} x^2 = \frac{3}{4}y^2 \\ \frac{3}{4}y^2 = \frac{7}{9} \end{cases}$ $\begin{cases} x^2 = \frac{7}{9} \\ y^2 = \frac{28}{27} \end{cases}$

$\begin{cases} x = \sqrt{\frac{7}{9}} \\ y = \sqrt{\frac{28}{27}} \end{cases}$

4) $S_{\triangle ABC} = S_{\triangle APD} + S_{\triangle BPD} + S_{\triangle BTD} + S_{\triangle DTC}$

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Чистовик

№1 (продолжение)

$$S_{\triangle APD} = \frac{AP\cdot PD}{2} = \frac{y\cdot x}{4} = \frac{\sqrt{\frac{28}{27}}\cdot\sqrt{\frac{7}{9}}}{4} = \frac{\sqrt{\frac{4\cdot 7^2}{9^2\cdot 3}}}{4} = \frac{\frac{2\cdot 7}{9}\sqrt{3}}{4}$$

$$S_{\triangle BPD} = S_{\triangle BTD} = \frac{x\cdot y}{2} = \frac{\sqrt{\frac{28}{27}}\cdot\sqrt{\frac{7}{9}}}{2} = \frac{\sqrt{\frac{4\cdot 7^2}{9^2\cdot 3}}}{2} = \frac{\frac{14}{9}\sqrt{3}}{2}$$

$$S_{\triangle DTC} = \frac{y\cdot 2x}{2} = \sqrt{\frac{28}{27}}\cdot\sqrt{\frac{7}{9}} = \frac{14}{9}\sqrt{3}$$

$$\cancel{S_{\triangle ABC} = \frac{\frac{14}{9}\sqrt{3}}{4} + \frac{\frac{14}{9}\sqrt{3}}{2} + \frac{\frac{14}{9}\sqrt{3}}{2} + \frac{14}{9}}$$

$$S_{\triangle ABC} = \frac{\sqrt{\frac{196}{243}}}{4} + \frac{\sqrt{\frac{196}{243}}}{2} + \frac{\sqrt{\frac{196}{243}}}{2} + \sqrt{\frac{196}{243}} =$$

$$= \frac{\sqrt{\frac{196}{243}}}{4} + 2\sqrt{\frac{196}{243}} = \frac{9\sqrt{\frac{196}{243}}}{4} =$$

$$= \frac{9\cdot\frac{14}{9}\cdot\sqrt{\frac{1}{3}}}{4} = \frac{14\cdot\sqrt{\frac{1}{3}}}{4} = \frac{7}{2}\sqrt{\frac{1}{3}} = 3{,}5\sqrt{\frac{1}{3}}.$$

Ответ: а) $\angle ABC = 90^\circ$

б) $S_{\triangle ABC} = 3{,}5\sqrt{\frac{1}{3}}$.

Черновик

№2

1) $\sqrt{x+1}-\sqrt{4-x}+3=2\sqrt{4+3x-x^2}$

$\sqrt{x+1}-\sqrt{4-x}=2\sqrt{-x^2+3x+4}-3$

$x+1+4-x-2\sqrt{-x^2+3x+4}=16+12x-4x^2+9-12\sqrt{-x^2+3x+4}$

$10\sqrt{-x^2+3x+4}=-4x^2+12x+20$

$2,5\sqrt{-x^2+3x+4}=-x^2+3x+4+1$

$\sqrt{-x^2+3x+4}=t$, $t\geq 0$

$2,5t=t^2+1$

$t^2-2,5+1=0$

$D=\frac{25}{4}-4^{16}=\frac{9}{4}$

$\left[\begin{array}{l} t=\dfrac{\frac{5}{2}+\frac{3}{2}}{2}=2 \\ t=\dfrac{\frac{5}{2}-\frac{3}{2}}{2}=\dfrac{1}{2} \end{array}\right.$

$\left[\begin{array}{l} -x^2+3x+4=4 \quad ① \\ -x^2+3x+4=\frac{1}{4} \quad ② \end{array}\right.$

① $x^2-3x=0$

$\left[\begin{array}{l} x=3 \\ x=0 \end{array}\right.$

② $x^2-3x-4+\frac{1}{4}=0$

$4x^2-12x-15=0$

$\frac{D}{4}=36+60=96$

$x=\frac{12\pm\sqrt{96}}{4}$

2) ОДЗ: $\begin{cases} x+1\geq 0 \\ 4-x\geq 0 \\ 4+3x-x^2\geq 0 \end{cases}$ $\cancel{x\geq -1}$ $\cancel{x\leq 4}$ $\begin{cases} x\geq -1 \\ x\leq 4 \\ x^2-3x-4\leq 0 \end{cases}$ $x\in[-1;4]$

Чистовик

№2 (продолжение)

3) $\begin{cases} \left[\begin{array}{l} x=3 \\ x=0 \\ x=\frac{12+\sqrt{96}}{4} \\ x=\frac{12-\sqrt{96}}{4} \end{array}\right. \\ x\in[-1;4] \end{cases}$

$x=\frac{12+\sqrt{96}}{4}=\frac{12+4\sqrt{6}}{4}=3+\sqrt{6}$

$\sqrt{6}>2 \Rightarrow 3+\sqrt{6}>5 \Rightarrow x=3+\sqrt{6}$ не подходит по ОДЗ

$x=\frac{12-\sqrt{96}}{4}=3-\sqrt{6}$

$\sqrt{6}<3 \Rightarrow 3-\sqrt{6}>0 \Rightarrow x=3-\sqrt{6}$ – подходит по ОДЗ.

Ответ: $0;\ 3-\sqrt{6};\ 3$.

Чистовик №3 (продолжение

$\frac{2}{a} > 3+2a$ $\Rightarrow$ $2 > 3a+2a^2$

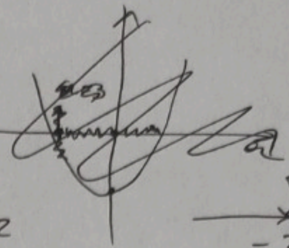
$\uparrow$

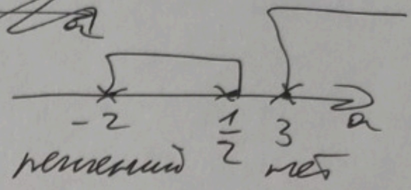
для B (при $a>3$)

$2a^2+3a-2<0$

$D=9+16=25$

$a=\frac{-3\pm 5}{4}$ $\left[\begin{array}{l} a=\frac{1}{2} \\ a=-2 \end{array}\right.$



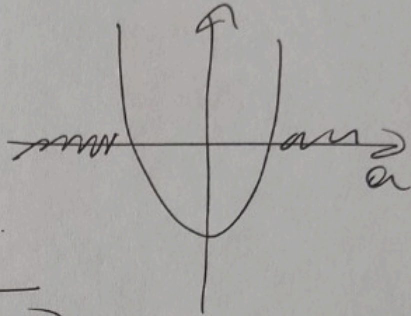


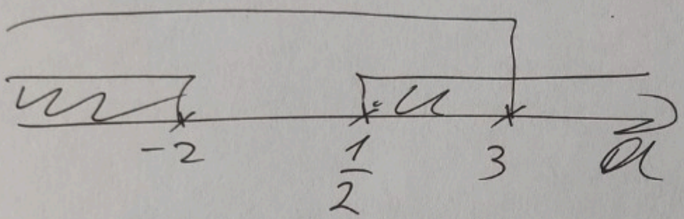
② A и B лежат ниже $y=x-3$

из A: $a<3$

из B: $\frac{2}{a}<3+2a$

$\begin{cases} 2a^2+3a-2>0 \\ a<3 \end{cases}$





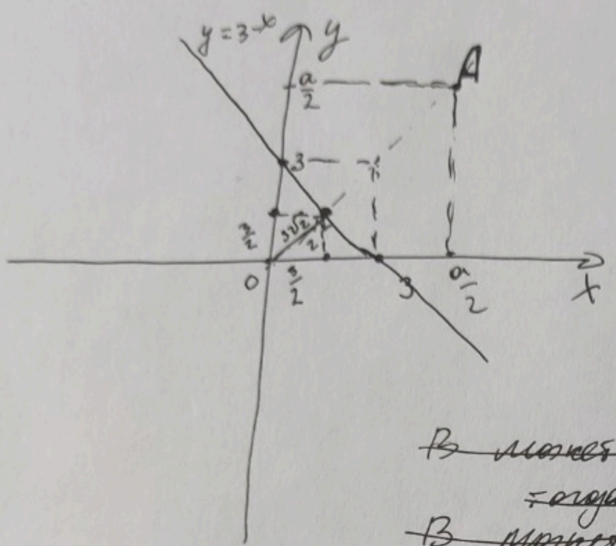
Ответ: $a\in(-\infty;-2)\cup\left(\frac{1}{2};3\right)$.

Чистовик

№3 A:

$A: 2a^2 - 2ax - 6ay + x^2 + 2xy + 5y^2 = 0$

$B: ax^2 + 4a^2x - ay + 4a^3 + 2 = 0$



1) $B: ay = ax^2 + 4a^2x + 4a^3 + 2$

$y = x^2 + 4ax + 4a^2 + \frac{2}{a}$ – парабола „ветви вверх".

$x_в = \frac{-4a}{2} = -2a.$

~~$-2a \geq 3$ (по графику)~~

$y_в = 4a^2 - 2a \cdot 4a + 4a^2 + \frac{2}{a} = \frac{2}{a}$

Координаты точки $B: \left(-2a; \frac{2}{a}\right).$

~~В может лежать выше $y = 3-x$, тогда: $-2a > 3$, $\frac{2}{a} > 3-x$~~

~~В может лежать ниже $y = 3-x$, тогда $-2a < 3$, $\frac{2}{a} < 3-x$~~

2) $A: x^2 + x(2y - 2a) + 5y^2 - 6ay + 2a^2 = 0$

$D = 4y^2 + 4a^2 - 8ay - 20y^2 + 24ay - 8a^2 = -16y^2 + 16ay - 8a^2 = -(16y^2 - 16ay + 4a) =$

$= -(4y - 2a)^2 \geq 0 \Rightarrow 4y = 2a$

$y = \frac{a}{2}$

$x^2 + x(a - 2a) + 5 \cdot \frac{a^2}{4} - 6a \cdot \frac{a}{2} + 2a^2 = 0$

$x^2 - ax + \frac{5}{4}a^2 - 3a^2 + 2a^2 = 0$

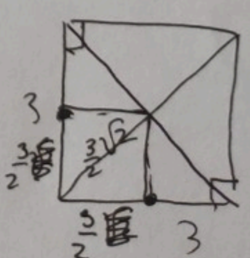
$x^2 - ax + \frac{1}{4}a^2 = 0$

$D = a^2 - a^2 = 0$

$\left(x - \frac{1}{2}a\right)^2 = 0 \Rightarrow x = y = \frac{a}{2}$

3) Координаты точки $B\left(-2a; \frac{2}{a}\right)$

Координаты точки $A\left(\frac{a}{2}; \frac{a}{2}\right)$



① A и B лежат выше $y = x - 3$

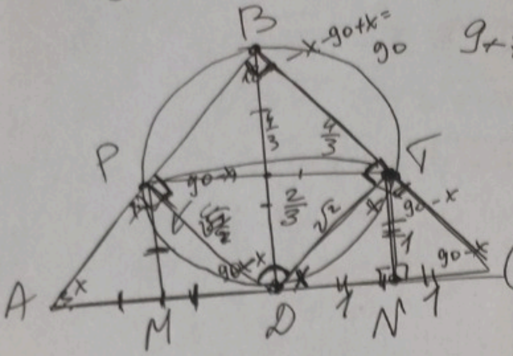
у A, имеем, что ~~$a > 3$~~

$a > 3$

~~Но тогда В~~ ~~$x_в < -6$~~ ~~$y_в < \frac{2}{3}$~~ – лежит ~~ниже~~ $y = x - 3$, т.к. ~~$\frac{2}{3} < 3$~~ $\frac{a}{2}$

5

$\frac{16}{9}-1^{9}=\frac{7}{9}$ Черновик



$\angle ABC$

$9+7=16$

$243/81$

81

$MP=0{,}5$ $\frac{1}{2}$

$NT=1$

$BD=\frac{4}{3}$

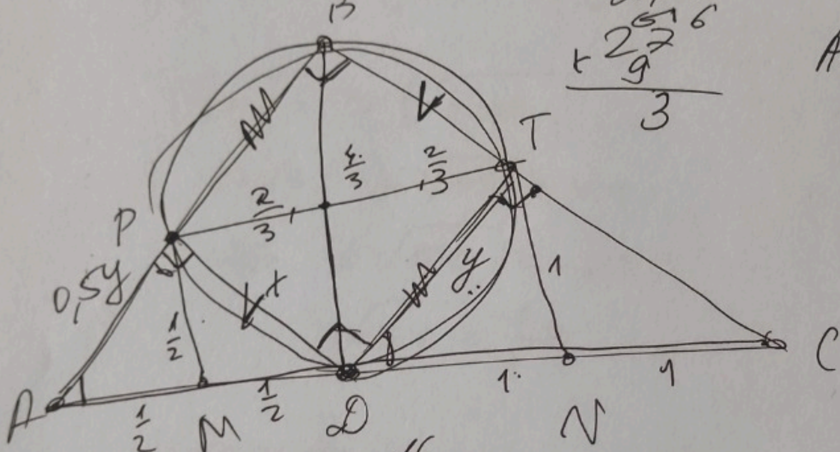
$S_{\triangle ABC}-?$

$PD=\sqrt{\frac{1}{4}+\frac{1}{4}}=\sqrt{\frac{1}{2}}$

$DP=\sqrt{2}$

$27\cdot 9=63+180=243$

$2f=$



$AD^2=AP\cdot AB$

$AP\cdot AB=1$

$DC^2=CT\cdot BC$

$CT\cdot BC=4$

$x^2+y^2=\frac{16}{9}$

$\frac{AM}{AM}=2 \Rightarrow AP=\frac{y}{2}$

$$\begin{cases}\frac{1}{4}y^2+x^2=1\\ x^2+y^2=\frac{16}{9}\end{cases}$$

$x^2=1-\frac{1}{4}y^2$

$1-\frac{1}{4}y^2+y^2=\frac{16}{9}$

$\frac{3}{4}y^2=\frac{16}{9}-1^{9}=\frac{7}{9}$

$\cancel{1-\frac{1}{4}y^2=\frac{16}{9}}$

$x^2=\frac{16}{9}-y^2$

$\frac{1}{4}y^2+\frac{16}{9}-y^2=1$

$y^2=\cancel{\frac{7}{9}\cdot\frac{3}{4}}$ $\frac{7\cdot 4}{9\cdot 3}=\frac{28}{27}$

$y=\frac{2}{3}\sqrt{}$

$\cancel{x=-1}$ Черновик

$x=4$ $\quad 0$

$\sqrt{5}+3=\cancel{2\cdot\sqrt{4+12}}$

$\sqrt{x+1}-\sqrt{4-x} \geq 0$

$\sqrt{x+1} \geq \sqrt{4-x}$

$\begin{cases} x+1 \geq 4-x \\ x \geq -1 \\ x \leq 4 \end{cases}$ $\quad 2x \geq 3 \quad x \geq 1{,}5$

$\cancel{x=}$

$\sqrt{x+1}-\sqrt{4-x}+3 \geq 0$

$\sqrt{x+1}+3 \geq \sqrt{4-x}$

$x+1+\cancel{3}+6\sqrt{x+1} \geq 4-x$

$\cancel{x}$ $6\sqrt{x+1} \geq -2x$

$\sqrt{x+1} \geq -\frac{1}{3}x$

$\cancel{x}x=\frac{3}{2}$

$\sqrt{2{,}5}-\sqrt{2{,}5}+3=2\cdot\sqrt{4+3\cdot\frac{5}{2}-\frac{25}{4}}$

$\sqrt{\cancel{4+\frac{5}{4}}}$

$4+\frac{15}{2}-\frac{25}{4}=\frac{5}{4}+4=\frac{21}{4}=$

$=$

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Черновик

№2.

$\sqrt{x+1} - \sqrt{4-x} + 3 = 2\sqrt{4+3x-x^2} - 3$

$x+1+4-x-2\sqrt{x+1}\cdot\sqrt{4-x} = 2\sqrt{4+3x-x^2} - 3$

$x+1+4-x-2\sqrt{4+3x-x^2} = 2\sqrt{4+3x-x^2} - 3$

$\cancel{(x+1)(4-x) = -x^2+4+3x}$

$\cancel{8 = 4\sqrt{4+3x-x^2}}$

$\cancel{2 = \sqrt{4+3x-x^2}}$

$\cancel{4 = 4+3x-x^2}$

$\cancel{x = 3}$

$x = 0$

$5 - 2\sqrt{4+3x-x^2} = 4\cdot(4+3x-x^2)+9-12\sqrt{4+3x-x^2}$

$(x+1)(4-x) = -x^2+4+4x-x$

$2a^2 - 20ax - 6ay + x^2 + 2xy + 5y^2 = 0$

$\cancel{-2ax - 4ay}$

$-2ax - 2ay + x^2 + 2xy + y^2 + 4y^2 - 4ay + a^2$

$-2a(x+y) + (x+y)^2 \; \cancel{+ay(y+a)} + (2y-a)^2 = -a^2 = a^2$

$(x-4)(x+1)$

$x^2 - 3x - 4 \leq 0$

$+ \quad -1 \quad - \quad 4 \quad +$

$4(4+3x-x^2)^2$

$= 16$

25

$16+9-5 = 20$

$-4x^2+12x+20$

$-\sqrt{5}+3 = 2$

$\frac{10}{4} = 2{,}5$

$10\sqrt{-x^2+3x+4} =$

$\cancel{5\sqrt{-x^2+3x+4}}$

$\frac{5}{2} = \frac{25}{4}$

$-x^2+3x+4 = \frac{25}{4}$

$-4x^2+12x+16 = 25$

$4x^2-12x+9 = 0$

~~чистовик~~ Черновик

№1

$ay = ax^2 + 4a^2x + 4a^3 + 2$

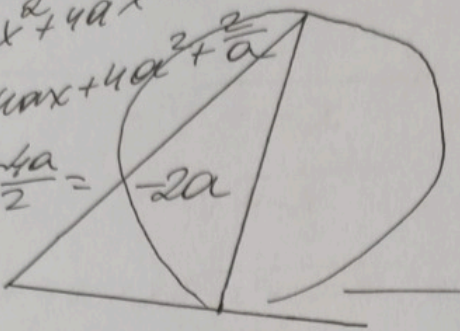
$ax^2 + 4a^2x - ay + 4a^3 + 2 = 0$

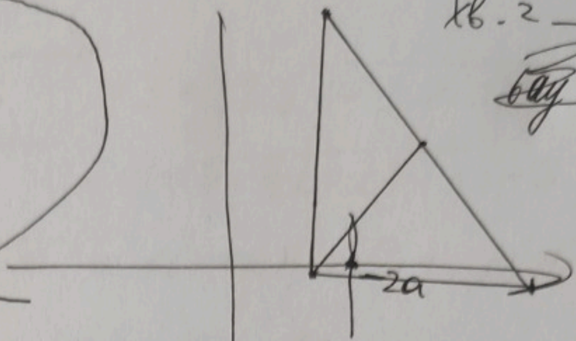
$y = x^2 + 4ax + 4a^2 + \frac{2}{a}$

$x_{в.} = \frac{-4a^2}{\cancel{6ay}}$

$x_{в.} = \frac{-4a}{2} = -2a$

$y_{в} =$





$\sqrt{x+1} - \sqrt{4-x} + 3 = 2\sqrt{4+3x-x^2}$

$\cancel{x+1-4-x+27}$

$\cancel{x}+1+4-\cancel{x} - 2\sqrt{(x+1)(4-x)} = 4\cdot(4+3x-x^2)+9 - 12\sqrt{4+3x-x^2}$

$5 - 2\sqrt{-x^2+3x+4} = 16+12x-4x^2+9-12\sqrt{-x^2+3x+4}$

$10\sqrt{-x^2+3x-4} = 16+12x-4x^2+9-5$

$10\sqrt{-x^2+3x+4} = -4x^2+12x+20$

$2{,}5\sqrt{-x^2+3x+4} = -x^2+3x+5$

$96 = 2\cdot 48 =$

$\times\begin{array}{r}16\\6\end{array}$

$\frac{12+\sqrt{96}}{4} = 3 + \frac{\sqrt{96}}{4} = 3 + \frac{4\sqrt{6}}{4} =$

$3+\sqrt{6} > 4$

$x^2 + 2xy + y^2 + 2a^2 - 2ax - 6ay + 4y^2$

$(x+y)^2 + 2a(\cancel{a}-x) - 6y(a-2)$

$2a^2 - 2ax - 6ay + x^2 + 2xy + 5y^2 = 0$

$-2a(x+3y) + x^2 + 6xy + 9y^2 - 4xy - 4y^2 + 2a^2$

$(x+3y)(-2a + x + 3y) - \cancel{4y(x+y)} + 2 =$

~~Чистовая~~ черновик

№2

$\sqrt{x+1} - \sqrt{4-x} + 3 = 2\sqrt{4+3x-x^2}$

1) $\sqrt{x+1} - \sqrt{4-x} = 2\sqrt{4+3x-x^2} - 3$

$x+1+4-x-2\sqrt{-x^2+3x+4} = 16+12x-4x^2+9-12\sqrt{-x^2+3x+4}$

$10\sqrt{-x^2+3x+4} = -4x^2+12x+20$

$2,5\sqrt{-x^2+3x+4} = -x^2+3x+4$

$y = 3-x$

$t = \sqrt{-x^2+3x+4}, \quad t \geq 0$

$2,5t = t^2$

$t^2 - 2,5t = 0$

$t(t-2,5) = 0$

$\left[\begin{array}{l} t=0 \\ t=2,5 \end{array}\right.$

$t^2 - 2,5t + 1 = 0$

$D =$

2 и $\frac{1}{2}$

$2 \cdot \frac{1}{2} = 1$

$2,5t = t^2+1$

$D = \frac{25}{4} - 4$

$\frac{\frac{5}{2}+\frac{3}{2}}{2} = 2$

$\left[\begin{array}{l} -x^2+3x+4=0 \\ -x^2+3x+4=\frac{25}{4} \end{array}\right.$

$\left[\begin{array}{l} x^2-3x-4=0 \quad а) \\ 4x^2-12x+9=0 \quad б) \end{array}\right.$

$\frac{5}{2}+\frac{3}{2} = \frac{8}{2}$

а) $x^2-3x-4=0$

$\left[\begin{array}{l} x=-1 \\ x=4 \end{array}\right.$

б) $4x^2-12x+9=0$

$\frac{D}{4} = 36-36=0$

$(2x-3)^2=0$

$x = 1,5$

$\frac{5}{4}a^2 - a^2 = \frac{1}{4}a^2$

2) ОДЗ:

$\begin{cases} x+1 \geq 0 \\ 4-x \geq 0 \\ -x^2+3x+4 \geq 0 \end{cases}$ $\begin{cases} x \geq -1 \\ x \leq 4 \\ x \in [-1;4] \end{cases}$ $x \in [-1;4]$.

$\cancel{2a^2 - 2ay}$

$x^2 + x(2y-2a) + 5y^2 - 6ay + 2a^2$

$x^2 - 3x - 4 + \frac{1}{4} = 0$

$D = 4y^2+4a^2-8ay-20y^2+24ay-8a^2 = -16y^2+16ay-4a^2 = -(16y^2-16ay+4a^2)$

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Часть 2

Олимпиада: **Математика, 10 класс (2 часть)**

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ID профиля: **195539**

Вариант 12

Чистовик

№4.

1) $\begin{cases} \frac{1}{x^2+y^2} + x^2y^2 = \frac{5}{4} \\ 2x^4 + 2y^4 + 5x^2y^2 = \frac{9}{4} \end{cases}$

$\begin{cases} \frac{1}{x^2+y^2} + x^2y^2 = \frac{5}{4} & (1) \\ 2(x^2+y^2)^2 + x^2y^2 = \frac{9}{4} & (2) \end{cases}$

(2) - (1):

$2(x^2+y^2)^2 - \frac{1}{x^2+y^2} = 1$

2) пусть $t = x^2+y^2$, $t \neq 0$, тогда:

$2t^2 - \frac{1}{t} = 1$

$2t^3 - t - 1 = 0$

	2	0	-1	-1
$t=1$	2	2	1	0

$(t-1)(2t^2+2t+1) = 0$

$\left[\begin{array}{l} t = 1 \\ 2t^2+2t+1 = 0 \end{array} \right.$

$\frac{D}{4} = 1 - 2 = -1$ $\cancel{\quad}$ $< 0 \Rightarrow$ решений нет $\Rightarrow$

$\Rightarrow x^2+y^2 = 1$, тогда $x^2y^2 = \frac{5}{4} - 1$ (из уравнения (1))

$x^2y^2 = \frac{1}{4} \Rightarrow \left[\begin{array}{l} xy = \frac{1}{2} \\ xy = -\frac{1}{2} \end{array} \right.$

3) Имеем:

$\begin{cases} x^2+y^2 = 1 \\ \left[\begin{array}{l} xy = \frac{1}{2} \\ xy = -\frac{1}{2} \end{array} \right. \end{cases}$

$(x+y)^2 - 2xy = x^2+y^2$, т.е. $(x+y)^2 - 2xy = 1$.

тогда: $(x+y)^2 = 1 + 2xy$

$\cancel{(x+y)^2 = \frac{3}{2}}$

$\left[\begin{array}{l} (x+y)^2 = 2 \\ (x+y)^2 = 0 \end{array} \right.$

4) $\left[\begin{array}{l} \begin{cases} x+y = 0 \\ xy = -\frac{1}{2} \end{cases} \\ \begin{cases} x+y = \sqrt{2} \\ xy = \frac{1}{2} \end{cases} \\ \begin{cases} x+y = -\sqrt{2} \\ xy = \frac{1}{2} \end{cases} \end{array} \right.$

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чистовик

5) а) $\begin{cases} x+y=0 \\ xy=-\frac{1}{2} \end{cases}$

$t^2-\frac{1}{2}=0$

$t^2=\frac{1}{2}$

$t=\pm\sqrt{\frac{1}{2}} \Rightarrow \begin{cases} x=\sqrt{\frac{1}{2}} \\ y=-\sqrt{\frac{1}{2}} \end{cases}$ $\begin{cases} x=-\sqrt{\frac{1}{2}} \\ y=\sqrt{\frac{1}{2}} \end{cases}$

б) $\begin{cases} x+y=\sqrt{2} \\ xy=\frac{1}{2} \end{cases}$

$t^2-\sqrt{2}t+\frac{1}{2}=0$

$D=2-2=0$

$\left(t-\frac{\sqrt{2}}{2}\right)^2=0$

$t=\frac{\sqrt{2}}{2} \Rightarrow \begin{cases} x=\frac{\sqrt{2}}{2} \\ y=\frac{\sqrt{2}}{2} \end{cases}$

в) $\begin{cases} x+y=-\sqrt{2} \\ xy=\frac{1}{2} \end{cases}$

$t^2+\sqrt{2}t+\frac{1}{2}=0$

$\left(t+\frac{\sqrt{2}}{2}\right)^2=0 \Rightarrow \begin{cases} x=-\frac{\sqrt{2}}{2} \\ y=-\frac{\sqrt{2}}{2} \end{cases}$

б) уравнение ① и ② не изменятся, если мы заменим y_0 на $-y_0$ и x_0 на $-x_0$ $\Rightarrow$

$\Rightarrow$ тоже подходят решения:

$\begin{cases} x=-\frac{\sqrt{2}}{2} \\ y=\frac{\sqrt{2}}{2} \end{cases}$

$\begin{cases} x=\frac{\sqrt{2}}{2} \\ y=-\frac{\sqrt{2}}{2} \end{cases}$

$\begin{cases} x=\sqrt{\frac{1}{2}} \\ y=\sqrt{\frac{1}{2}} \end{cases}$

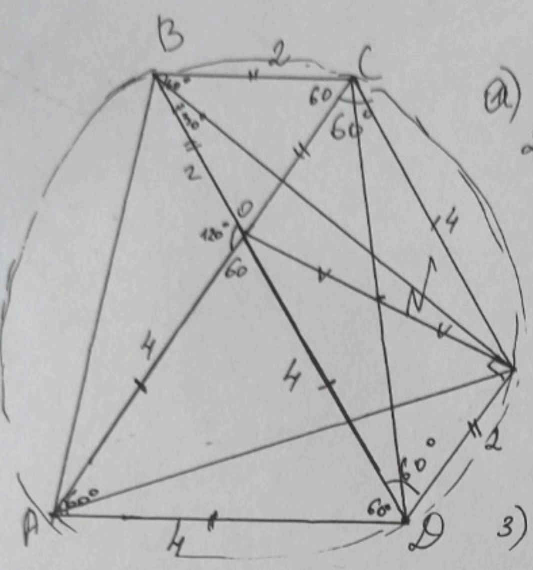
$\begin{cases} x=-\sqrt{\frac{1}{2}} \\ y=-\sqrt{\frac{1}{2}} \end{cases}$

~~Ответ: $x=\sqrt{}$~~

Ответ: $\left(\sqrt{\frac{1}{2}}; -\sqrt{\frac{1}{2}}\right)$; $\left(-\sqrt{\frac{1}{2}}; \sqrt{\frac{1}{2}}\right)$; $\left(\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2}\right)$; $\left(-\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2}\right)$; $\left(-\sqrt{\frac{1}{2}}; -\sqrt{\frac{1}{2}}\right)$; $\left(\sqrt{\frac{1}{2}}; \sqrt{\frac{1}{2}}\right)$; $\left(-\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2}\right)$; $\left(\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2}\right)$.

Чистовик

№6. Дано: четырёхугольник $ABCD$, $BD \cap AC = O$, $\triangle BOC$ и $\triangle AOD$ – правильные, T симметрична O относ. N – средина CD.



а) Д-ть: $\triangle ATB$ – правильный

Д-во: 1) $\triangle BOC$ – правильный $\Rightarrow \angle BOC = \angle BCO = \angle CBO = 60°$, $BC = CO = OB$ – $\Downarrow$ $\angle BOA = \angle COD = 120°$.

$\triangle AOD$ – правильный $\Rightarrow \angle OAD = \angle AOD = \angle ODA = 60°$, $AO = OD = AD$.

2) N – середина CD, $ON = NT$, $CN = ND \Rightarrow OCTD$ – параллелограмм $\Rightarrow CO = DT$, $CO \parallel DT$, $OD = CT$, $OD \parallel CT$

3) $TD = CO = BC$, $BC \nparallel TD$, $CT \parallel BD$ $\Big\} \Rightarrow BCTD$ – равнобокая трапеция $\Rightarrow$ вокруг $BCTD$ можно описать окружность ($\angle CBD + \angle CTD = 180°$)

$CT = AD$, $CT \nparallel AD$, $CA \parallel TD$ (т.к. $CO \parallel TD$) $\Rightarrow ADTC$ – равнобокая трапеция $\Rightarrow$ вокруг $ADTC$ можно опис. окружность, т.к. ~~$\angle BCT + \angle B$~~ ($\angle CAD + \angle CTD = 180°$); $BC \parallel AD$ (т.к. $\angle BCA = \angle CAD$), $AB = CD$ (т.к. $\triangle ABO = \triangle CDO$) $\Rightarrow ABCD$ – равнобокая трапеция $\Rightarrow$ вокруг $ABCD$ можно описать окружность.

4) Из пункта 3 следует, ~~что~~ что A, B, C, T, D лежат на одной окружности (вокруг $ABCTD$ можно описать окружность) $\Rightarrow \angle BTA = \angle BDA = 60°$ (т.к. опираются на одну дугу – $\smile AB$).

$\angle BAT = \angle BDT$, т.к. они опираются на дугу $\smile BT$

$\angle BDT = \angle CBD$ (т.к. $BCTD$ – равнобокая трапеция) $= 60°$ $\Big\} \Rightarrow$

$\Rightarrow \angle BAT = 60°$

$\angle BTA = 60°$ $\Big\} \Rightarrow \triangle ABT$ – правильный, ч.т.д.

б) Найти: $\dfrac{S_{\triangle ABT}}{S_{ABCD}}$ – ? Дано: $BC = 2$, $AD = 4$.

~~Трапеция $BCTD$ равнобокая $\Rightarrow BT$ – дно биссектриса $\angle CBD \Rightarrow \angle TBD = 30°$, $\angle TDB = 60° \Rightarrow \angle BTD = 90° = \angle BAD$.~~

~~$\triangle TBD$: $\angle BTD = 90°$, $\angle TBD = 30° \Rightarrow TD = \frac{1}{2} BD$~~

1) $\angle BOA = 120°$, $\angle BTA = 60°$, $\angle BOA$ и $\angle BTA$ опираются на одну дугу – $\smile AB \Rightarrow \angle BOA$ – центральный $\Rightarrow$ BD и AC – диаметры $\Rightarrow \angle BTD = 90°$

3

№6 (продолжение) Чистовик

б) $BC=2$, $AD=4$, $\frac{S_{\triangle ABT}}{S_{ABCD}}$ – ?

1) $\triangle BCT$:
по Т. косинусов

$BT^2 = BC^2 + CT^2 - 2\cdot BC\cdot CT\cdot \cos 120°$

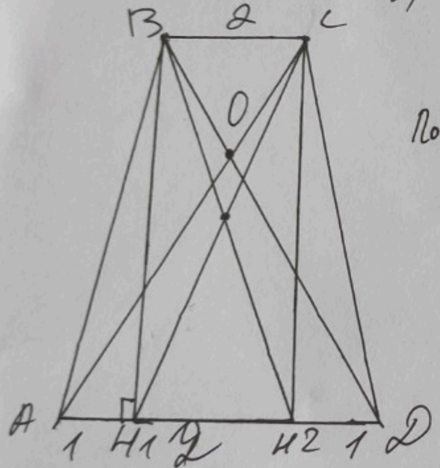
$BT = \sqrt{4+16+2\cdot 2\cdot 4\cdot \frac{1}{2}} = \sqrt{28} \Rightarrow$

$\Rightarrow S_{\triangle ABT} = \frac{BT^2\cdot\sqrt{3}}{4} = \frac{28\sqrt{3}}{4} = 7\sqrt{3}$.

2) $\angle BCA = \angle CAD \Rightarrow BC \parallel AD \Rightarrow ABCD$ – трапеция

$\left.\begin{array}{l} OB = OC \\ AO = OD \\ \angle BOA = \angle COD \end{array}\right\} \Rightarrow \triangle BOA = \triangle COD \Rightarrow AB = CD \Bigg\} \Rightarrow$

$\Rightarrow ABCD$ – равнобокая трапеция



3) BH_1 – высота трапеции $ABCD$

BCH_2H_1 – прямоугольник $\Rightarrow H_1H_2 = 2$

$AH_1 = H_2D$, $AD = 4 \Rightarrow AH_1 = 1$.

По Т. cos: $AB = \sqrt{4+16+2\cdot 2\cdot 4\cdot \frac{1}{2}} = \sqrt{28}$ $\Bigg\} \Rightarrow$

$\Rightarrow BH_1 = \sqrt{27} \Rightarrow$

$\Rightarrow S_{ABCD} = \frac{BC+AD}{2}\cdot BH_1 =$

$= 3\sqrt{27} = 9\sqrt{3}$

4) $\frac{S_{\triangle ABT}}{S_{ABCD}} = \frac{7\sqrt{3}}{9\sqrt{3}} = \frac{7}{9}$.

Ответ: $\frac{S_{\triangle ABT}}{S_{ABCD}} = \frac{7}{9}$.

4

a) $\begin{cases} x+y=0 \\ xy=-\frac{1}{2} \end{cases}$

$t^2-\frac{1}{2}\cancel{t}=0$

$t^2=\frac{1}{2} \Rightarrow$ $x=\frac{1}{2}$, $y=-$

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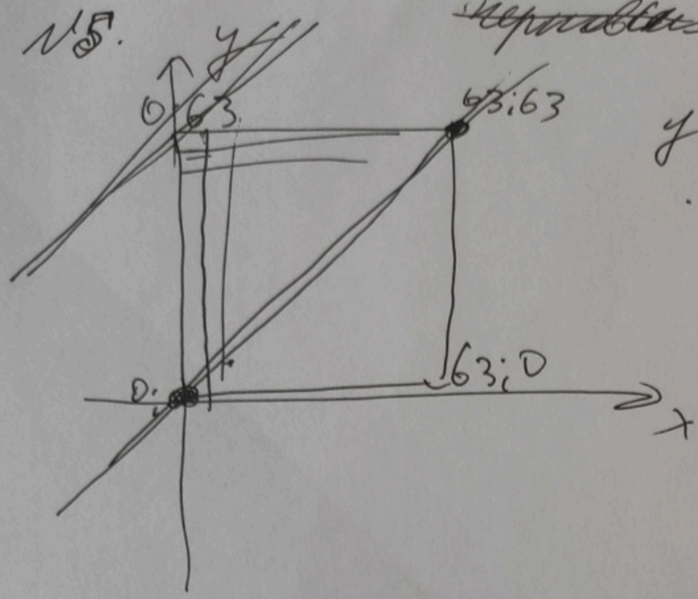
№5.

~~Черновик~~ Черновик



$y=x$

$y=63-x$

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~~Чистовик~~ черновик

2) $\triangle BTD$: $\angle BTD = 90^\circ \Rightarrow BT = \sqrt{BD^2 - TD^2} = \sqrt{36-4} = \sqrt{32} \Rightarrow$

$\Rightarrow S_{\triangle ABT} = \frac{BT^2\sqrt{3}}{4} = \frac{32\sqrt{3}}{4} = 8\sqrt{3}$

3) $S_{ABCD} = S_{\triangle ABD} + S_{\triangle BCD}$

$\angle BAD = 90^\circ$; $\angle BCD = 90^\circ$

$BT^2 = BC^2 + CT^2 - 2\,BC \cdot CT \cdot \cos 120^\circ$

$BT^2 =$

$AB = \sqrt{20}$

$CD = \sqrt{20}$

ЧЕРНОВИК

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Черновик

№4

$$\begin{cases} \frac{1}{x^2+y^2} + x^2y^2 = \frac{5}{4} \\ 2x^4+8y^4+5x^2y^2 = \frac{9}{4} \end{cases}$$

$$\begin{cases} \frac{1}{x^2+y^2} + x^2y^2 = \frac{5}{4} \quad (1) \\ 2(x^2+y^2)+x^2y^2 = \frac{9}{4} \quad (2) \end{cases}$$

(2) - (1):

$2(x^2+y^2)^2 - \frac{1}{x^2+y^2} = 1$

$x^2+y^2 = t$, $t \neq 0$ $\cancel{x \neq 0}$ $\cancel{y \neq 0}$

$2t^2 - \frac{1}{t} = 1$

$2t^3 - t - 1 = 0$

	2	0	-1	-1
$t=1$	2	2	1	0

$(t-1)(2t^2+2t+1) = 0$

$D<0$

$t=1$

$x^2+y^2=1$

$1+x^2y^2 = \frac{5}{4}$

$x^2y^2 = \frac{1}{4}$

$$\begin{cases} x^2+y^2=1 \\ x^2y^2 = \frac{1}{4} \end{cases}$$

$xy = \pm\frac{1}{2}$

$\left(-\frac{\sqrt{2}}{2}\right)^2$

$\frac{2}{4}$

$(x^2+y^2)^2 = x^4+y^4+2x^2y^2 = 1$

$\cancel{(x+y}$ $x^2+y^2+2xy-2xy = (x+y)^2 - 2xy = 1$

$x+y = 1+2xy$

$\cancel{x+y=}$

$\cancel{x+y=}$

$$\left[\begin{array}{l} \begin{cases} x+y = \frac{3}{2} \\ xy = \frac{1}{2} \end{cases} \\ \begin{cases} x+y = \frac{1}{2} \\ xy = -\frac{1}{2} \end{cases} \end{array}\right.$$

$t^2 - \frac{3}{2}t + \frac{1}{2} = 0$

$D = \frac{9}{4} - 2 = \frac{1}{4}$

$\cancel{t = \frac{\frac{3}{2} \pm \frac{1}{2}}{2}}$

$t=1$

$t=\frac{1}{2}$

$t^2 - \frac{1}{2}t - \frac{1}{2} = 0$

$\cancel{D = \frac{1}{4} + 2 = \frac{9}{4}}$

$t=1$

$t=-\frac{1}{2}$

$x = \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{1}{2}$

проверка

$x = \sqrt{\frac{1}{2}}$

$y = -\sqrt{\frac{1}{2}}$

$\frac{1}{\frac{1}{2}+\frac{1}{2}} + \cancel{\left(-\frac{1}{4}\right)} =$

$= \frac{1}{1} + \frac{1}{4} = \frac{5}{4}$ (+)

$x = \frac{\sqrt{2}}{2}$

$y = \frac{\sqrt{2}}{2}$

$\frac{1}{\frac{2}{4}+\frac{2}{4}} + \frac{2}{4}\cdot\frac{2}{4} =$

$= 1 + \frac{1}{4} = \frac{5}{4}$ (+)

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2) $\triangle BTD$: $\angle BTD = 90° \Rightarrow BT = \sqrt{BD^2 - DT^2} = \sqrt{36-4} = \sqrt{32} \Rightarrow$

$\Rightarrow S_{\triangle ABT} = \frac{BT^2\sqrt{3}}{4} = \frac{32\sqrt{3}}{4} = 8\sqrt{3}$.

черновик

3) $CD = BT = \sqrt{32}$

BD - диаметр $\Rightarrow \angle BAD = 90° \Rightarrow AB = \sqrt{BD^2 - AD^2} = \sqrt{36-16} =$
$= \sqrt{20}$

$S_{ABCD} = S_{\triangle ABC} + S_{\triangle ADC}$.

$\angle ABC = 90°$, $\angle ADC = 90°$ (т. к. AC - диаметр) $\Rightarrow$

$\Rightarrow S_{\triangle ABC} = \frac{AB \cdot BC}{2} = \frac{\sqrt{20} \cdot 2}{2} = \sqrt{20} = 2\sqrt{5}$

$S_{\triangle ADC} = \frac{CD \cdot AD}{2} = \frac{\sqrt{32} \cdot 4}{2} = 2\sqrt{32} = 8\sqrt{2}$

4) $\frac{S_{\triangle ABT}}{S_{ABCD}} = \frac{8\sqrt{3}}{2\sqrt{5} + 8\sqrt{2}} = \frac{8\sqrt{3}(8\sqrt{2} - 2\sqrt{5})}{128 - 28} = \frac{64\sqrt{6} - 16\sqrt{15}}{100} =$

$= \frac{16\sqrt{6} - 4\sqrt{15}}{25}$.

Ответ: $\frac{16\sqrt{6} - 4\sqrt{15}}{25}$.

черновик

$4+16+2\cdot 2\cdot 4\cdot\frac{1}{2}$ ~~[illegible]~~ Черновик!

№4. $8+2$

$$\begin{cases}\frac{1}{x^2+y^2}+x^2y^2=\frac{5}{4}\\ 2x^4+2y^4+5x^2y^2=\frac{9}{4}\end{cases}$$

$$\begin{cases}\frac{1}{x^2+y^2}+x^2y^2=\frac{5}{4} \quad (1)\\ 2(x^2+y^2)^2+x^2y^2=\frac{9}{4} \quad (2)\end{cases}$$

$(2)-(1)$:

$$2(x^2+y^2)^2-\frac{1}{x^2+y^2}=1$$

Пусть $t=x^2+y^2$, тогда; $t>0$.

~~$2t^2-t=1$~~ $2t^2-\frac{1}{t}=1$

$2t^3-t-1=0$

	2	0	−1	−1
$t=1$	2	2		

CA – диаметр $\Rightarrow \angle CTA=90^\circ \Rightarrow$

$\Rightarrow AT=\sqrt{36-16}=20$

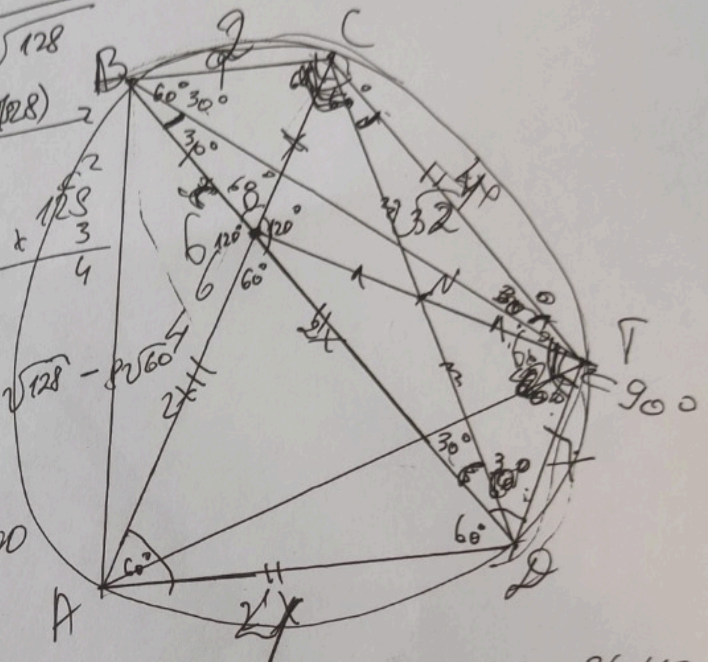
~~$BT^2=16+4+4\cdot\frac{1}{2}$~~ $32\cdot 4=120+8$ $BT^2=5x^2$

$$\frac{8\sqrt3}{\sqrt{20}+\sqrt{128}}^2=\frac{8\sqrt3(\sqrt{20}-\sqrt{128})}{\cancel{128-20}}^2$$

$x \; \frac{128\,3}{4}$

$=8\sqrt3\cdot\sqrt{128}-8\sqrt{60}$

$128-100$

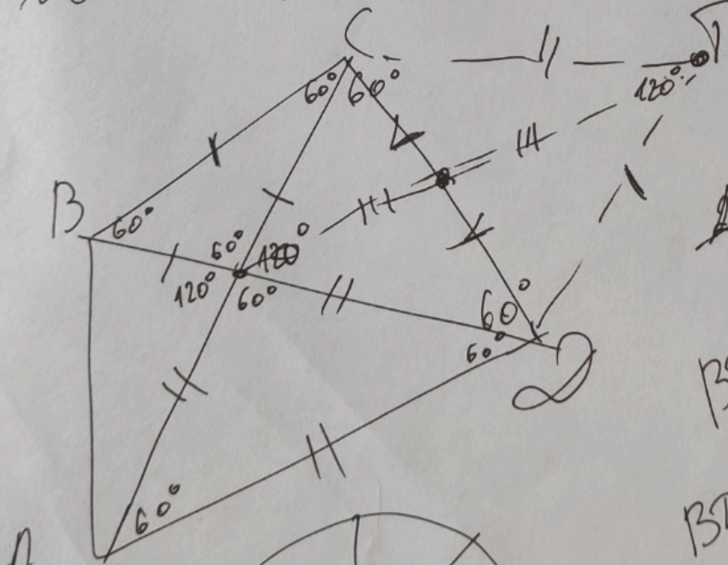


№6. $S=\frac{32\cdot\sqrt3}{4}=8\sqrt3$

$CD=36-4=\sqrt{32}$

$\frac{a^2\sqrt3}{4}$

~~4 5 6 64.8~~



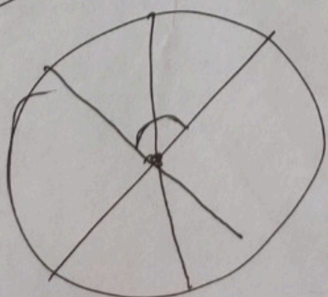
$BT^2=BC^2+CT^2-2BC\cdot CT\cdot\cos 120^\circ$

$BT^2=4+16+2\cdot 2\cdot 4\cdot\frac{1}{2}$

$BT^2=4+16+8=28$

$=4+16+8=28$

$BT=\sqrt{28}$



$4+16+8$

$CD^2=36-4=32$

$BT^2=32$

$642\;\; 45$

12

$4\,(\sqrt5-2\sqrt5)$

$100\;\; 25$

№1 Черновик

Чер

$$\begin{cases} \frac{1}{x^2+y^2}+x^2y^2=\frac{5}{4} \quad /5 \\ 2x^4+2y^4+5x^2y^2=\frac{9}{4} \end{cases}$$

$\cancel{\frac{1}{x^2+y^2}+x^4y^4+2x^2y^2=x^2y^2}$

$$\begin{cases} \frac{5}{x^2+y^2}+5x^2y^2=\frac{25}{4} \\ - \\ 2x^4+2y^4+5x^2y^2=\frac{9}{4} \end{cases}$$

$$\frac{5}{x^2+y^2}-2(x^4+y^4)=\frac{16}{4}$$

$$\frac{1}{x^2+y^2}+x^2y^2=\frac{5}{4}$$

$$2x^4+2y^4+5x^2y^2=\frac{9}{4}$$

$$x^4+y^4+2x^2y^2+x^4+y^4+2x^2y^2+x^2y^2=\frac{9}{4}$$

$$2(x^2+y^2)^2+x^2y^2=\frac{9}{4}$$

$\cancel{\frac{1}{x^2+y^2}}$ $2(x^2+y^2)^2-\frac{1}{x^2+y^2}=1$

$$2t^2-\frac{1}{t}-1=0$$

$$2t^3-t-1=0$$

$t=1$

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