Часть 1

Олимпиада: **Математика, 10 класс (1 часть)**

Шифр: **211005708**

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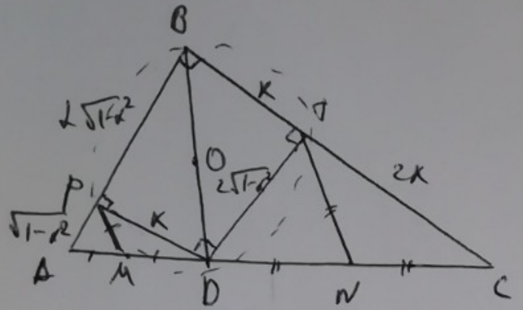
Вариант 12

Тогда получаем: Чистовик 3, задача 2.

$$\left[\begin{array}{l} x=3 \\ x=\frac{6-\sqrt{66}}{4} \end{array}\right.$$

Ответ: $3;\ \frac{6-\sqrt{66}}{4}$

Задача 1. Чистовик 1.



Дано: $\triangle ABC$,

окр $(O; OB)$, $AM = MD$, $DN = NC$

$PM \parallel TN$

1) $\angle ABC$ - ?

2) $MP = \frac{1}{2}$; $NT = 1$, $BD = \frac{4}{3}$.

S_{ABC} - ?

① $\angle BPD = \angle BTD = 90°$, т.к. они опираются на диаметр.

② Тогда $PM = AM = MD$, а $TN = ND = NC$, как медианы в пр. треуг.

③ Пусть $\angle MPD = \alpha$, тогда $\angle PDM = \alpha$, т.к. $\triangle PMD$ – равнобедренный.

④ $\angle PMA = 2\alpha$ (как внешний), $\angle TND = PMA = 2\alpha$, т.к. $TN \parallel PM$.

⑤ $\angle NDT = \angle NTD = 90° - \alpha$, т.к. $\triangle TND$ – равнобедренный.

⑥ $\angle PDT = 180° - (\angle PDM + \angle TDN) = 180° - 90° = 90°$

⑦ Тогда у $BPDT$, $\angle ABC = 360° - 270° = 90°$

⑧ $\triangle APD \sim \triangle DTC$ (по двум углам)

⑨ Пусть $PD = x$, тогда $TC = 2x$ $\left(\frac{PD}{TC} = \frac{AD}{CD} = 2\right.$, т.к. $PM = \frac{1}{2}AD$, $\left.TN = \frac{1}{2}CD\right)$

⑩ Тогда по th. Пифагора. $AP = \sqrt{1-x^2}$, $TD = 2\sqrt{1-x^2}$

⑪ $PD = BT = x$

⑫ По th. Пифагора в $\triangle BTD$:

$$\left(\frac{4}{3}\right)^2 = x^2 + \left(2\sqrt{1-x^2}\right)^2$$

$$\frac{16}{9} = x^2 + 4 - 4x^2, \quad 3x^2 = 4 - \frac{16}{9} = \frac{36}{9} - \frac{16}{9} = \frac{20}{9}, \quad x = \sqrt{\frac{2\cdot 2\cdot 5}{3\cdot 3\cdot 3}} = \frac{2}{3}\sqrt{\frac{5}{3}}$$

⑬ $AB = 3\sqrt{1-x^2} = 3\cdot\sqrt{1-\frac{4}{9}\cdot\frac{5}{3}} = 3\sqrt{\frac{7}{27}} = \sqrt{\frac{7}{3}}$

⑭ $BC = 3x = 2\sqrt{\frac{5}{3}}$

⑮ $S_{ABC} = \frac{1}{2}\cdot AB\cdot BC = \frac{1}{2}\cdot\sqrt{\frac{7}{3}}\cdot 2\sqrt{\frac{5}{3}} = \sqrt{\frac{35}{9}} = \frac{\sqrt{35}}{3}$

Ответ: $90°$; $\frac{\sqrt{35}}{3}$

$\sqrt{x+1} - \sqrt{4-x} + 3 = 2\sqrt{4+3x-x^2}$ Чистовик. 2., Задача 2.

$-2\sqrt{-(x-4)(x+1)} = \sqrt{4-x} - \sqrt{x+1} - 3$

$-2\sqrt{(4-x)(x+1)} = \sqrt{4-x} - \sqrt{x+1} - 3$

$(\sqrt{4-x})^2 - 2\sqrt{(4-x)(x+1)} + (\sqrt{x+1})^2 = \sqrt{4-x} - \sqrt{x+1} - 3 + (4-x) + (x+1)$

$\left(\sqrt{4-x} - \sqrt{x+1}\right)^2 = (\sqrt{4-x} - \sqrt{x+1}) + 2$

$\sqrt{4-x} - \sqrt{x+1} = y,$

$y^2 - y - 2 = 0, \quad (y-2)(y+1) = 0$

$$\left[\begin{array}{l} y = 2 \\ y = -1 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \sqrt{4-x} - \sqrt{x+1} = 2 \\ \sqrt{4-x} - \sqrt{x+1} = -1 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \sqrt{4-x} = 2 + \sqrt{x+1} \\ \sqrt{x+1} = \sqrt{4-x} + 1. \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left\{\begin{array}{l} \left[\begin{array}{l} 4-x = (2+\sqrt{x+1})^2 \\ x+1 = (\sqrt{4-x}+1)^2 \end{array}\right. \\ 4-x \geq 0 \\ x+1 \geq 0 \end{array}\right. \Leftrightarrow \left\{\begin{array}{l} \left[\begin{array}{l} \cancel{4}-x = \cancel{4} + 4\sqrt{x+1} + x+1 \\ x+\cancel{1} = 4-x + 2\sqrt{4-x} + \cancel{1} \end{array}\right. \\ x \leq 4 \\ x \geq -1 \end{array}\right. \Leftrightarrow \left\{\begin{array}{l} \left[\begin{array}{l} 4\sqrt{x+1} = -2x-1 \\ \sqrt{4-x} = x-2 \end{array}\right. \\ x \leq 4 \\ x \geq -1 \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left\{\begin{array}{l} \left[\begin{array}{l} \left\{\begin{array}{l} 16(x+1) = 4x^2 + 4x + 1 \\ -2x-1 \geq 0 \end{array}\right. \\ \left\{\begin{array}{l} \cancel{4}-x = x^2 - 4x + \cancel{4} \\ x \geq 2 \end{array}\right. \end{array}\right. \\ -1 \leq x \leq 4 \end{array}\right. \Leftrightarrow \left\{\begin{array}{l} \left[\begin{array}{l} \left\{\begin{array}{l} 4x^2 - 12x - 15 = 0 \\ x \leq -\frac{1}{2} \end{array}\right. \\ \left\{\begin{array}{l} x(x-3) = 0 \\ x \geq 2 \end{array}\right. \end{array}\right. \\ -1 \leq x \leq 4 \end{array}\right. \Leftrightarrow \left\{\begin{array}{l} \left[\begin{array}{l} \left\{\begin{array}{l} 4x^2 - 12x - 15 = 0 \\ x \leq -\frac{1}{2} \end{array}\right. (A) \\ x = 3 \end{array}\right. \\ -1 \leq x \leq 4 \end{array}\right.$$

Р. (A): $4x^2 - 12x - 15 = 0$

$\frac{D}{4} = 36 - 4 \cdot (-15) = 66$

$x_{1,2} = \frac{6 \pm \sqrt{66}}{4}$; $x = \frac{6+\sqrt{66}}{4}$ – не удовлетворяет условию $x \leq -\frac{1}{2}$

Оценим $\frac{6-\sqrt{66}}{4}$. $\sqrt{66} > \sqrt{64} > 8$, $\frac{6-\sqrt{66}}{4} < \frac{6-8}{4} < -\frac{1}{2}$.

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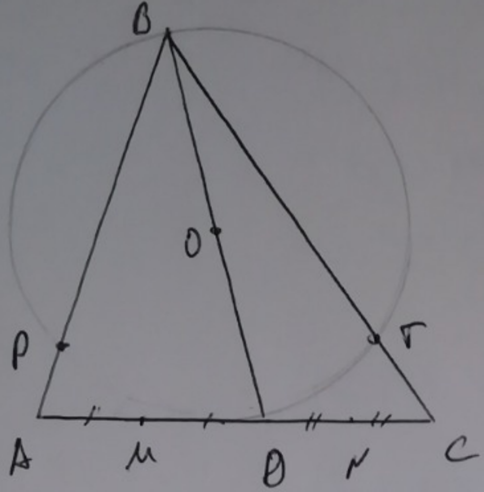
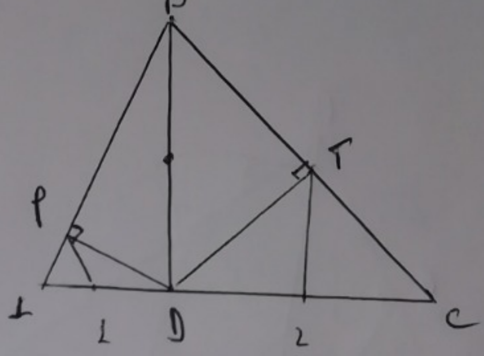
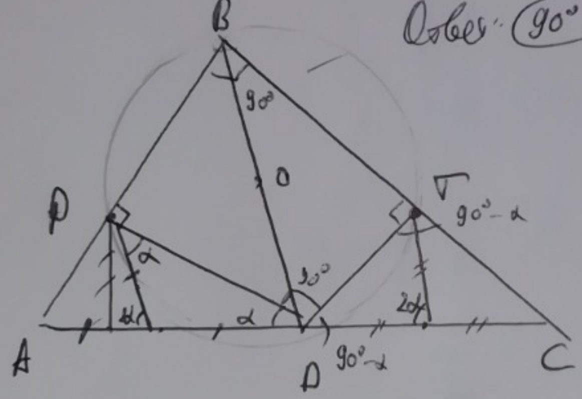
Черновик 1.

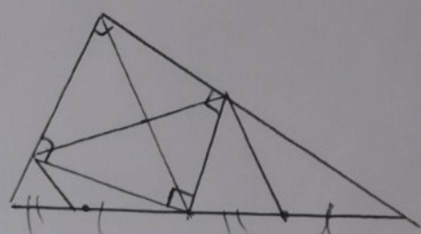
PMTN



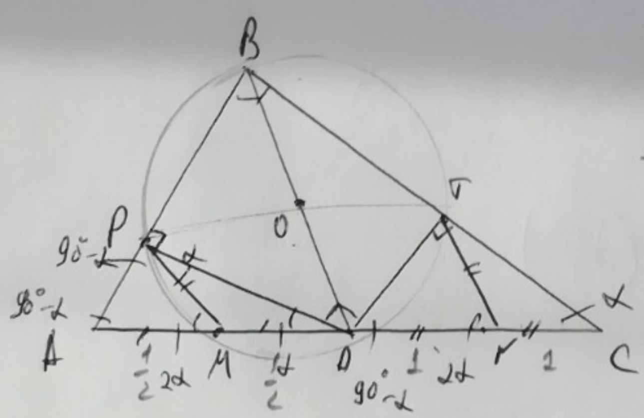


Ответ: 90°





$MP = \frac{1}{2}$ $NT = 1$, $BD = \frac{4}{3}$



$AC = 3$

$BD = \frac{4}{3}$

$180° - (90° + \alpha)$

$= 90 - \alpha$

$180° - (90° - \alpha) - 2\alpha =$

$= 180° - 90° + \alpha - 2\alpha =$

$90° - \alpha$

Чернетка 4.

$\sqrt{x+1} - \sqrt{4-x} + 3 = 2\sqrt{4+3x-x^2}$

$(x+1) + (4-x) + \sqrt{x+1} - \sqrt{4-x} + 3 = (\sqrt{x+1})^2 + 2\sqrt{(x+1)(4-x)} + (\sqrt{4-x})^2$

$5 + 3 + \sqrt{x+1} - \sqrt{4-x} = (\sqrt{x+1} + \sqrt{4-x})^2$

$8 + \sqrt{x+1} - \sqrt{4-x} = (\sqrt{x+1} + \sqrt{4-x})^2$

$\sqrt{x+1} = y, \quad \sqrt{4-x} = t$

$\begin{cases} 8 + y - t = (y+t)^2 \\ y - t + 3 = 2\cancel{\sqrt{yt}} \end{cases}$

$\begin{cases} 8 + y - t = (y+t)^2 \\ y - t + 3 = 2yt \end{cases}$

$\begin{cases} \cancel{8 + y - t} = (y+t)^2 \\ y \cancel{- t + 3} = 2\sqrt{yt} \end{cases}$

$\begin{cases} 5 = (y+t)^2 - 2yt \end{cases}$

$\cancel{5 = (y+t)^2 - 2\sqrt{yt}}$

$2\sqrt{yt} = (y+t)^2 - 5$

$4yt = ((y+t)^2 - 5)^2$

$4yt =$

$5 =$

$5 = y^2 + \cancel{2yt} + t^2 - \cancel{2yt}$

$5 = y^2 + t^2$

$5 = x + 1$

$2 + 1 + 3 \geq 0$

4

$\left[\begin{array}{l} \sqrt{4-x} = 2 + \sqrt{x+1} \\ \sqrt{x+1} = \sqrt{4-x} + 1 \end{array}\right.$

$\left[\begin{array}{l} 4-x = (2+\sqrt{x+1})^2 \\ x+1 = (\sqrt{4-x}+1)^2 \end{array}\right.$

$\left[\begin{array}{l} 4-x = 4 + 4\sqrt{x+1} + x + 1 \\ x+1 = 4-x + 2\sqrt{4-x} + 1 \end{array}\right.$

$\left[\begin{array}{l} 4\sqrt{x+1} = -2x - 1 \\ \sqrt{4-x} = 2 - x \end{array}\right.$

$\left[\begin{array}{l} 16(x+1) = (2x+1)^2 \\ 4-x = 4 - 4x + x^2 \end{array}\right.$

$x^2 - 3x = 0$

$x(x-3) = 0$

$\left[\begin{array}{l} x = 0 \text{ — не задов.} \\ x = 3 \end{array}\right.$

$\begin{cases} x \geq -1 \\ x \leq 4 \end{cases}$

$-1 \leq x \leq 4$

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$\sqrt{x+1} - \sqrt{4-x} + 3 = 2\sqrt{4+3x-x^2}$

При $x=0$

$1 - 2 + 3 = 2 \cdot 2.$

$\sqrt{x+1} - \sqrt{4-x} + 3 = 2\sqrt{-(x-4)(x+1)}$

$\sqrt{x+1} - \sqrt{4-x} + 3 = 2\sqrt{(4-x)(x+1)}$

$\sqrt{x+1} - \sqrt{4-x} + 3 = 2\sqrt{4-x}\sqrt{x+1}$

$$\begin{cases} \sqrt{x+1} = y, & y \geq 0 \\ \sqrt{4-x} = t, & t \geq 0 \end{cases}$$

$\cancel{y - t + 3 = 2\sqrt{yt}}$

$y - t + 3 = 2yt$

$\cancel{y - 2yt}$

$y + 3 = 2yt + t$

$y + 3 = t(2y+1)$

$t = \dfrac{y+3}{2y+1}$

$x^2 - 3x - 4 \geq 0$

$(x-4)(x+1) \geq 0$

Черновик 3

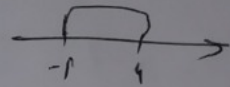
$x^2 - 3x - 4$

$(x-4)(x+1)$

ОДЗ:

$$\begin{cases} x+1 \geq 0 \\ 4-x \geq 0 \\ \cancel{4+3x-x^2 \geq 0} \\ \sqrt{x+1} - \sqrt{4-x} + 3 \geq 0 \end{cases}$$

$x^2 - 3x - 4 \leq 0$



$$\begin{cases} x \geq -1 \\ x \leq 4 \\ x \in [0; 3] \end{cases}$$

$\sqrt{x+1} \geq \sqrt{4-x}$

$\sqrt{x+1} + 3 \geq \sqrt{4-x}$

$(\sqrt{x+1} + 3)^2 \geq 4 - x$

$x + 1 + 9 + 6\sqrt{x+1} \geq 4 - x$

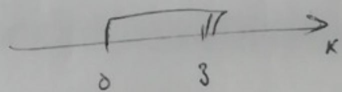
$6\sqrt{x+1} \geq -2x - 6$

$36(x+1) \geq 4(x+3)^2$

$36x + 36 \geq 4x^2 + 24x + 36$

$4x^2 - 12x \leq 0$

$4x(x-3) \leq 0$

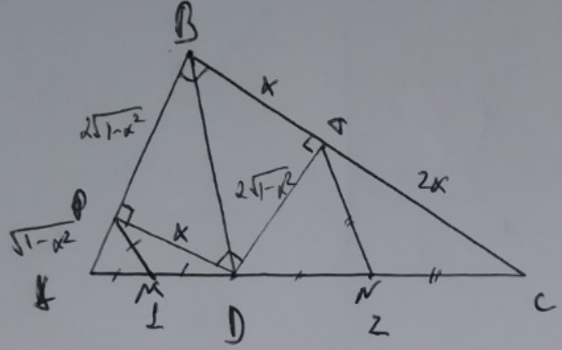


Черновик.

$$4 = 4x^2 + \left(2\sqrt{1-x^2}\right)^2$$

$$4 = 4x^2 + 4(1-x^2)$$

$$\cancel{1 = x^2 + 1 - x^2}$$



$$\frac{4}{3} = x^2 + \left(2\sqrt{1-x^2}\right)^2$$

$$\frac{16}{9} = x^2 + 4(1-x^2)$$

$$\frac{16}{9} = x^2 + 4 - 4x^2$$

$$3x^2 = 4 - \frac{16}{9}$$

$$3x^2 = \frac{9\cdot 4}{9} - \frac{16}{9}$$

$$3x^2 = \frac{36}{9} - \frac{16}{9} = \frac{20}{9}$$

$$x^2 = \frac{20}{27}$$

$$x^2 = \sqrt{\frac{20}{27}} = \sqrt{\frac{4\cdot 5}{3^2\cdot 3}}$$

$$AB = 3\sqrt{1-x^2} =$$

$$= 3\sqrt{1-\frac{20}{27}} = 3\sqrt{\frac{7}{27}} = 3\sqrt{\frac{7}{3\cdot 3\cdot 3}} = \sqrt{\frac{7}{3}}$$

$$BC = 3x = 3\cdot\sqrt{\frac{20}{27}} = 3\cdot\sqrt{\frac{20}{3\cdot 3\cdot 3}} = \sqrt{\frac{20}{3}}$$

$$\frac{7}{3} + \frac{20}{3} = 9$$

$$\frac{27}{3} = 9, \quad 9 = 9$$

$$S_{ABC} = \sqrt{\frac{20}{3}}\cdot\sqrt{\frac{7}{3}}\cdot\frac{1}{2} = \frac{1}{2}\sqrt{\frac{2\cdot 5\cdot 2\cdot 7}{3\cdot 3}} = \frac{1}{2}\cdot\frac{2\sqrt{35}}{3} = \frac{\sqrt{35}}{3}$$

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$16x+16=4x^2+4x+1$

$4x^2-12x-15=0$

$\frac{D}{4}=36-4\cdot(-15)=66$

$x_{1,2}=\frac{6\pm\sqrt{66}}{4}$

$\sqrt{66}\approx\sqrt{64}\approx 8$

$\frac{6\pm 8,\ldots}{4}$

$\left[\begin{array}{l}\frac{14,\ldots}{4}\\ \frac{-2,\ldots}{4}\end{array}\right.$ $\left[\begin{array}{l}3,\frac{\ldots}{4}\ldots\\ -\frac{1}{2},\ldots\end{array}\right.$

$\begin{cases}-2x-1\geq 0\\ x\geq -1\\ x\leq 4\end{cases}$ $2x\leq -1$

$\underline{-1\leq x\leq -\frac{1}{2}}$

Черновик. 5.

$x+1=(\sqrt{4-x}+1)^2$

$\cancel{x}+\cancel{1}=4-x+2\sqrt{4-x}+\cancel{1}$

$2\sqrt{4-x}=2x-4$

$\sqrt{4-x}=x-2 \Leftrightarrow \begin{cases}4-x=x^2-4x+4\\ x\geq 2\end{cases}$

$x^2-3x=0$

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Часть 2

Олимпиада: **Математика, 10 класс (2 часть)**

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Вариант 12

Чистовик 1.

Задача 4.

$$\begin{cases} \frac{1}{x^2+y^2} + x^2y^2 = \frac{5}{4} \\ 2x^4+2y^4+5x^2y^2 = \frac{9}{4} \end{cases} \Leftrightarrow \begin{cases} 1 + x^2y^2(x^2+y^2) = \frac{5}{4}(x^2+y^2) \\ 2(x^2+y^2)^2 + x^2y^2 = \frac{9}{4} \end{cases} \Leftrightarrow$$

Пусть $x^2y^2 = t$, а $x^2+y^2 = z$, $t \geq 0$, $z > 0$

$$\begin{cases} 1+tz = \frac{5}{4}z \\ 2 \cdot z^2 + t = \frac{9}{4} \end{cases} \Leftrightarrow \begin{cases} t = \frac{9}{4} - 2z^2 \\ 1 + z\left(\frac{9}{4} - 2z^2\right) = \frac{5}{4}z \quad (1) \end{cases}$$

Р.(1): $1 + \frac{9}{4}z - 2z^3 = \frac{5}{4}z$

$2z^3 - z - 1 = 0$

$(z-1)(2z^3+2z+1) = 0$

$z = 1$, т.к. $2z^2+2z+1=0$ не имеет решений.

$$\begin{cases} x^2+y^2 = 1 \\ x^2y^2 = \frac{1}{4} \end{cases} \Leftrightarrow \begin{cases} x^2 = \frac{1}{4y^2} \\ \frac{1}{4y^2} + y^2 = 1 \end{cases}$$

$4y^4 - 4y^2 + 1 = 0$

$(2y^2-1) = 0$

$y^2 = \frac{1}{2}$

$x^2 = \frac{1}{2}$

Ответ: $\left(-\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}\right)$; $\left(-\frac{1}{\sqrt{2}}; -\frac{1}{\sqrt{2}}\right)$; $\left(\frac{1}{\sqrt{2}}; -\frac{1}{\sqrt{2}}\right)$; $\left(\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}\right)$

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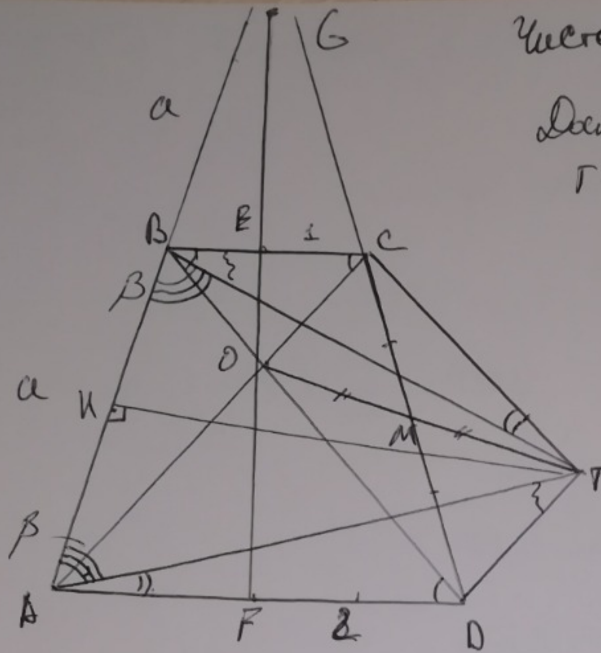

Чистовик 2, задача 6.

Дано: $\triangle BOC$ и $\triangle AOD$ – правильные.
Т – симметрия O отн-но середины CD.

а) Док-ть, что $\triangle ABT$ – правильный.

б) $BC = 2$, $AD = 4$,
$\frac{S_{ABT}}{S_{ABCD}}$ – ?

а)

Решение:

① Пусть точка M – середина CD. Тогда OM – медиана. Воспользуемся методом удвоения медианы. Тогда OCTD – параллелограмм, т.к. диагонали делятся точкой пересечения пополам.

② Т.к. $\angle BCO = \angle CAD = 60^\circ$, $BC \parallel AD$, ABCD – трапеция ($\triangle BOC$ и $\triangle AOD$ – правильные)

③ $\triangle ABO = \triangle COD$ по 2 дум сторонам и углу между ними, следовательно трапеция равнобедренная.

④ Рассмотрим $\triangle BCT$ и $\triangle ATD$.

1. $DT = CO$, $CO = BC$
2. $AO = OD$, $OD = CT$ (треугольники равнобедренные, а CODT – параллелограмм)
3. $\angle BCT = \angle BCO + \angle OCT = 120^\circ$
   $\angle ADT = \angle ADB + \angle BDT = 120^\circ$ $\Big\} \Rightarrow \angle ADT = \angle BCT$.

⑤ Из ④, $\triangle BCT = \triangle ADT$, $AT = TB$

⑥ Пусть $\angle TAB = \angle TBA = \beta$ ($\triangle ABT$ – равнобедренный)
Тогда $\angle CBT + 2\beta + \angle TAD = 180^\circ$, т.к. $BC \parallel AD$, AB – секущая.
$\angle CBT + \angle TAD = 180^\circ - 2\beta$

⑦ Из равенства $\triangle ADT$ и $\triangle BCT$,
$\angle CBT + \angle TAD = \angle CTB + \angle ATD$.

⑧ $\angle COD = \angle CTD = 180^\circ - 60^\circ = 120^\circ$ (т.к. OCTD – параллелограмм)

⑨ $\angle BTA = \angle CTD - (\angle CTB + \angle ATD) = 120^\circ - (180^\circ - 2\beta) \Rightarrow \angle BTA = 2\beta - 60^\circ$

Чистовик 3, задача 6.

⑤ Тогда по сумме углов $\triangle AB\Gamma$:

$2\beta + (4\beta - 60^\circ) = 180^\circ$, $\beta = 60^\circ$, $\triangle AB\Gamma$ – правильн, ч.т.д.

б)

① Пусть $AB = CD = a$.

Проведем высоту ΓK. Тогда $S_{AB\Gamma} = \frac{1}{2} \cdot a \cdot \Gamma K = \frac{1}{2} a\, a \cos 30^\circ =$

$= \frac{\sqrt{3}}{4} a^2$

② $S_{ABCD} = \frac{1}{2}(AD + BC) \cdot EF = \frac{6}{2} \cdot EF = 3EF$

Из $\triangle OEC$: $\operatorname{tg} 60^\circ = \frac{OE}{EC}$, $OE = EC \operatorname{tg} 60^\circ$

Из $\triangle OFD$: $\operatorname{tg} 60^\circ = \frac{OF}{FD}$, $OF = FD \operatorname{tg} 60^\circ$

$EF = OE + OF = 3 \operatorname{tg} 60^\circ$

③ Пусть G – точка пересечения боковых сторон трапеции.

BC – ср. лин $\triangle ADG$. $DG = AB = a$

Из $\triangle AFG$:

$(2a)^2 = 2^2 + (2EF)^2$

$a^2 = 1 + EF^2$

④ $\frac{S_{AB\Gamma}}{S_{ABCD}} = \frac{\frac{\sqrt{3}}{4} a^2}{3EF} = \frac{\sqrt{3}(1 + EF^2)}{4 \cdot 3 \cdot EF} = \frac{\sqrt{3}(1 + 9 \operatorname{tg}^2 60^\circ)}{12 \cdot 3 \cdot \operatorname{tg} 60^\circ} =$

$= \frac{\cancel{\sqrt{3}}(1 + 9 \cdot 3)}{12 \cdot 3 \cdot \cancel{\sqrt{3}}} = \frac{28}{36} = \frac{7}{9}$

Ответ: $\frac{7}{9}$.

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Черновик 3.

$$y^4 - y^2 + \frac{5}{4} = 0$$

$$y^2 = n$$

$$n^2 - n + \frac{5}{4} = 0$$

$$D = b^2 - 4ac = 1 -$$

$$\begin{cases} x^2 + y^2 = 1 \\ x^2 y^2 = \frac{1}{4} \end{cases}$$

$$\begin{cases} x^2 = \frac{1}{4y^2} \\ \frac{1}{4y^2} + y^2 = 1. \end{cases}$$

$$4y^4 - 4y^2 + 1 = 0$$

$$(2y^2 - 1) = 0$$

$$2y^2 = 1.$$

$$y^2 = \frac{1}{2}$$

$$y = \pm \frac{\sqrt{2}}{2}$$

$$x^2 = \frac{1}{4 \cdot \left(\frac{1}{2}\right)} = \frac{1}{2}$$

$$x = \pm \frac{\sqrt{2}}{2}$$

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№4 Черновик.

$$\begin{cases} \frac{1}{x^2+y^2} + x^2y^2 = \frac{5}{4} \\ 2x^4 + 2y^4 + 5x^2y^2 = \frac{9}{4} \end{cases} \Leftrightarrow \begin{cases} 1 + x^2y^2(x^2+y^2) = \frac{5}{4}(x^2+y^2) \\ 2(x^4+y^4+2x^2y^2-2x^2y^2) + 5x^2y^2 = \frac{9}{4} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 1 + x^2y^2(x^2+y^2) = \frac{5}{4}(x^2+y^2) \\ 2((x^2+y^2)^2 - 2x^2y^2) + 5x^2y^2 = \frac{9}{4} \end{cases} \Leftrightarrow \begin{cases} 1 + x^2y^2(x^2+y^2) = \frac{5}{4}(x^2+y^2) \\ 2(x^2+y^2)^2 + x^2y^2 = \frac{9}{4} \end{cases}$$

Пусть $x^2y^2 = t$, $x^2+y^2 = z$, $t \geq 0$, $z > 0$

$$\begin{cases} 1 + t \cdot z = \frac{5}{4}z \\ 2 \cdot z^2 + t = \frac{9}{4} \end{cases} \quad \begin{cases} t = \frac{9}{4} - 2z^2 \\ 1 + z\left(\frac{9}{4} - 2z^2\right) = \frac{5}{4}z. \quad (1) \end{cases}$$

Р.(1): $1 + \frac{9}{4}z - 2z^3 = \frac{5}{4}z$

$2z^3 - z - 1 = 0$

$(z-1)(2z^2+2z+1) = 0$

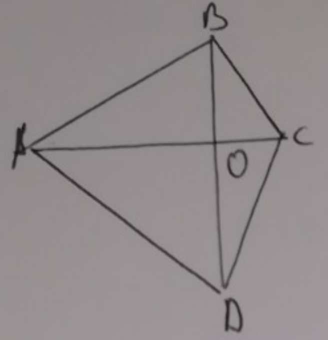
$z = 1$, т.к. $2z^2+2z+1=0$ не имеет решений.

$$\begin{cases} z = 1 \\ t = \frac{1}{4} \end{cases} \Leftrightarrow \begin{cases} x^2+y^2 = 1 \\ x^2y^2 = \frac{5}{4} \end{cases} \Leftrightarrow \begin{cases} (x^2+y^2)^2 = 1. \\ x^2y^2 = \frac{5}{4} \end{cases} \Leftrightarrow \begin{cases} x^4+y^4 = 1 - 2x^2y^2 \\ x^2y^2 = \frac{5}{4} \end{cases}$$

$$\begin{cases} x^4+y^4 = 1 - 2\cdot\frac{5}{4} \\ x^2y^2 = \frac{5}{4} \end{cases} \quad x^4+y^4 = -\frac{3}{2}, \varnothing$$

Ответ: решений нет.

Черновик 5.

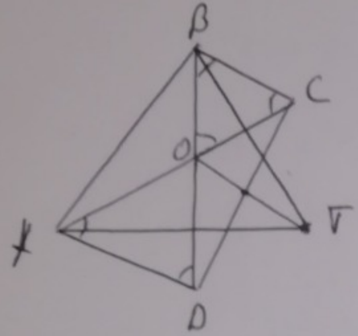


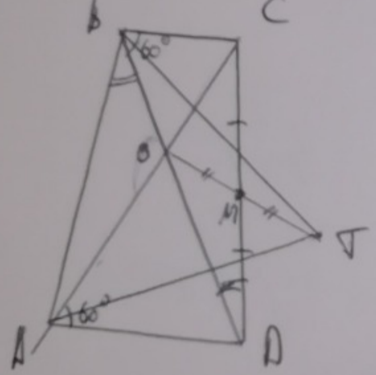
$BC = 2$ $\frac{ABT}{ABCD} = ?$

$AD = 4$

$S_{ABCD} = \frac{(BC + AD)}{2} \cdot$

$\frac{ABCD}{ABT}$

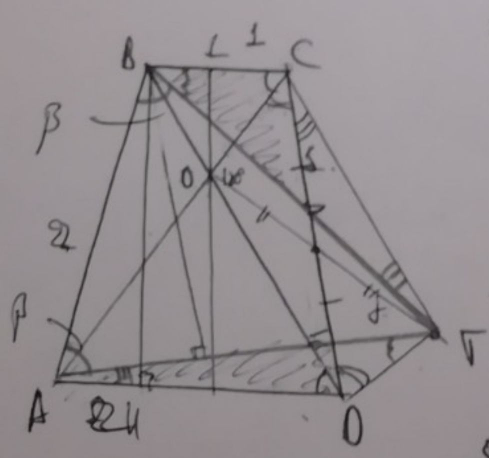




ABCD – равнобок. трапеция.

$BK = \cancel{\sqrt{4-1} = \sqrt{3}}$

$\frac{S_{ABCD}}{S_{ABT}} = S_{ABT} + S$



$AD = CT$ ($AD = OD$, $OCTD$ – парал.)

$OT = BC$ ($BC = OC$, $OCTD$ – параллел.)

$\triangle OTD = \triangle BCT$,

$BT = AT$

$\gamma = 120° - (\xi +)))$

$\xi +))) + 2\beta = 180°$

$\gamma = 120° - (180° - 2\beta)$

$\gamma = -60° + 2\beta$

$\beta + \beta + (-60° + 2\beta) = 180°$

$\beta = 60°$

$x^2y^2 = t,\ x^2+y^2 = z$ Черновик 2

$$\begin{cases} 1+tz = \frac{5}{4}z \\ 2z^2+t = \frac{9}{4} \end{cases} \qquad \begin{cases} t = \frac{9}{4}-2z^2 \\ 1+\left(\frac{9}{4}-2z^2\right)z = \frac{5}{4}z \end{cases}$$

$1+\frac{9}{4}z-2z^3 = \frac{5}{4}z$

$2z^3-z-1=0$

$z=1$, корн.

$$\begin{array}{l|l} 2z^3-z-1 & z-1 \\ \hline -\ 2z^3-2z & 2z^2+2z+1 \\ \quad -2z^2-z & \\ -\ -2z^2+2z & \\ \quad\quad z-1 & \\ \quad -\ z-1 & \end{array}$$

$(z-1)(2z^2-2z+1)=0$

$2z^2-2z+1=0$

$\frac{D}{4} = 1-2<0,\ \varnothing$

$z=1,$

$t = \frac{9}{4}-1 = \frac{5}{4}$

$$\begin{cases} (x^2+y^2)^2 = 1 \\ x^2y^2 = \frac{5}{4} \end{cases} \qquad x^4+y^4+2x^2y^2=1.$$

$(z-1)(2z^2+2z+1)$

$x^2 = \frac{5}{4y^2}$

$\frac{5}{4y^2}+y^2=1. \quad |\cdot y^2$

$\frac{5}{4}+y^4=y^2.$

$$\begin{cases} x^2+y^2=1 \\ x^2y^2 = \frac{5}{4} \end{cases} \qquad \begin{cases} (x+y)^2-2xy=1. \\ xy = \pm\frac{\sqrt{5}}{2} \end{cases}$$

$$\left[\begin{array}{l} \begin{cases} (x+y)^2 = 1+2xy \\ xy = \frac{\sqrt{5}}{2} \end{cases} \\ \begin{cases} (x+y)^2 = 1+2xy \\ xy = -\frac{\sqrt{5}}{2} \end{cases} \quad \varnothing \end{array}\right.$$

$$\left[\ \begin{cases} (x+y)^2 = 1+\sqrt{5} \\ xy = \frac{\sqrt{5}}{2} \end{cases}\right.$$

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№4 Черновик 1.

$$\begin{cases} \frac{1}{x^2+y^2} + x^2y^2 = \frac{5}{4} \\ 2x^4 + 2y^4 + 5x^2y^2 = \frac{9}{4} \end{cases}$$

$$\begin{cases} 1 + x^2y^2(x^2+y^2) = \frac{5}{4}(x^2+y^2) \\ 2(x^4+y^4) + 5x^2y^2 = \frac{9}{4} \end{cases}$$

$$\begin{cases} 1 + x^2y^2(x^2+y^2) = \frac{5}{4}(x^2+y^2) \\ 2((x^2+y^2)^2 - 2x^2y^2) + 5x^2y^2 = \frac{9}{4} \end{cases}$$

$$\begin{cases} 1 + x^2y^2(x^2+y^2) = \frac{5}{4}(x^2+y^2) \\ 2(x^2+y^2)^2 + x^2y^2 = \frac{9}{4} \end{cases}$$

$\exists\, x^2y^2 = t, \quad x^2+y^2 = z,$

$$\begin{cases} 1 + tz = \frac{5}{4}z \\ 2z + t = \frac{9}{4} \end{cases},$$

$$\begin{cases} t = \frac{9}{4} - 2z \\ 1 + (\frac{9}{4} - 2z)_z = \frac{5}{4}z \end{cases}$$

$$\begin{cases} t = \frac{9}{4} - 2z \\ 1 + \frac{9}{4}z - 2z^2 = \frac{5}{4}z \end{cases}$$

$$\begin{cases} t = \frac{9}{4} - 2z \\ 1 + z - 2z^2 = 0 \end{cases}$$

$$\begin{cases} 2z^2 - z - 1 = 0 \\ t = \frac{9}{4} - 2z \end{cases}$$

$$\begin{cases} \cancel{x^2} \end{cases} \quad \cancel{x^2y^2 = z}$$

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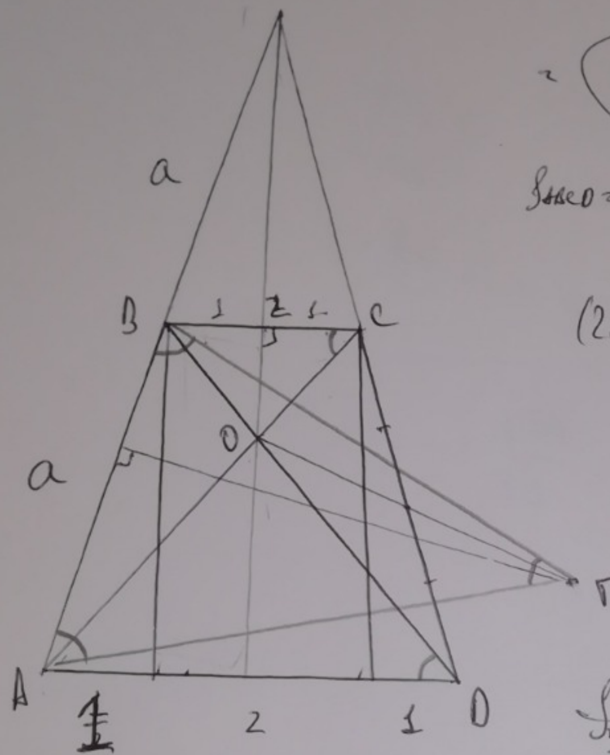
Черновик 6.

$AB = a$

$S_{ABГ} = \frac{1}{2} \cdot a \cdot h = \frac{1}{2} a \cdot \cos 30° a$

$= \frac{a^2 \cdot \sqrt{3}}{4}$

$S_{ABCD} = \frac{1}{2}(BC + AD) \cdot h$



$(2a)^2 = (2)^2 + (2h)^2$

$4a^2 = 4 + 4h^2$

$h^2 = a^2 - 1$

$h = \sqrt{a^2 - 1}$

~~$\frac{S_{ABГ}}{S_{ABCD}} = \frac{\frac{a^2\sqrt{3}}{4}}{}$~~

$tg 60° = \frac{h_1}{1}$, $h_1 = tg 60° \cdot 1$

$\cancel{h_2 =}$ $tg 60° = \frac{h_2}{2}$, $h_2 = 2 \cdot tg 60°$

$h = h_1 + h_2 = 3 tg 60°$

$tg 60° = \sqrt{3}$

$a^2 = h^2 + 1$

$\frac{S_{ABГ}}{S_{ABCD}} = \frac{\frac{a^2\sqrt{3}}{4}}{3 \cdot h} = \frac{a^2\sqrt{3}}{4 \cdot 3h} = \frac{(h^2+1)\sqrt{3}}{12 \cdot h} = \frac{(9 tg^2 60° + 1)\cancel{\sqrt{3}}}{12 \cdot 3 \cancel{tg 60°}} =$

$= \frac{9 \cdot 3 + 1}{36} = \frac{28}{36} = \frac{14}{18} = \frac{7}{9}$

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