Часть 1

Олимпиада: **Математика, 10 класс (1 часть)**

Шифр: **211005555**

ID профиля: **166783**

Вариант 12

Черновик

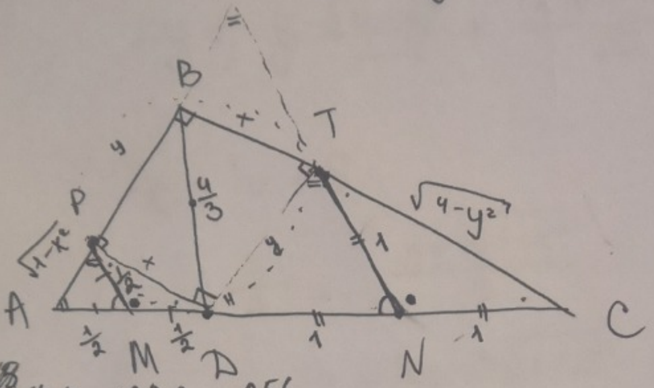
$x+y=3$

$2a^2-2ax-6ay+x^2+2xy+5y^2=0$

$ax^2+4a^2x-ay+4a^3+2=0$

$(x-a)^2-6ay+a^2+2xy+5y^2=0$

$(x-a)^2+(y-3a)^2+4y^2-8a^2+2xy=0$



$x^2+y^2=\frac{4}{3}$

$4-y^2=TC^2$

$1-x^2$

$y^2=\frac{4}{3}-x^2$

$11520 \mid 2$, $5760 \mid 2$

$5760 \mid 2$, 2880

$64\cdot 4=128\cdot 2=256$

$256 \times 45 = 1280 + 1024 = 11520$

$4-\frac{4}{3}$

$\frac{8}{3}$

$81-36=45$

$240 \times 240 = 96 \; 48 = 57600$

$57600-11520=46080$

$46080 \mid 2$, $23040 \mid 2$, $11.520 \mid 2$, $5760 \mid 2$, $2880 \mid 2$, $1440 \mid 2$, $720 \mid 2$, $360 \mid 2$, $180 \mid 2$, $90 \mid 2$, 45

$32 \times 9 = 288$

$288-48=240$

$45x^4-240x^2+64=0$

$D=$

$x^2+y^2=\frac{4}{3}$

$(y+\sqrt{1-x^2})^2+(x+\sqrt{4-y^2})^2=9$

$\cancel{y^2}+2y\sqrt{1-x^2}+1-\cancel{x^2}+\cancel{x^2}+2x\sqrt{4-y^2}+4-\cancel{y^2}=9$

$y\sqrt{1-x^2}+x\sqrt{4-y^2}=2$

$y^2-y^2\cancel{x^2}=4-2x\sqrt{4-y^2}+4x^2-x^2\cancel{y^2}$

$-y^2-4x^2+4=2x\sqrt{4-y^2}$

$-3x^2+\frac{8}{3}=2x\sqrt{4-y^2}$

$\frac{64}{9}-16x^2+9x^4=16x^2-4x^2y^2$

$\frac{64}{9}=16x^2-4x^2(\frac{4}{3}-x^2)$

$\frac{64}{9}-32x^2+9x^4+\frac{16}{3}x^2-4x^4=0$

$64-288x^2+81x^4+48x^2-36x^4=0$

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$$\left(y+\sqrt{1-x^2}\right)\cdot\left(x+\sqrt{4-y^2}\right)=$$

$$=xy+y\sqrt{4-y^2}+x\sqrt{1-x^2}+\sqrt{1-x^2}\sqrt{4-y^2}$$

~~$y+\frac{\sqrt{4-y^2}}{2}$~~ ~~y^2+y-y^2~~

$$xy+\frac{y\sqrt{4-y^2}}{2}+\frac{x\sqrt{1-x^2}}{2}$$

$$\frac{h}{2}+h=S$$

$$\frac{y}{2}=\frac{y+\sqrt{1-x^2}}{3}$$

$$3y=2y+2\sqrt{1-x^2}$$

$$y=2\sqrt{1-x^2}$$

$$x^2+y^2=\frac{4}{3}$$

$$x^4+2x^2y^2+y^4=\frac{16}{9}$$

$$\frac{x}{1}=\frac{x+\sqrt{4-y^2}}{3}$$

$$\boxed{\begin{aligned}2x&=\sqrt{4-y^2}\\ \frac{y}{2}&=\sqrt{1-x^2}\end{aligned}}$$

$$4x^2=4-y^2$$

$$4x^2+y^2=4$$

$$4x^2+\frac{4}{3}-x^2=4$$

$$3x^2=\frac{8}{3}$$

$$x^2=\frac{8}{9}$$

$$x=\frac{2\sqrt{2}}{3}$$

$$\frac{\frac{3y}{2}\cdot 3x}{2}=S$$

$$\frac{9xy}{4}=S$$

$$\left(y+\frac{y}{2}\right)^2+9x^2=3$$

$$\frac{9y^2}{4}+9x^2=3$$

$$\frac{\sqrt{1-x^2}}{1}=\frac{\sqrt{1-x^2}+y}{3}$$

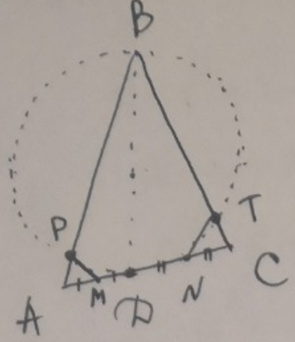
$$\frac{9xy}{4}=\sqrt{2}$$

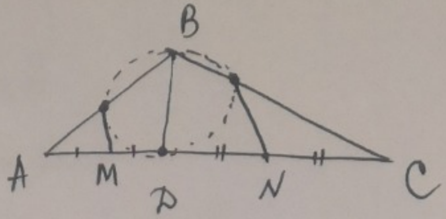
$$\frac{8}{9}+y^2=\frac{4}{3}$$

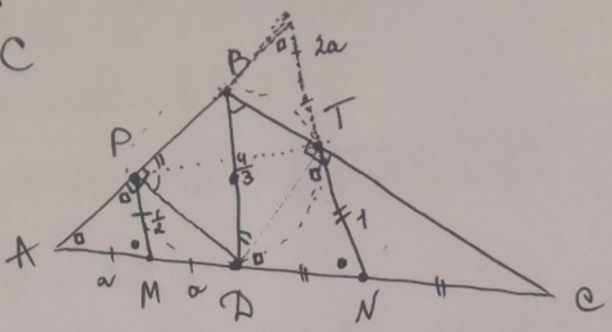
$$y^2=\frac{4}{9}$$

$$y=\frac{2}{3}$$

Черновик







$\sqrt{x+1} - \sqrt{4-x} + 3 = 2\sqrt{4+3x-x^2}$

$4+3x-x^2$ $-x^2+3x+4$

$D = 9+16 = 25$

$(x+1)(x-4)$

$x = \frac{-3+5}{-2} = -1$

$x = \frac{-3-5}{-2} = 4$

$\sqrt{x+1} - \sqrt{4-x} + 3 = 2\sqrt{(4-x)(x+1)}$

$\sqrt{x+1} = a$

$-a = \sqrt{-x-1}$

$-a+5 = \sqrt{-x+4}$

~~$a - \sqrt{5-a} + 3 =$~~

$a - 5 + a + 3 = 2 a(5-a)$

$2a-5+3 = 10a-2a^2$

$2a^2-8a-2=0$

$a^2-4a-1=0$

$D = 16+4=20$

$a = \frac{4+\sqrt{20}}{2} = 2+\sqrt{5}$

Черновик

$-2\sqrt{x+1}\sqrt{4-x}+\sqrt{x+1}-\sqrt{4-x}+3=0$

$a-b+3=2ab$

~~$a-b-2ab+3=0$~~

$a^2-2ab+b^2+3=2ab$

$a^2-4ab+b^2+3=0$

~~$x+1+4-x=4\sqrt{(4-x)(x+1)}$~~

$a-b=2ab-3$

$a^2-2ab+b^2=4a^2b^2-12ab+9$

$4a^2b^2-a^2-b^2+9=10ab$

$4(4-x)(x+1)-x-1-4+\cancel{x}+9=10\sqrt{(4-x)(x+1)}$

$16x+16-4x^2-4x+4=10\sqrt{(4-x)(x+1)}$

$-4x^2+12x+20=10\sqrt{(4-x)(x+1)}$

$-x^2+3x+5=10\sqrt{(4-x)(x+1)}$

$(4-x)(x+1)+1=10\sqrt{x+1}\sqrt{4-x}$

$(-x^2+3x+5)(-x^2+3x+5)+2(4-x)(x+1)+1=100(x+1)(4-x)$

x^4

$a^2+1=10a$

$\sqrt{x+1}-\sqrt{4-x}+3=2\sqrt{4+3x-x^2}$

~~$a^2-4ab+b^2+3=0$~~

$1-2\frac{b}{a}+\frac{b}{a}=4\frac{b}{a}-$

$$\begin{array}{r} 16 \\ \times 15 \\ \hline 80 \\ 16 \\ \hline 240 \end{array}$$

$$\begin{array}{r|l} 384 & 2 \\ 192 & 2 \\ 96 & 2 \\ 48 & 2 \\ 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \end{array}$$

$$\begin{array}{r|l} 96 & 2 \\ 48 & 2 \\ 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & \end{array}$$

$32\cdot3$

$16\cdot6$

Чистовик

$3x - x^2 = 0$

$x(3x-1)=0$ $x=0$ $x=\frac{1}{3}$

$-x^2+3x+\frac{15}{4}=0$

$-4x^2+12x+15=0$

$D=144+240=384$

$x=^{-12+}$

④

Чистовик

№2

$\sqrt{x+1}-\sqrt{4-x}+3=2\sqrt{4+3x-x^2}$

$\sqrt{x+1}-\sqrt{4-x}+3=2\sqrt{(4-x)(x+1)}$

~~Проведём замену~~

~~$\sqrt{x+1}=a$~~

$\cancel{x}+1-2\sqrt{x+1}\sqrt{4-x}+4-\cancel{x}=4(4-x)(x+1)-12\sqrt{4-x}\sqrt{x+1}+9$

$16x+16-4x^2-4x+9-5=10\sqrt{x+1}\sqrt{4-x}$

$-4x^2+12x+20=10\sqrt{x+1}\sqrt{4-x}$

$-x^2+3x+5=\frac{10\sqrt{x+1}\sqrt{4-x}}{4}=\frac{5\sqrt{x+1}\sqrt{4-x}}{2}$

$(4-x)(x+1)+1=\frac{10\sqrt{x+1}\sqrt{4-x}}{4}$

Заменим

$\sqrt{x+1}\sqrt{4-x}=a$

$a^2+1=\frac{10a}{4}$

$4a^2+4=10a$

$4a^2-10a+4=0$

$D=100-64=36$

$a=\frac{10+6}{8}=2$

$a=\frac{10-6}{8}=\frac{1}{2}$

~~$a^2-10a+1=0$~~

~~$D=100-4=96$~~

~~$a=\frac{10+4\sqrt{6}}{2}=5+2\sqrt{6}$~~

~~$a=\frac{10-4\sqrt{6}}{2}$ – пост. кор. $a\geq 0$~~

~~$\sqrt{x+1}\sqrt{4-x}=5+2\sqrt{6}$~~

~~$(x+1)(4-x)=25-20\sqrt{6}+24$~~

~~$4+3x-x^2=49-20\sqrt{6}$~~

~~$x^2-3x+45-20\sqrt{6}=0$~~

$\left[\begin{array}{l}\sqrt{x+1}\sqrt{4-x}=2\\ \sqrt{x+1}\sqrt{4-x}=\frac{1}{2}\end{array}\right.\quad\left[\begin{array}{l}4+3x-x^2=4\\ 4+3x-x^2=\frac{1}{4}\end{array}\right.$

Чистовик

Также из (1)

$$y^2 = \frac{16}{9} - x^2$$

$$4 - 4x^2 = \frac{16}{9} - x^2$$

$$3x^2 = \frac{20}{9}$$

$$x^2 = \frac{20}{27} \qquad x = \frac{2\sqrt{5}}{3\sqrt{3}}$$

$$y^2 = \frac{16}{9} - \frac{20}{27} = \frac{28}{27} \qquad y = \frac{2\sqrt{7}}{3\sqrt{3}}$$

$$S = \frac{(y + \sqrt{1-x^2})(x + \sqrt{4-y^2})}{2}$$

$$y = 2\sqrt{1-x^2}$$

Рас-м $\angle PAD$

$\sin\angle PAD = \frac{x}{1}$ (из $\triangle PAD$) $\sin\angle PAD = \frac{x + \sqrt{4-y^2}}{3}$

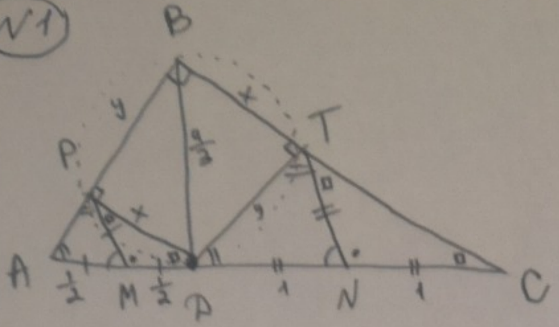
$$2x = \sqrt{4-y^2}$$

$$S = \frac{\frac{3y}{2} \cdot 3x}{2} = \frac{9xy}{4}$$

$$S = \frac{\frac{\cancel{9} \cdot 4\sqrt{35}}{2\cancel{7}^{3}}}{4} = \frac{\sqrt{35}}{3}$$

Чистовик

№1



а) Проведём PD и DT

$PD \perp AB$ $DT \perp AB$ ($\angle BTD$ и $\angle BPD$ опир. на DB - диам.)

PM и TN - медиана в прям. Δ

$\Downarrow$

$PM = AM = MD$ $TN = NC = DN$

$\angle PMA = \angle TND$, ΔTND и ΔPMA - равноб.
(PM‖TN)

$\Downarrow$

$\angle APM = \angle PAM = \angle NTD = \angle NDT$

$\angle PMD = \angle TNC$ (PM‖TN), Δ-ки равн.

$\Downarrow$

$\angle MPD = \angle PDM = \angle NTC = \angle TCN$

т.к. $DT \perp BC \Rightarrow \angle DTC = 90^\circ = \angle DTN + \angle NTC$

$\angle PDT = 180^\circ - \angle PDM - \angle TDN = 180^\circ - \underbrace{\angle DTN - \angle NTC}_{90^\circ} = 90^\circ$

PBTD - четыр., впис. в окр.

$\angle PBT + \angle PDT = 180^\circ \Rightarrow \angle PBT = 180^\circ - \angle PDT = 90^\circ$

б) Обозначим $PB = DT = y$ $BT = PD = x$ (PBDT - прямоуг.)

Запишем имеющиеся данные

$$x^2 + y^2 = \frac{16}{9} \quad (1)$$

$$CT = \sqrt{4 - y^2} \quad AP = \sqrt{1 - x^2}$$

$$S_{ABC} = \frac{(y + \sqrt{1 - x^2}) \cdot (x + \sqrt{4 - y^2})}{2}$$

Рас-м $\angle TCD$

$\sin \angle TCD = \frac{y}{2}$ (из ΔTDC) $\sin \angle TCD = \frac{y + \sqrt{1 - x^2}}{3}$ (из ΔABC)

$$\frac{y}{2} = \frac{y + \sqrt{1 - x^2}}{3} \quad y = 2\sqrt{1 - x^2}$$

$$1 - x^2 = \frac{y^2}{4} \quad 4 - 4x^2 = y^2$$

Часть 2

Олимпиада: **Математика, 10 класс (2 часть)**

Шифр: **211005555**

ID профиля: **166783**

Вариант 12

Чистовик

№1

$$\begin{cases} \frac{1}{x^2+y^2} + x^2y^2 = \frac{5}{4} \\ 2x^4 + 2y^4 + 5x^2y^2 = \frac{9}{4} \end{cases}$$

$$2(x^4+y^4+2x^2y^2)+x^2y^2=\frac{9}{4}$$

$$\begin{cases} \frac{1}{x^2+y^2} + x^2y^2 = \frac{5}{4} \\ 2(x^2+y^2)^2 + x^2y^2 = \frac{9}{4} \end{cases}$$

Заменим:

$a = x^2+y^2$

$b = x^2y^2$

$$\begin{cases} \frac{1}{a} + b = \frac{5}{4} & (2) \\ 2a^2 + b = \frac{9}{4} & (1) \end{cases}$$

$(1)-(2) = 2a^2 - \frac{1}{a} = \underline{1}$

$2a^3 - a - 1 = 0$

Подбором $a = \underline{1}$ $\nearrow$

$$\begin{array}{l|l} 2a^3 \dots\dots -a-1 & a-1 \\ \cline{2-2} \underline{2a^3-2a^2} & 2a^2+2a+1 \\ \quad 2a^2-a & \\ \quad \underline{2a^2-2a} & \\ \qquad \underline{a-1} & \\ \qquad\quad 0 & \end{array}$$

$2a^2+2a+1=0$

$D=4-8$ корней нет

Из (2) $\frac{1}{1}+b=\frac{5}{4}$ $b=\frac{1}{4}$

$$\begin{cases} x^2+y^2=1 \\ x^2y^2=\frac{1}{4} \end{cases} \qquad x^2=1-y^2$$

$(1-y^2)y^2=\frac{1}{4}$ $\qquad y^2-y^4=\frac{1}{4}$

$4y^4-4y^2+1=0$

$(2y^2-1)^2=0$ $\qquad y^2=\frac{1}{2}$ $\quad y=\pm\frac{\sqrt{2}}{2}$

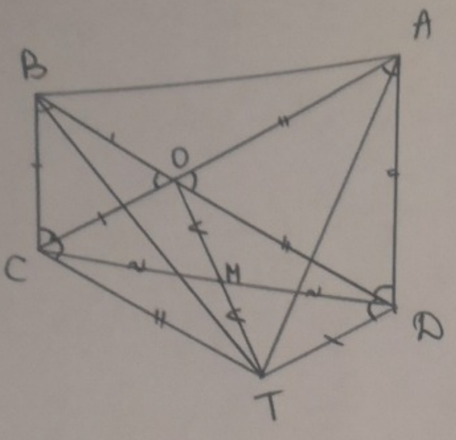
$y=\frac{\sqrt{2}}{2} \Rightarrow x^2=1-y^2=\frac{1}{2}$ $\quad x=\pm\frac{\sqrt{2}}{2}$

$y=-\frac{\sqrt{2}}{2} \Rightarrow x=\pm\frac{\sqrt{2}}{2}$

Ответ: $\left(\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$ $\left(\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2}\right)$ $\left(-\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$ $\left(-\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2}\right)$

Чистовик

№6



Проведём CT и DT

$OM = MT \quad CM = MD$

$\angle OMC = \angle DMT$

$\Downarrow$

$\triangle OMC = \triangle MDT$

$\angle MDT = \angle OCM$

$OC \parallel DT$, $OM = MT \quad CM = MD$

$\Downarrow$

ODTC – парал.

$OC = DT \quad OD = CT$

Рас-м $\triangle ABD$ и $\triangle ACT$

Обоз-м $BC = BO = OC = DT = x \quad AD = AO = OD = CT = y$

$BD = x + y = AC$

$CT = AD$

$60° = \angle OCT = \angle AOD$ ([illegible] ODTC – парал.)

$\Downarrow$

$\triangle = \triangle \Rightarrow AT = AB$

Рас-м $\triangle ABC$ и $\triangle BDT$

$DB = x + y = AC$

$BC = DT$

$\angle ODT = \angle BOC = 60°$ (ODTC – парал.)

$\Downarrow$

$\triangle = \triangle \Rightarrow BT = AB$

$BT = AB = AT \Rightarrow$ $\triangle ABT$ – равност.

Рас-м $\triangle ADT$

$AT^2 = AD^2 + DT^2 - 2AD \cdot DT \cdot \cos\angle ADT \qquad \angle ADT = 60° + 60° = 120°$

$AT^2 = 16 + 4 + \cancel{2} \cdot 2 \cdot 4 \cdot \frac{1}{\cancel{2}} = 28 \qquad AT = \sqrt{28} = 2\sqrt{7}$

$S_{ABT} = \frac{\sqrt{3}\,AT^2}{4} = \frac{28 \cdot \sqrt{3}}{4} = 7\sqrt{3}$

($\angle AOB = 180 - 60 = 120°$)

$S_{ABCD} = S_{AOD} + S_{BOC} + 2S_{AOB} = \frac{\sqrt{3}\,AD^2}{4} + \frac{\sqrt{3}\,BC^2}{4} + \cancel{2} \cdot AO \cdot OB \cdot \frac{1}{\cancel{2}} \cdot \sin\angle AOB$

$S_{ABCD} = 4\sqrt{3} + \sqrt{3} + 8 \cdot \frac{\sqrt{3}}{2} = 9\sqrt{3}$

$\frac{S_{ABT}}{S_{ABCD}} = \frac{7\sqrt{3}}{9\sqrt{3}} = \frac{7}{9}$

Ответ: отнош. – $\frac{7}{9}$

Чистовик

№5

Представим координаты двух узлов в виде $(a;b)$ и $(c;d)$

Чтобы оба узла не лежали на прямой, парал. какой-либо оси, нужно соблюдать условие

$a \neq c \quad b \neq d$

Начнём выбирать узлы:

a может быть с любой x-й коорд. внутри кв.

$\Downarrow$

62 выбора

b в зависимости от a может быть

$b=a \quad b=63-a$ (2 выбора всегда – пересечение графиков не находится в узле, т.к. $x_{пер.}=\frac{63}{2}=31,5$)

$\Downarrow$

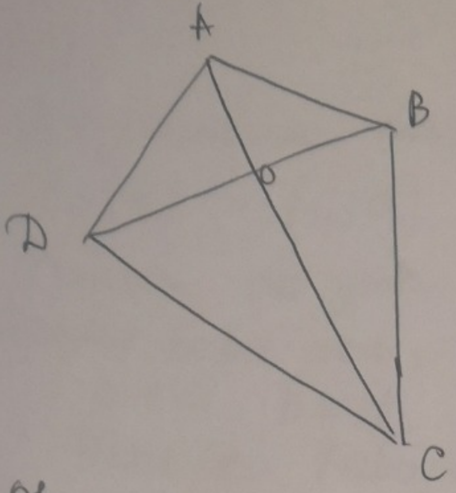
2 выбора

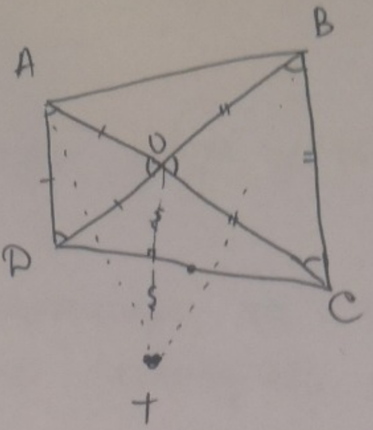
Второй же узел не обязательно должен располагаться на прямых $y=x$ $y=63-x$, но его координаты не должны совпадать с $(a;b)$

$c \to 61$ точка (выбор) $d \to 61$ точка (выбор)

Колво способов: $62 \cdot 2 \cdot 61 \cdot 61 = 461.404$

Черновик





61
× 61
61
366
3721
× 62
7442
22326
230702
2
461404

61
× 61
61
366
3721
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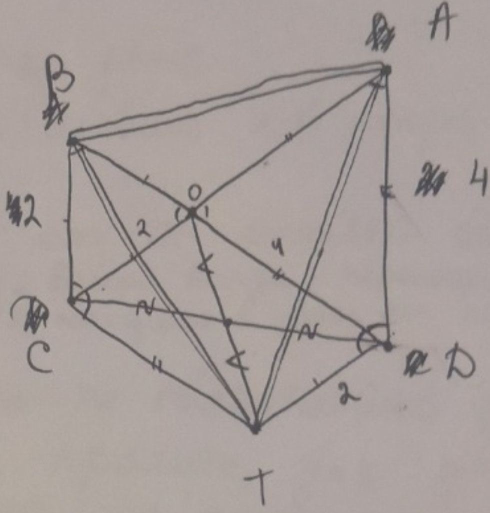
230702
461404



$$S = \cancel{2} \cdot 4 \cdot \frac{1}{\cancel{2}} \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}$$

$$4\sqrt{3} + 5\sqrt{3}$$

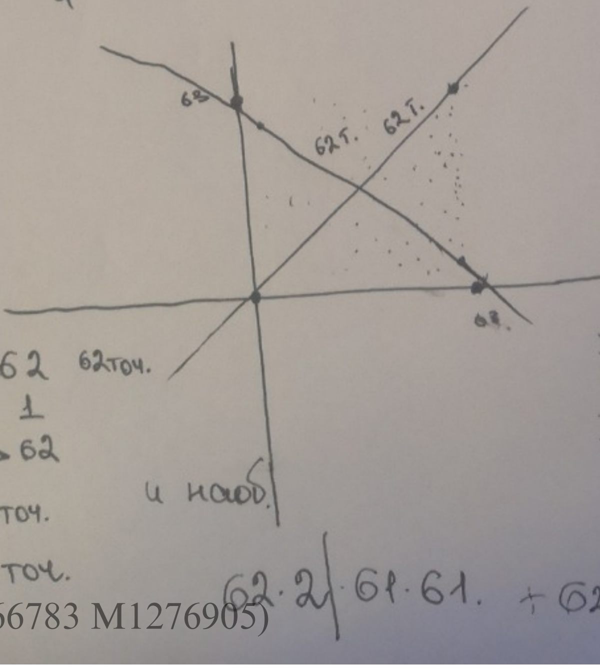
$$\frac{4 \cdot \sqrt{3}}{4} = \sqrt{3} \qquad \frac{16\sqrt{3}}{4} = 4\sqrt{3}$$

$$a = 16 + 4 + \cancel{2} \cdot 8 \cdot \frac{1}{\cancel{2}} =$$

$$= 28 \qquad a = \sqrt{28} = 2\sqrt{7}$$

$$S_{ABT} = \frac{28 \cdot \sqrt{3}}{4} = 7\sqrt{3}$$

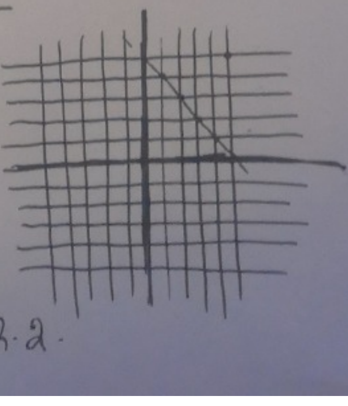
$$\frac{7\sqrt{3}}{9\sqrt{3}} = \frac{7}{9}$$



$(a, b) \quad (c, d)$

$a \neq c$

$b \neq d$



$a \to 1 \ldots 62$ 62точ.

$b \to 1 \rightrightarrows \begin{matrix} 1 \\ 62 \end{matrix}$

и наоб.

$c \to$ 61 точ.

$d \to$ 61 точ.

$62 \cdot 2 \mid 61 \cdot 61. \; + 62 \cdot 2.$

Черновик

$$\begin{cases} \frac{1}{x^2+y^2} + x^2y^2 = \frac{5}{4} \\ 2x^4 + 2y^4 + 5x^2y^2 = \frac{9}{4} \end{cases}$$

$(2a^2+2a+1)(a-1) =$

$= 2a^3 \cancel{-2a^2} + \cancel{2a^2} - 2a + a - 1$

$$\begin{cases} \frac{1}{x^2+y^2} + x^2y^2 = \frac{5}{4} \\ 2(x^2+y^2)^2 + x^2y^2 = \frac{9}{4} \end{cases}$$

$x^2+y^2 = a$

$x^2y^2 = b$

$2(x^2+y^2)^2 - \frac{1}{x^2+y^2} = 1$

$2(x^2+y^2)^3 - 1 = x^2+y^2$

$$\begin{array}{l|l} 2a^3 - a - 1 & a-1 \\ \underline{2a^3 - 2a^2} & \overline{2a^2+2a+1} \\ \quad 2a^2 - a & \\ \quad \underline{2a^2 - 2a} & \\ \qquad a - 1 & \end{array}$$

$$\begin{cases} \frac{1}{a} + b = \frac{5}{4} \\ 2a^2 + b = \frac{9}{4} \end{cases}$$

[illegible]

$\frac{1}{1} + b = \frac{5}{4}$

$b = \frac{1}{4}$

$8a^2 + 4b = \frac{9}{4}$

$2a^2 - \frac{1}{a} = 1$

$2a^3 - 1 = a$

$2a^3 - a - 1 = 0$

$2a^2 + 2a + 1 = 0$

$D = 4 - 8$ …

$x^2 = \frac{1}{2}$

$\boxed{x = \frac{\sqrt{2}}{2}}$

[illegible]

$1 + \frac{1}{4}$

$$\begin{cases} x^2+y^2 = 1 \\ x^2y^2 = \frac{1}{4} \end{cases}$$

$x^2 = 1 - y^2$

$(1-y^2)y^2 = \frac{1}{4}$ $\quad y^2 - y^4 = \frac{1}{4}$

$y^4 - y^2 + \frac{1}{4} = 0$

$4y^4 - 4y^2 + 1 = 0$

[illegible] $(2y^2 - 1)^2 = 0$

$2y^2 = 1$ $\quad y^2 = \frac{1}{2}$ $\quad \boxed{y = \frac{\sqrt{2}}{2}}$