Часть 1

Олимпиада: **Математика, 10 класс (1 часть)**

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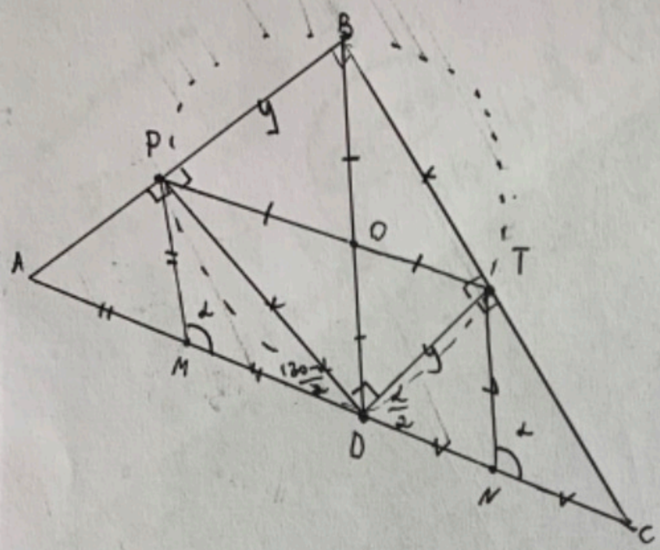
Вариант 11



пусть O – середина BD, тогда

O – центр окружности

⇓

OP = OT = OB = OD

рассмотрим ∠BTD – он вписанный и опир. на диаметр BD

⇓

$\angle BTD = 90°$

⇓

$\angle DTC = 180° - 90° = 90°$ (как смежные)

аналогично

$\angle BPD = \angle APD = 90°$

⇓

△DTC и △APD – прямоуг.

$\angle PMD = \angle TNC =$ (как соответств. $= \alpha$ при PM ∥ TN)

а т.к. △DTC и △APD прямоуг. и TN и PM – медианы то

TN = DN = NC

PM = AM = MD

⇓

△PMD, △AMP, △DTN, △TNC – равнобедр.

⇓

$\angle NTC = \angle NCT = \frac{180° - \alpha}{2} =$

$= \angle MDP = \angle MPD$

$\angle NDT = 90° - \angle NTC =$

$= 90° - 90° + \frac{\alpha}{2} = \frac{\alpha}{2}$

⇓

$\angle PDT = 180° - 90 + \frac{\alpha}{2} - \frac{\alpha}{2} = 90°$

1

⇓

$\angle PBT = 360° - 90° - 90° - 90° = 90° = \angle ABC$

2) т.к все углы $PBTD = 90°$, то он явл. прямоуг.
⇓ и т.к $PO = OT = OB = OD$

O – точка пересеч. его диагоналей.

⇓

$PB = DT = y$

$PD = BT = x$

$PT = BD = \frac{\sqrt{3}}{2} \cdot 2 = \sqrt{3}$

$PM = AM = MO = \frac{1}{2}$ ~~DC = DT~~ =

$DN = NC = NT = 2$

⇓

$AC = 5$

$x^2 + y^2 = 3$ (из $\triangle PBT$)

$S_{ABC} = \frac{AB \cdot BC}{2}$.

$\triangle APD \sim \triangle DTC$ (из п.1)

⇓

$\frac{AP}{DT} = \frac{AD}{DC}$.

$\frac{AP}{y} = \frac{1}{4}$ $AP = \frac{y}{4}$

аналогично

2

$\triangle$ $\frac{PD}{TC} = \frac{AD}{DC} = \frac{1}{4}$

$\frac{x}{TC} = \frac{1}{4}$ $TC = 4x$

⇓

$BC = x + 4x = 5x$ $AB = y + \frac{y}{4} = \frac{5}{4}y$

$$\frac{AB \cdot BC}{2}$$

$$\Downarrow$$

$$S_{ABC} = \frac{5x \cdot \frac{5}{4}y}{2} = \frac{25xy}{8}$$

$y^2 - \frac{y^2}{16}$

из т. Пиф. для $\triangle ABC$

$$\frac{25}{16}y^2 + 25x^2 = 25$$

$$\frac{y^2}{16} + x^2 = 1 \qquad x^2 + y^2 = 3$$

$$x^2 = 3 - y^2$$

$$\frac{y^2}{16} + 3 - y^2 = 1$$

$$2 = \frac{15}{16}y^2$$

$$32 = 15y^2$$

$$y = \sqrt{\frac{32}{15}}$$

$$x = \sqrt{3 - \frac{32}{15}} = \sqrt{\frac{45-32}{15}} = \sqrt{\frac{13}{15}}$$

$$S_{ABC} = \frac{25 \cdot \sqrt{\frac{13}{15}} \cdot \sqrt{\frac{32}{15}}}{8} =$$

$$= \frac{25\sqrt{\frac{13 \cdot 32}{15^2}}}{2} =$$

$$= \frac{25\sqrt{13 \cdot 32}}{8 \cdot 150} = \frac{5\sqrt{416}}{24}$$

(3)

Ответ: $S_{ABC} = \frac{5\sqrt{416}}{24}$

$\angle ABC = 90°$

$\sqrt{x+2} - \sqrt{3-x} + 3 = 2\sqrt{6+x-x^2}$

заметим что $\sqrt{x+2}\sqrt{3-x} = \sqrt{6+x-x^2}$

пусть $\sqrt{x+2} = a$, $a \geq 0$

$\sqrt{3-x} = b$, $b \geq 0$

$\Downarrow$

$$\begin{cases} a-b+3 = 2ab \\ a^2+b^2 = x+2+3-x = 5 \end{cases}$$

$a = \sqrt{5-b^2}$

$\sqrt{5-b^2} - b + 3 = 2b\sqrt{5-b^2}$

$\sqrt{5-b^2}(1-2b) = b-3$

$(5-b^2)(1-4b+4b^2) = b^2-6b+9$

$5-20b+20b^2-b^2+4b^3-4b^4 = b^2-6b+9$

$4b^4-4b^3-12b^2+14b+4=0$

$b=1$ яв. корнем тогда

$4b^4-4b^3-12b^2+14b+4 \,|\, b-1$

$-4b^4-4b^3$ | $4b^3-12b-4$

$-12b^2$

$-12b^2+12b$

$-4b+4$

$-4b+4$

0

(В)

$(4b^3-12b-4)(b-1)=0$

$\checkmark$

$b=-2$ яв. корнем.

$(4b^2-8b-2)(b+2)(b-1)=0$

$2(2b^2-4b-1)(b+2)(b-1)$

$D = 16+8=24$

$b = \frac{4 \pm \sqrt{24}}{4} = 1 \pm \frac{\sqrt{6}}{2}$

т.к $b \geq 0$, то

$b_1 = 1; \quad b_2 = 1+\frac{\sqrt{6}}{2}$

$\sqrt{3-x} = 1$ $\quad\quad$ $\sqrt{3-x} = 1+\frac{\sqrt{6}}{2}$

$3-x = 1$ $\quad\quad$ $3-x = 1+\frac{6}{4}+\sqrt{6}$

$x=2$ $\quad\quad$ $0,5-\sqrt{6} = x$

Ответ: $x=2$

$x = 0,5-\sqrt{6}$

(5)

№3 $ax^2 - 2a^2x - ay + a^3 + 4 = 0$

$y = x^2 - 2ax + \left(a^2 + \frac{4}{a}\right)$

$B(x_0; y_0)$ $x_0 = \frac{2a}{2} = a$

(5)

$y_0 = a^2 - 2a^2 + a^2 + \frac{4}{a} = \frac{4}{a}$

$B\left(a; \frac{4}{a}\right)$

$a(5a + 12x + 4y) + (4x^2 + 8xy + 4y^2) + 4x^2 = 0$

$a(5a + 12x + 4y) + 4x^2 + ~~(2x+2y)^2~~ \; 4(x+y)^2 = 0$

a ~~(5a+12~~ . $5a^2 + 12ax + 4x^2 + 4ya + 4(x+y)^2 = 0$

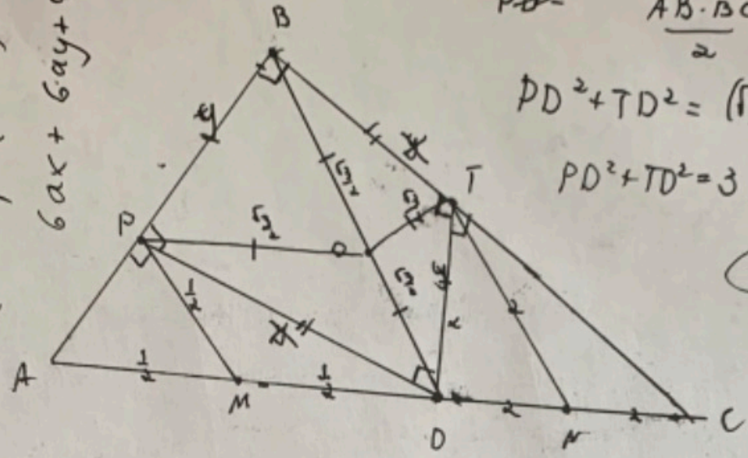
~~5a~~$(9a^2 + 12ax + 4x^2) - 4a^2 + 4ay + 4(x+y)^2 = 0$

$(3a + 2x)^2 + 4(x+y)^2 - 4(a^2 - ay) = 0$

$(3a + 2x)^2 + 4(x+y)^2 - 4a(a - y) = 0$

$3(a+2x)\cdot(2x+2y)$

$6ax+6ay+4x^2+4xy$



P

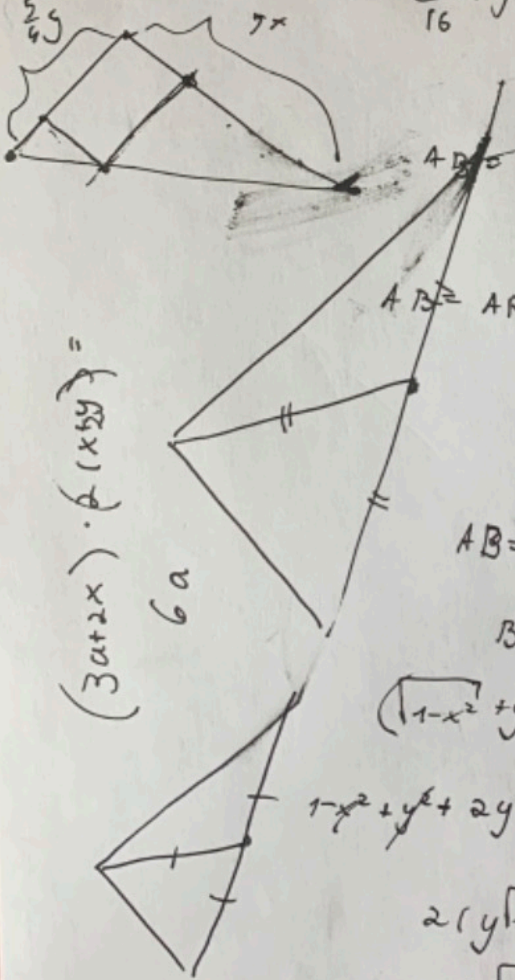
$PD = \frac{AB\cdot BC}{2}$

$PD^2+TD^2=(\sqrt{3})^2$

$PD^2+TD^2=3$ $AD=1$ $DC=4$

$AC=5$

$\frac{25}{16}y^2+25x^2=25$



$AD = AP+PB = 1-x$

$PD=x$ $TD=y$

$AB = AP+PB = AP+TD =$ $x^2+y^2=3$

$x=\sqrt{3-y^2}$

$= 1-$

$(3a+2x)\cdot 2(x+2y) =$

$6a$

(AP)

$AB=\sqrt{1-x^2}+y$

$BC=\sqrt{16-y^2}+x$

$(\sqrt{1-x^2}+y)^2+(\sqrt{1-y^2}+x)^2=25$

$1-x^2+y^2+2y\sqrt{1-x^2}+16-y^2+x^2+2x\sqrt{16-y^2}=25$

$2(y\sqrt{1-x^2}+x\sqrt{16-y^2})=23$

$y\sqrt{1-x^2}+x\sqrt{1-y^2}=11,5$

$y\sqrt{1-3+y^2}+\sqrt{3-y^2}\sqrt{1-y^2}=11,5$

$$y\sqrt{y^2-2} + \sqrt{y^4-4y^2+3} = 11{,}5$$

$$y\sqrt{1-x^2} + x\sqrt{16-y^2} = 4$$

$$\left(\sqrt{16-y^2}+x\right)^2 + \left(y+\sqrt{1-x^2}\right)^2 = 25$$

$$x^2 + 16 - y^2 + 2x\sqrt{16-y^2} + y^2 + 2y\sqrt{1-x^2} + 1 - x^2 = 25$$

$$\begin{array}{r} 16 \\ \times\ 3 \\ \hline 48 \end{array}$$

$$2\left(x\sqrt{16-y^2} + y\sqrt{1-x^2}\right) = 4$$

$$\sqrt{3-y^2}\sqrt{16-y^2} + y\sqrt{1-3+y^2} = 4$$

$$\sqrt{y^4-19y^2+48} + y\sqrt{y^2-2} = 4$$

$$y\sqrt{1-x^2} + x\sqrt{1-y^2} = 11{,}5$$

$$y\sqrt{1-3+y^2} + \sqrt{3-y^2}\sqrt{1-y^2} = 11{,}5$$

$= 11,5$

$\sqrt{1,5-\sqrt{6}} - \sqrt{1,5+\sqrt{6}} = 2$

$(4b^3-12b-4)(b-1)=0$ $\quad 4b^4.$

$4b^3-12b-4=0$ $\quad 3\cdot2-36-4$ $\quad 3$

$-32+12-4$

$4b^3-12b-4 \,|\, b+2$

$-4b^3+8b^2 \,|\, 4b^2-8b-2$ $\quad -32+36-4$

$-8b^2-12b$

$-8b^2-16b$

$-2b-4$

$-2b+4$

0

$(4b^2-8b-2)(b+2)$

$4b^3+8b^2-8b^2-16b-2b-4$

$4b$

$4b^2-8b-2=0$

$2b^2-4b-1=0$

$D=16+8=24$

$b=\frac{4\pm\sqrt{24}}{4}=1\pm\frac{2}{4}\sqrt{6}=$

$=1\pm\frac{\sqrt{6}}{2}$

$\sqrt{3-x}=1$

$3-x=1$

$x=2$

$\sqrt{3-x}=1+\frac{\sqrt{6}}{2}$

$\sqrt{3-x}$ $\quad 3-x=1+\sqrt{6}+\frac{6}{4}$

$x=3-1-\sqrt{6}$

$2-x=\sqrt{6}+\frac{6}{4}=\sqrt{6}+1,5$

$0,5-\sqrt{6}=x$

$y\sqrt{y}$

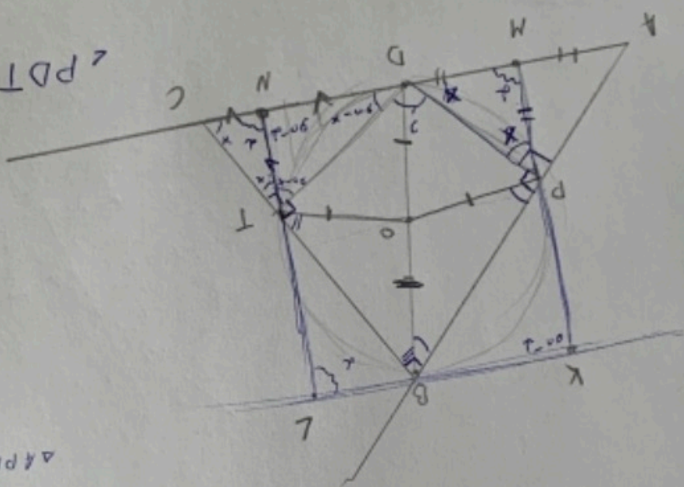
$(4b^2 - 8b - 2)(b^2 + b - 2)$

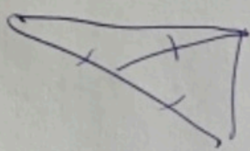
$4b^4 + 4b^3 - 8b^2 - 8b^3 - 8b^2 + 16b - 2b^2 - 2b + 4 = 0$

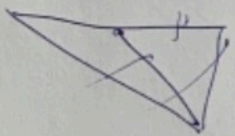
$4b^4 - 4b^3 - 18b^2 + 14b + 4$

$x^2 +$

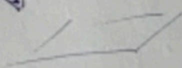
$\angle PDT = 180 - x - 90 + x = 90°$

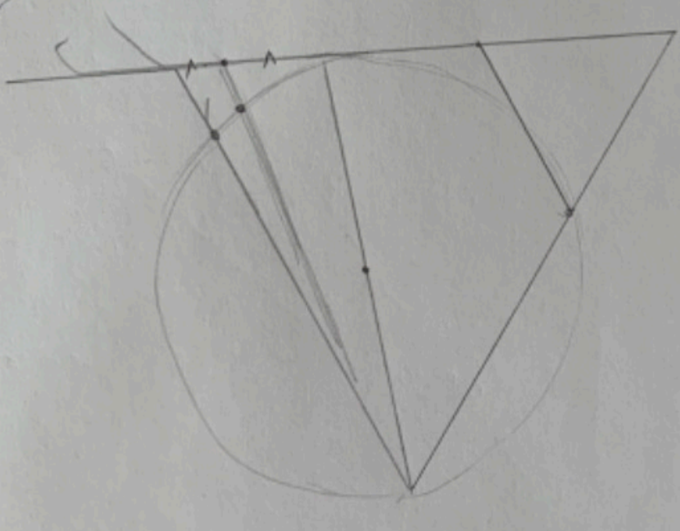






$\triangle KPD$





$y\sqrt{1-3+y^2}$

$a \geq 0 \quad b \geq 0$

$a^2 + b^2 = 5$

$a - b + 3 = 2ab$

$a = \frac{b-3}{1-2b}$

$a - 2ab - b = -3$

$a + 3 = 2ab + b$

$a^2 + 6a + 9 = 4a^2b^2 + 4ab^2 + b^2$

$a^2 + 6a + 9 = b^2(4a^2 + 4a + 1)$

~~$a = b +$~~

$a = \sqrt{5 - b^2}$

$\sqrt{5-b^2} - b + 3 = 2\sqrt{5-b^2}\, b$

~~$\sqrt{5-b^2}$~~ $\sqrt{5-b^2}(1 - 2b) = b - 3$

$(5-b^2)(1 - 4b + 4b^2) = b^2 - 6b + 9$

$5 - 20b + 20b^2 - b^2 + 4b^3 - 4b^4 = b^2 - 6b + 9$

$4b^4 - 4b^3 - 18b^2 + 14b + 4 = 0$

$b = 1$ явл. корнем.

$4b^4 - 4b^3 - 18b^2 + 14b + 4 \;|\; b - 1$

$- \; 4b^4 - 4b^3 \qquad\qquad 4b^3 - 18b - 4$

$0 \quad -18b^2 + 14b$

$- \; -18b + 18b$

$-4b + 4$

0

$ax^2-2a^2x-ay+a^3+4=0$

$B=(x_0;y_0)$ $\quad ax^2-2a^2x=-(ay-a^3-4)$

$x_0=\frac{2a^2}{}$

$\sqrt{16}\sqrt{2\sqrt{2}}$

$ay=ax^2-2a^2x+(a^3+4)$

$2\cdot2\cdot2\sqrt{2}$ $\quad 8\sqrt{2}$

$y=x^2-2ax+\left(a^2+\frac{4}{a}\right)$

$\sqrt{5}\cdot2$

$x_0=\frac{2a}{2}=a$

$y_0=a^2-2a^2+a^2+\frac{4}{a}=\boxed{\frac{4}{a}}$

$\sqrt{5}\cdot\sqrt{8}\,\sqrt{24}$

$B\left(a;\frac{4}{a}\right)$ $\quad 2\sqrt{2}x \quad 2y$

$\sqrt{8}\cdot2\cdot2^2=\sqrt{2\cdot16}=4\sqrt{8}$

$5a^2+12ax+4ay+3x^2+8xy+4y^2=0$ $\quad +8xy-8xy$

$a(5a+12x+4y)+$

$4(2x^2+2xy+1)=0$

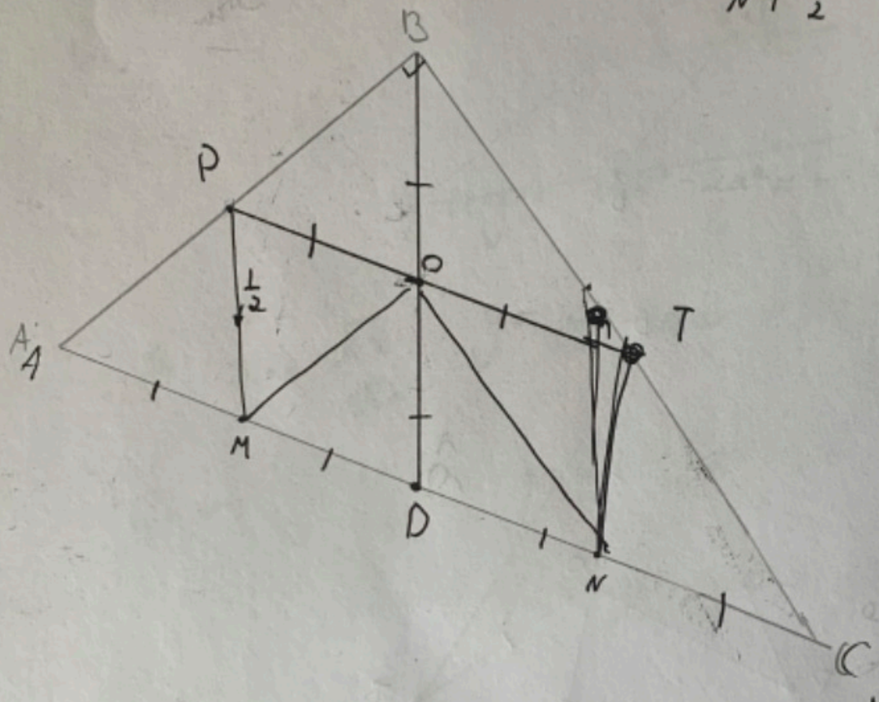
$3x^2+12ax+5a^2$

~~$5a^2+12ax+4ay+3x^2+16xy+4y^2=2xy$~~

$y(1-3+y^2$ + ...

$MP = \frac{1}{2}$

$NT\ 2$



$8 = 2 \cdot 2 \cdot 2$

Часть 2

Олимпиада: **Математика, 10 класс (2 часть)**

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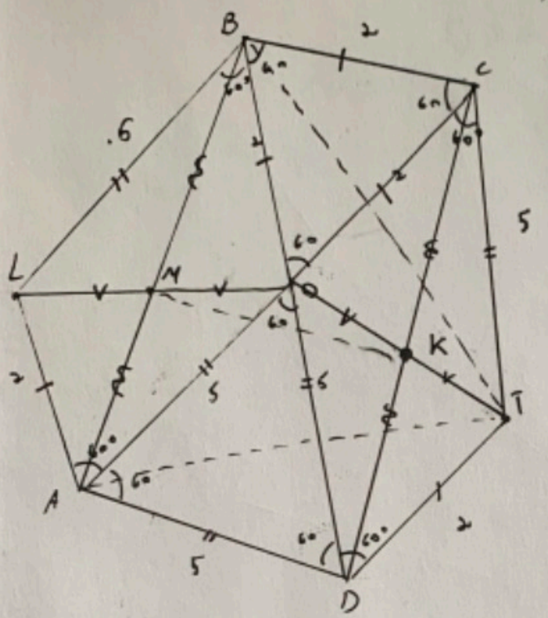
ID профиля: **816226**

Вариант 11

№6



1) т.к $ABCD$ вып.

и $\angle CBD = \angle BDA$ (как [illegible] скрещив.)

⇓

$BC \parallel AD$

⇓

$ABCD$ – равнобедр. трапец.

(т.к $\triangle BOA = \triangle COD$ по 2 ст. и углу между ними) ⇒ $AB = CD$

рассмотрим $OCTD$:

$OK = KT$ $DK = KC$

DC и OT – диагонали

⇓

$OCDT$ – пар-м.

⇓

$OC = DT$

$DO = CT$

$\angle COD = \angle CTD = 120°$

$\angle ODT = \angle OCT = 60°$

$\triangle BCT$; $\triangle ADT$:

$AD = CT$ $DT = BC$ (т выше док.)

$\angle ADT = 120° = \angle BCT$

⇓

$\triangle BCT = \triangle ADT$ (по 2 ст. и ∠ между ними)

⇓

$AT = TB$

⇓

$\triangle ABT$ – равнобедр.

т.к M – середина AB

$ABCD$ – равн. трап. то $MK \parallel AD \parallel BC$

$\angle M = MO = OK = KT$ (из прав. трапеции)

$MA = MB = KC = CK$

$\angle BOA$ – пар-м

⇓

$\angle B = AO$

$AL = OB$

$\angle LAO = \angle LBO = 60°$

⇓

$\Delta ALB = \Delta ADT$ ($LB = AD$; $DT = LA$; $\angle ALB = \angle ADT = 120°$)

⇓

$\Delta ALB = \Delta ADT$ (по 2 ст. и ∠ между ними)

⇓

$AB = AT = TB$

⇓

Δ BAT – правильный.

ч.т.д

2) $S_{ABCD} = \frac{BC + AD}{2} \cdot h = 3,5h$

$BT = AT = AB$ (найдем их по т. косинусов для ΔBCT):

$BT^2 = 4 + 25 + 2 \cdot 10 \cdot \cos 120°$

$BT^2 = 29 + 20 \cos(90° + 30°)$

$BT^2 = 29 - 20 \sin 30° = 29 - 10 = 19$

$BT = \sqrt{19} = AB = AT = CD$ (т.к ABCD – равн. трапец.)

⇓

$S_{ABT} = \frac{1}{2} BT^2 \cdot \sin 60° = \frac{1}{2} \cdot 19 \cdot \frac{\sqrt{3}}{2} = \frac{19\sqrt{3}}{4}$

$h = \sqrt{AB^2 - \left(\frac{AD - BC}{2}\right)^2} = \sqrt{19 - 1,5^2} =$

$= \sqrt{19 - \frac{9}{4}} = \sqrt{\frac{19 \cdot 4 - 9}{4}} = \frac{\sqrt{67}}{2}$

$S_{ABCD} = \frac{7}{2} \cdot \frac{\sqrt{67}}{2} = \frac{7\sqrt{67}}{4}$ $\Rightarrow$ $\frac{S_{ABT}}{S_{ABCD}} = \frac{19\sqrt{3} \cdot 4}{4 \cdot 7\sqrt{67}} =$

-3+15-4

8

$\frac{19}{7}\sqrt{\frac{3}{67}}$

Ответ:

1) г.т.:

2) $\frac{S_{ABT}}{S_{ABCD}} = \frac{19}{7}\sqrt{\frac{3}{67}}$

№5 Всего способов выбрать 2 точки внутри квадрата

$$n_1 = C^2_{64^2} = \frac{(64^2)!}{(64^2-2)!\,2!} = \frac{64\cdot 63}{2} = 32\cdot 63 = \frac{(64^2)!}{(64^2-2)!\,2}$$

узлов прин. прямым $y = x$ и $y = 65 - x$ (диагонали)

$$k = 64\cdot 2 - 1 = 127$$

тогда остальных точек

$$t = 64^2 - 127$$

тогда узлов не принадл. обеим из прямых мы посчитали

$$n_2 = C^2_{64^2-127} = \frac{(64^2-127)!}{(64^2-129)!\,2!}$$

также мы посчитали случаи когда 2 точки лежат на парал. осям прямых. всего таких пар мы насчитали

$$n_3 = 2\cdot\left(63\cdot\left((65-2)+(65-3)\right)+(65-2)\right) =$$

$$= 2\cdot 63\cdot 125 + 126.$$

тогда всего способов:

$$N = n_1 - n_2 - n_3 = \frac{(64^2)!}{(64!-2)!\cdot 2!} - \frac{(64^2-127)!}{(64^2-129)!\,2!} - 126\cdot 125 - 126$$

④ Ответ: $N = \frac{(64^2)!}{(64!-2)!\cdot 2!} - \frac{(64^2-127)!}{(64^2-129)!\,2!} - 126\cdot 125 - 126$

№4

$x^2+y^2>0$

$$\begin{cases} \frac{4}{x^2+y^2} + x^2y^2 = 5 \\ (x^2+y^2)^2 + x^2y^2 = 20 \end{cases}$$

$a = (x^2+y^2)^2 \quad a \geq 0$

$\frac{4}{a} - a^2 = -15$

$4 - a^3 = -15a$

$a^3 - 15a - 4 = 0$

~~$a^3 - 15a = 4$~~

~~$a(a^2-15) = 4$~~

$4^3 = 64$

$a^3 - 15a - 4 \mid a-4$

$-\ a^3 - 4a^2 \quad \mid a^2 + 4a + 1$

$-4a - 15a$

$-4a + 16$

$a - 4$

$(a^2-4a+1)(a-4) =$

$(a^2-4a+1)(a-4) = 0$

$D = 16 - 4 = 12$

$a = \frac{4 \pm \sqrt{12}}{2} \quad a<0$ не уд. усл. $a \geq 0$

(5)

$x^2+y^2 = 4$

$x^2y^2 = 4$

$x = \pm 2 \quad y = \pm 2$

$x^2+y^2 = \frac{4+\sqrt{12}}{2} = 2+\sqrt{3}$

$x^2y^2 = 20 - 2 - \sqrt{3} = 13 - \sqrt{3}$

$x^2 = \frac{13-\sqrt{3}}{y^2}$

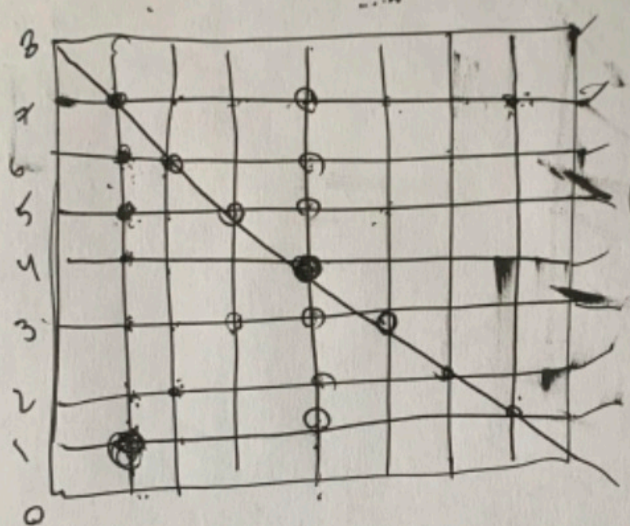
$\frac{13-\sqrt{3}}{y^2} + y^2 = 2+\sqrt{3}$

чуть чуть не успел, отдыхаем

Ответ! $x = \pm 2 \quad y = \pm 2$

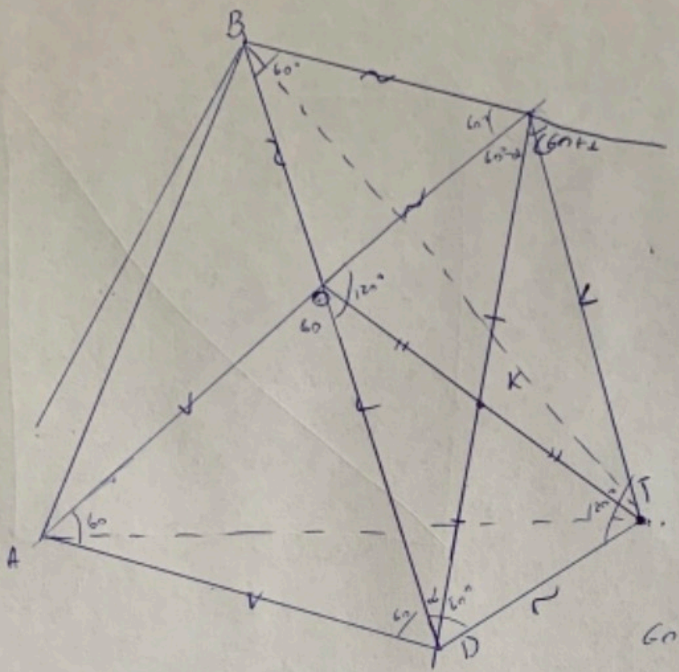
C^n_k

$64^2 = C^{64}_{2} = \frac{64!}{62!\,2!} = \frac{64 \cdot 63}{2}$



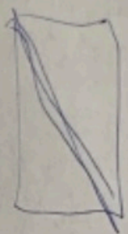
(13) $6+5$ (6)

$$\begin{array}{r} 63 \\ 62 \\ \hline 125 \end{array}$$



$120 - 60° - 60° - \alpha =$

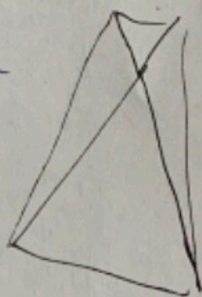
$= $ <s>120°</s> $60° - \alpha$

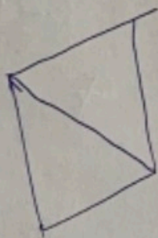


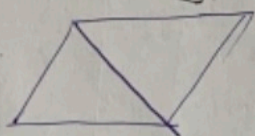
$60° - \alpha + \alpha + 120°$

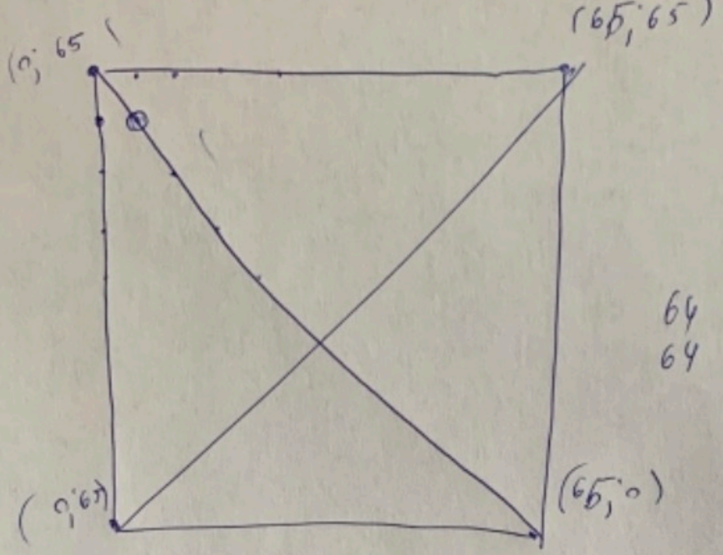
$\angle BOT = $ <s>120°</s> $60° + \angle COT$

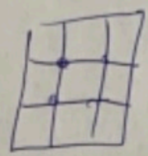
$\angle AOT = 60° \angle DOT$



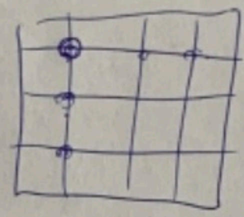


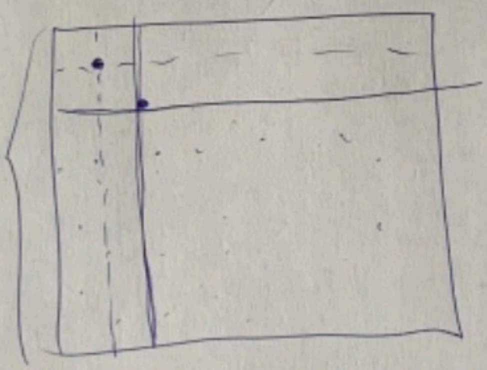






64
64



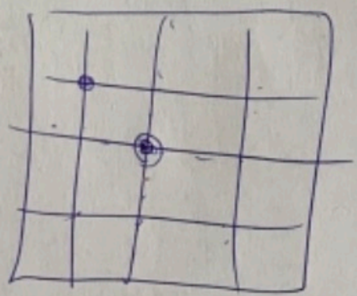


$64 \cdot 64 = (8^4) - 64 \cdot 2 =$

$= (8^4 - 128).$

64^2

$3^2 - 6 =$



$$x^2y^2 = \frac{25-\sqrt{241}}{2}$$

$$x^2+y^2 = \frac{15+\sqrt{241}}{2}$$

$$x^2 = \frac{15-\sqrt{241}}{2} - y^2$$

$$\frac{15-\sqrt{241}}{2}y^2 - y^4 = \frac{25-\sqrt{241}}{2}$$

$t^3 - 30t^2 + 225t - 434 = 0$

$225 \cdot 3 = 675$

$27 - 30 \cdot 9 + 225 \cdot 3 - 434 = 0$

$27 - 270 + 675 - 434 = 0$

$434 + 220 = 754$, $754 - 27 = 727$

~~$8 - 120 + 450 - 434 = 0$~~

16

15^2: $15 \cdot 15 = 225$, $225 + 16 = 241$

$$\begin{cases} \frac{4}{x^2+y^2} + x^2y^2 = 5 \\ (x^2+y^2) + x^2y^2 = 20 \end{cases}$$

$\frac{4}{x^2+y^2} - (x^2+y^2) = -15$, $x^2+y^2 = a$

$\frac{4}{a} - a = -15$

$\frac{4-a^2}{a} = -15$

$4 - a^2 = -15a$

$a^2 - 15a - 4 = 0$

$D = 15^2 + 4 \cdot 4 = 225 + 16 = 241$

$a = x^2+y^2 = \frac{15+\sqrt{241}}{2}$

$x^2y^2 =$

$\frac{15+\sqrt{241}}{2} + x^2y^2 = 20$

$x^2y^2 = 20 - \frac{15+\sqrt{241}}{2} = \frac{40-15-\sqrt{241}}{2} = \boxed{\frac{25-\sqrt{241}}{2}}$

$D = 25^2 - 96 \cdot 4$

$20 - x^2y^2 \geq 0$

$x^2y^2 \leq 20$

$(x^2+y^2)^2 + x^2y^2 = 20$

$x^2+y^2 = \pm\sqrt{20 - x^2y^2}$

$\frac{4}{\sqrt{20 - x^2y^2}} + x^2y^2 = 5$

$\frac{4}{\sqrt{20-a}} + a = 5$

$\frac{4 + \sqrt{20-a}\,a}{\sqrt{20-a}} = 5$

$4 + \sqrt{20-a}\,a = 5\sqrt{20-a}$

$4 = \sqrt{20-a}\,(5-a)$

$16 = (20-a)(25-10a+a^2)$

$16 = 500 - 200a + 20a^2 - 25a + 10a^2 - a^3$

$-a^3 - 30a^2 + 225a - 434 = 0$

$25 \cdot 25 = 625$

$625 - 384 = 241$

$96 \cdot 4 = 384$

$19 \cdot 19 = 261$

$19 \cdot 19 = 261$

$8 - 250 + 120 + 5m - 484$

$25 \cdot 20 = 500$

500

484

≤ 25

$(x^2+y^2)^2 + x^2y^2 = 20$

$\sout{x^2+y^2 = \sqrt{20 - x^2y^2}}$

$x^2+y^2 = \frac{4}{5-x^2y^2}$

$225 \cdot 3$ $\quad 225 \cdot 3 = 675$

$\left(\frac{4}{5-x^2y^2}\right)^2 + x^2y^2 = 20$

$27 - 270 + 675 - 434$

$x^2y^2 = t$

$675 - 270 = 405$ $\quad 25 \cdot 20 = 500$ $\quad 432$

$\left(\frac{4}{5-t}\right)^2 + t = 20$

$\frac{16}{25-10t+t^2} + t = 20$

$16 + 25t - 10t^2 + t^3 = 500 - 200t + 20t^2$

$t^3 - 30t^2 + 225t - 484 = 0$

$-8 - 120$

$\sout{-1 - 30 - 225 - 434}$

$1 - 30 + 225 - 434$

$$\begin{cases} \frac{4}{x^2+y^2} + x^2y^2 = 5 \\ x^4+y^4+3x^2y^2=20 \end{cases}$$

$x^2+y^2 \neq 0$

$4+x^4y^4 = 5x^2+5y^2$

$5x^2+5y^2 - x^4y^4 = 4$

$x^4+y^4+3x^2y^2=20$

$$\begin{cases} 10x^2+10y^2-2x^4y^4=4 \\ x^4+y^4+3x^2y^2=20 \end{cases}$$

$5x^2+5y^2=4+x^4y^4$

$x^4+y^4+3x^2y^2=20$

$(x-y$

$(x^2+y^2)^2+x^2y^2=20$

$5x^2+5y^2+x^4$

$(x^2+y^2)^2+x^2y^2=20$

$x^2+y^2= 20-x^2y^2$

$$\frac{4}{20-x^2y^2}+x^2y^2=5$$

$$\frac{4}{20-a}+a=5$$

$4+20a-a^2=100-5a$

$-a^2+25a-96=0$

$a^2-25a+96=0$