# Часть 1

Олимпиада: **Математика, 10 класс (1 часть)**

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Вариант 11

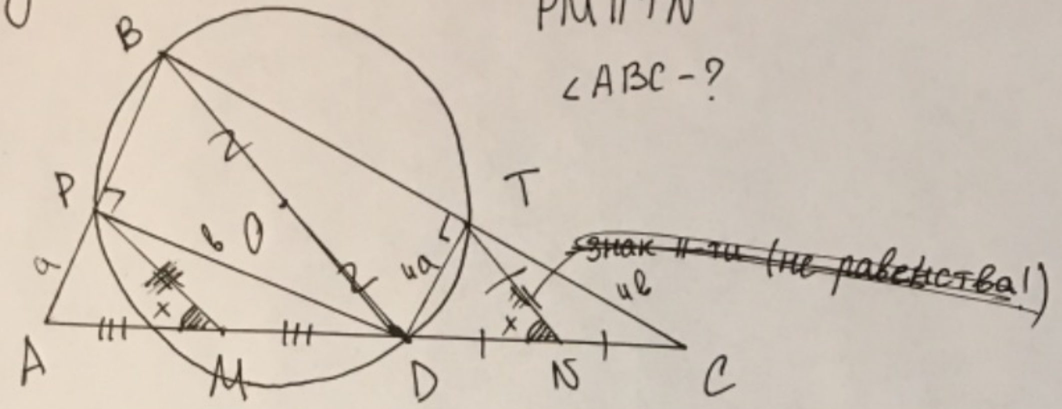
# Чистовик

## Задача 1

$AM = MD$; $DN = NC$

$PM \parallel TN$

$\angle ABC - ?$




$PM \parallel TN \Rightarrow \angle AMP = \angle DNT = x$

$BD$ - диаметр $\Rightarrow \angle BPD = \angle BTD = 90°$

$\Rightarrow \angle APD = 90°$, $\angle DTC = 90°$

$PM$ - медиана $\triangle APD$, $\triangle APD$ - прямоугольный $\Rightarrow PM = AM = MD$

аналогично $DN = TN = NC$

тогда $\triangle APM$ и $\triangle DTN$ - р/б

$\Rightarrow \angle PAM = \angle APM = \frac{180° - x}{2} = 90° - \frac{x}{2}$

$\Rightarrow \angle DPM = \angle PDM = \frac{x}{2}$

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~~аналогично~~

$\angle TDN = \angle DTN = 90° - \frac{x}{2}$

$\Rightarrow \angle PDT = 180° - \frac{x}{2} - (90° - \frac{x}{2}) = 90° \Rightarrow \angle ABC = 90°$ (т.к. они опираются на 1 хорду, PT)

$MP = \frac{1}{2}$, $NT = 2$, $BD = \sqrt{3}$

$S_{ABC} - ?$

$AD = 2MP = 1$, $CD = 2TN = 4$ $\Rightarrow AC = 5$

$\angle NCT = \angle ADP = \frac{x}{2}$

Пусть $AP = a$

$\sin\frac{x}{2} = a = \frac{DT}{4} \Rightarrow DT = 4a$

Пусть $PD = b$

аналогично $TC = 4b$

~~Пусть $AP = a$, $PD = b$~~

~~$\angle NCT = \angle ADP = \frac{x}{2} \Rightarrow \sin\frac{x}{2} = \frac{b}{4}$~~

$\begin{cases} 16a^2 + b^2 = 3 \\ a^2 + b^2 = 1 \end{cases} \Rightarrow a^2 = \frac{2}{15}$

$\Rightarrow b^2 = \frac{13}{15}$

$S_{ABC} = \frac{1}{2} \cdot 5a \cdot 5b = \frac{25}{2} \cdot \frac{\sqrt{2}}{\sqrt{15}} \cdot \frac{\sqrt{13}}{\sqrt{15}} = \frac{5}{6}\sqrt{26}$

Ответ: а) $90°$ б) $\frac{5}{6}\sqrt{26}$

Чистовик

Задача 2

$\sqrt{x+2}-\sqrt{3-x}+3=2\sqrt{6+x-x^2}$

ОДЗ: $x\in[-2;3]$

$a=\sqrt{x+2},\ a\geqslant 0$

$b=\sqrt{3-x},\ b\geqslant 0$

$\begin{cases} a-b+3=2ab \\ a^2+b^2=5 \end{cases} \Leftrightarrow \begin{cases} a-b=2ab-3 \\ (a-b)^2+2ab=5 \end{cases} \Leftrightarrow \begin{cases} 2ab=a-b+3 \\ 2ab=5-(a-b)^2 \end{cases}$

$a-b+3=5-(a-b)^2$

$t=a-b$

$t+3=5-t^2 \Leftrightarrow t^2+t-2=0 \Leftrightarrow t^2+2t-t-2=0 \Leftrightarrow (t+2)(t-1)=0$

$\left[\begin{array}{l} t=1 \\ t=-2 \end{array}\right. \Rightarrow \left[\begin{array}{l} a-b=1 \\ a-b=-2 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} a=b+1 \\ a=b-2 \end{array}\right.$

1) $a=b+1$,

$b+1-b+3=2(b^2+b)$

$4=2b^2+2b \Leftrightarrow 2=b^2+b \Leftrightarrow 0=b^2+b-2 \Leftrightarrow (b+2)(b-1)=0 \Leftrightarrow \left[\begin{array}{l} b=-2 \\ b=1 \end{array}\right.$

но $b\geqslant 0 \Rightarrow b=1$

$\sqrt{3-x}=1 \Leftrightarrow x=2$

2) $a=b-2$

$b-2-b+3=2b^2+2b \Leftrightarrow 1=2b^2+2b \Leftrightarrow 0=2b^2+2b-1$ 2

$b=-1\pm\sqrt{3}$

$b\geqslant 0 \Rightarrow b=\sqrt{3}-1$

$a=\sqrt{3}-1-2 \Leftrightarrow a=\sqrt{3}-3$

но $a\geqslant 0$!

Ответ: $\{2\}$

Чистовик

Задача 3

$$\left[\begin{aligned}&\begin{cases} 5a^2+12ax_1+4ay_1+8x_1^2+8x_1y_1+4y_1^2=0\\ y_1>3x_1+4\\ ax_2^2-2a^2x_2-ay_2+a^3+4=0\\ y_2<3x_2+4 \end{cases}\\ &\begin{cases} 5a^2+12ax_1+4ay_1+8x_1^2+8x_1y_1+4y_1^2=0\\ y_1<3x_1+4\\ ax_2^2-2a^2x_2-ay_2+a^3+4=0\\ y_2>3x_2+4 \end{cases}\end{aligned}\right.$$

3

$$\left[\begin{aligned}&\begin{cases} 5a^2+12ax_1+4a(3x_1+4)+8x_1^2+8x_1(3x_1+4)+4(3x_1+4)^2>0\\ ax_2^2-2a^2x_2-a(3x_2+4)+a^3+4<0 \end{cases}\\ &\begin{cases} 5a^2+12ax_1+4a(3x_1+4)+8x_1^2+8x_1(3x_1+4)+4(3x_1+4)^2<0\\ ax_2^2-2a^2x_2-a(3x_2+4)+a^3+4>0 \end{cases}\end{aligned}\right.$$

# Черновик

$\sqrt{x+2}-\sqrt{3-x}+3=2\sqrt{6+x-x^2}$

$\sqrt{x+2}-\sqrt{3-x}+3=2\sqrt{x+2}\sqrt{3-x}$

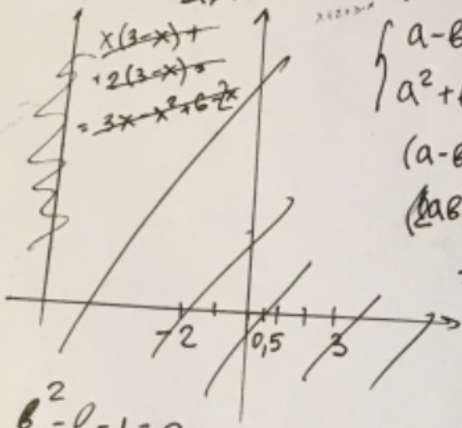
ОДЗ: $x\in[-2;3]$

$a-b+3=2ab$ $\qquad y_1=\sqrt{x+2}-\sqrt{3-x}$

$a-2ab-b+3=0$ $\qquad$ ② $2-1+3=2\cdot2$

$\cancel{a+ab}=3ab$ $\qquad 1-2+3=2\cdot2$

$\cancel{a-ab-2ab+}$ $\qquad \cancel{-\sqrt{3-x}(2\sqrt{x+2}+1)}$ $\qquad a-b=2ab-3$




$\begin{cases}a-b+3=2ab\\a^2+b^2=5\end{cases}$ $\qquad a-b=2ab-3$

$(a-b)^2+2ab=5$

$(2ab-3)^2+2ab=5$ $\qquad t=2ab$

$t^2-6t+9+2t=5$

$t^2-4t+4=0$

$(t-2)^2=0$

$t=2$

$b^2-b-1=0$

$2ab=5-(a-b)^2$ $\qquad \begin{cases}2ab=2\\a-b=-1\end{cases}$ $\qquad ab=1$ $\qquad a=b-1$

$(a-b)+3=5-(a-b)^2$

$t+3=5-t^2$ $\qquad b^2-b-1=0$

$t^2+t-2=0$ $\qquad b^2+b-2b-2=0$

$(b+1)(b-2)=0$

$\cancel{t^2+}$ $\qquad b=2$

$t^2-t+2t-2=0$ $\qquad \sqrt{3-x}=2$ $\qquad x=-1$

$(t-1)(t+2)=0$

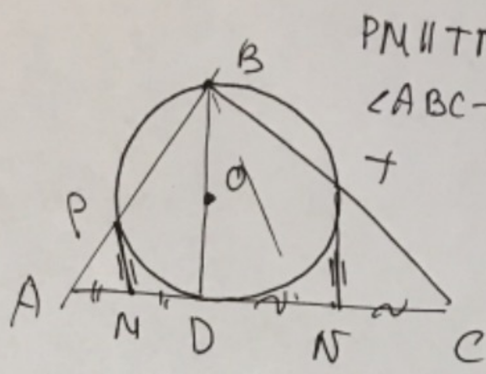
$t=$ $\qquad \dfrac{-2\pm\sqrt{4+4\cdot2}}{2}$

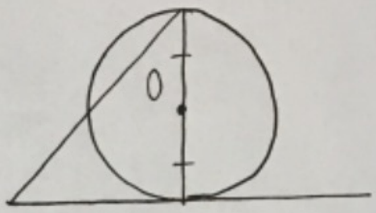
$\left[\begin{array}{l}a-b=1\\a-b=-2\end{array}\right.$ $\qquad \begin{cases}a=b+1\\a=b-2\end{cases}$ $\qquad \dfrac{-2\pm2\sqrt{3}}{2}$

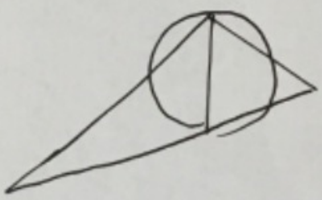
$-1\pm\sqrt{3}$

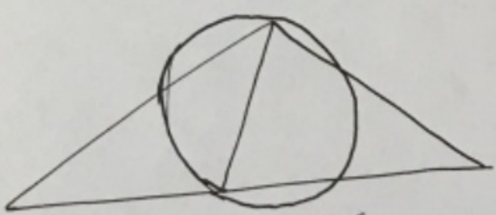
PN ∥ TN

∠ABC – ?

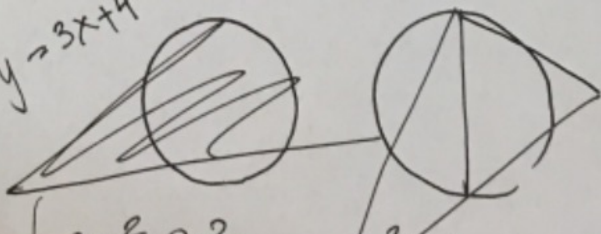




$y=3x+4$



$5a^2+12ax+4ay+8x^2+8xy+4y^2=0$

$\cancel{4(x^2+2xy}$ $\qquad 2\cancel{(4x}$ $\qquad 4xy+4xy$

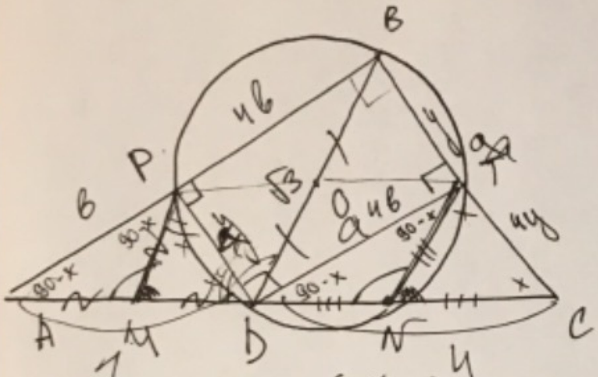
$\cancel{4a^2+a^2+4x(3a+1}$

$ax^2-2a^2x-ay+a^3+4=0$

$ax^2-2a^2x+a^3+4=ay$ $\qquad a\neq0$

$x^2-2ax+a^2+\frac{4}{a}=y$

Черновик




$180-\alpha+90-x+x=0$

$90-\alpha=0 \quad \alpha=90$

$MP=\frac{1}{2} \quad NT=2 \quad BD=\sqrt{3}$

$S_{ABC}$ - ?

$S_1=\frac{1}{2}\sqrt{3}\sin\gamma$

$S_2=\cancel{\quad}2\sqrt{3}\sin\gamma$

$S=2{,}5\sqrt{3}\sin\gamma$

$a^2+5^2$ $\quad \sin x=\frac{a}{4}=\frac{b}{a}$

$a=4b$

$3=16b^2+y^2 \quad 3=16$

$16=16b^2+16y^2 \quad 1=b^2+y^2$

$13=15y^2 \quad \frac{1}{14}=b^2$

$\frac{13}{14}=y^2$

$\frac{5b\cdot 5y}{2}=\cancel{\quad}\frac{25}{2}\cdot b\cdot y=$

$=\frac{25}{2}\cdot\frac{1}{\sqrt{14}}\cdot\frac{\sqrt{13}}{\sqrt{14}}=\frac{25}{2\cdot 2\cdot 7}\sqrt{13}=\frac{25}{28}\sqrt{13}$

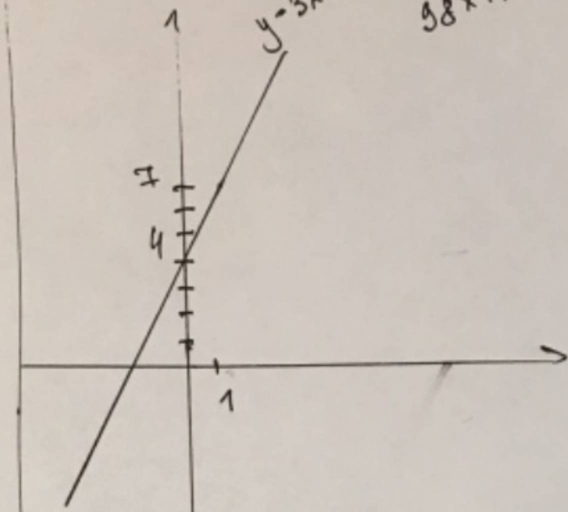
$\frac{1}{4}a^2+\frac{a^2}{2}+3ax_1+ay+2x^2+2xy+y^2=0$

$\frac{1}{4}a^2+\frac{1}{4}a^2 \quad b^2=1-a^2=1-\frac{2}{15}=\frac{13}{15}$

$3\left(\frac{1}{2}a+x\right)^2+\left(\frac{1}{2}a+y\right)^2$

$\frac{25}{\cancel{30}}\sqrt{26}$

$y=3x+4 \quad 98x^2+128x_1+64$




$$\begin{cases}5a^2+12ax_1+4ay_1+8x_1^2+8xy_1+4y_1^2=0\\ y_1>3x_1+4\\ ax_2^2-2a^2x_2-ay_2+a^3+4=0\\ y_2<3x_2+4\end{cases}$$

$\cancel{5a^2+12ax_1+4ax_1+8}$

$5a^2+12ax_1+4a(3x_1+4)+8x_1^2+8x_1(3x_1+4)+$
$+4(3x_1+4)^2>0$

$5a^2+12ax_1+12ax_1+16a+8x_1^2+24x_1^2+32x_1+$
$+4(9x_1^2+24x_1+16)>0$

$5a^2+24ax_1+16a+32x_1^2+36x_1^2+96x_1+64>0$

$\cancel{5a^2+98x_1^2}$

$5a^2+24ax_1+16a+98x_1^2+128x_1+64>0$

$5a^2+8a(3x+2)$

~~$b$ $\quad 2R=a \quad S=\frac{bc}{2}=\frac{b}{2}$~~

$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=2R=\frac{c}{\sin\gamma}$

$S=\frac{1}{2}ab\sin\gamma=\frac{1}{2}ab\cdot\frac{c}{2R}$

$\frac{abc}{4R}$

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# Часть 2

Олимпиада: **Математика, 10 класс (2 часть)**

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Вариант 11

Чистовик

Задача 4

ОДЗ: $\begin{cases} x \neq 0 \\ y \neq 0 \end{cases}$

$$\begin{cases} \frac{4}{x^2+y^2} + y^2x^2 = 5 \\ x^4+y^4+3x^2y^2 = 20 \end{cases}$$

$a = x^2,\ b = y^2,\ a>0,\ b>0$

$$\begin{cases} \frac{4}{a+b} + ab = 5 \\ (a+b)^2+ab = 20 \end{cases} \Leftrightarrow \begin{cases} ab = 5 - \frac{4}{a+b} \\ (a+b)^2+ab = 20 \end{cases}$$

4

$t = a+b,\ t>0$

$t^2+5-\frac{4}{t} = 20$

$t^3-15t-4 = 0$

$$\begin{array}{l|l} t^3-15t-4 & t-4 \\ \underline{-\ t^3-4t^2} & t^2+4t+1 \\ \quad 4t^2-15t & \\ \quad \underline{-\ 4t^2-16t} & \\ \qquad t-4 & \\ \qquad \underline{-\ t-4} & \\ \qquad\ 0 & \end{array}$$

$(t-4)(t^2+4t+1) = 0$

$$\begin{cases} \left[\begin{array}{l} t=4 \\ t=-2+\sqrt{3} \\ t=-2-\sqrt{2} \end{array}\right. \\ t>0 \end{cases} \Leftrightarrow t=4$$

$$\begin{cases} a+b=4 \\ ab=4 \end{cases} \Leftrightarrow \begin{cases} b=4-a \\ ab=4 \end{cases}$$

$4a-a^2=4 \Leftrightarrow 0=a^2-4a+4 \Leftrightarrow 0=(a-2)^2$

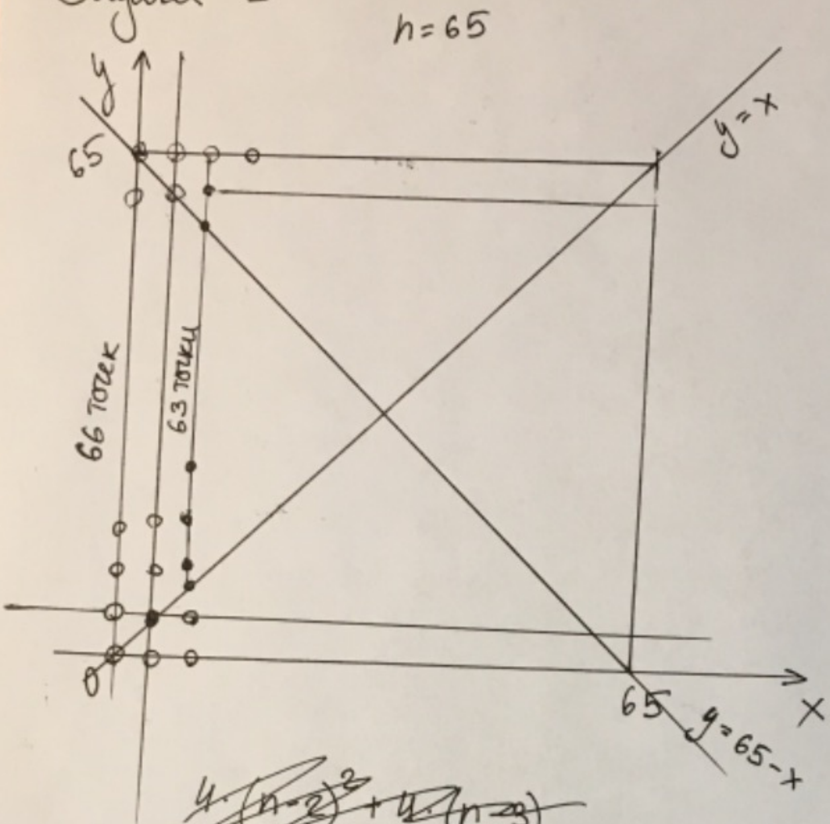
$a=2 \Rightarrow b=2$

$$\begin{cases} x^2=2 \\ y^2=2 \end{cases} \Leftrightarrow \left[\begin{array}{l} \begin{cases} x=-\sqrt{2} \\ y=-\sqrt{2} \end{cases} \\ \begin{cases} x=-\sqrt{2} \\ y=\sqrt{2} \end{cases} \\ \begin{cases} x=\sqrt{2} \\ y=-\sqrt{2} \end{cases} \\ \begin{cases} x=\sqrt{2} \\ y=\sqrt{2} \end{cases} \end{array}\right.$$

Ответ: $(-\sqrt{2};-\sqrt{2});\ (-\sqrt{2};\sqrt{2});\ (\sqrt{2};-\sqrt{2});\ (\sqrt{2};\sqrt{2})$

Чистовик

Задача 5




$\cancel{4\cdot(n-2)^2+4\cdot(n-3)}$

$4\cdot(n-2)^2+4\cdot\left((n-2)^2-1\right)+4\cdot\left((n-2)^2-2\right)+\ldots+1=$

$=\cancel{4\cdot(65-2)^2+4\cdot(}$

$=4\cdot 63^2+4\left(63^2-1\right)+4\left(63^2-2\right)+\ldots+1$

можно рассмотреть ситуацию с точкой (1;1), найти для неё 2 точку, соответствующую условию можно $(n-2)^2$ способами. Аналогично с остальными точками, учитывая повторы (то есть пара (1;1) и (2;2) должна учитываться всего 1 раз)

Ответ: $4\cdot 63^2+4\cdot\left(63^2-1\right)+4\cdot\left(63^2-2\right)+\ldots+1$

5

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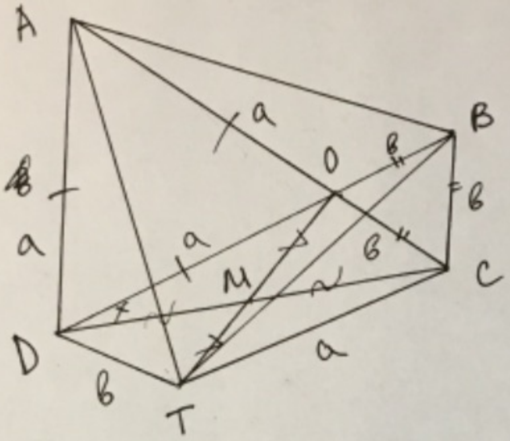
# Чистовик

**Задача 6**

$\triangle BOC$ и $AOD$ – р/с
$T$ симм. точке $O$ относит. середины $CD$ (точке $M$)
б) $BC = 2$, $AD = 5$
$(b = 2;\ a = 5)$

а) (!) $ABT$ – р/с
б) $\dfrac{S_{ABT}}{S_{ABCD}}$ – ?




$\triangle BOC$ и $AOD$ – р/с $\Rightarrow$
$\angle AOD = \angle ODA = \angle DAO = 60^\circ$,
$AO = OD = AD$;
$\angle BOC = \angle OCB = \angle CBO = 60^\circ$,
$OB = BC = OC$
пусть $AO = a$, $OB = b$

$(\cdot)T$ симм. $(\cdot)O$ относит. $(\cdot)M \Rightarrow TM = MO$
$DM = MC$ (т.к. $M$ – середина $CD$)
$\angle DMT = \angle OMC$ (вертик. $\angle$)
$\Rightarrow \triangle DMT = \triangle MOC$ по СУС

аналогично $\triangle DMO = \triangle TMC \Rightarrow CTDO$ – пар/м
пусть $\angle ODC = x$
$\angle DOC = 180^\circ - \angle AOD = 120^\circ$
$\angle DCO = 180^\circ - 120^\circ - x = 60^\circ - x$

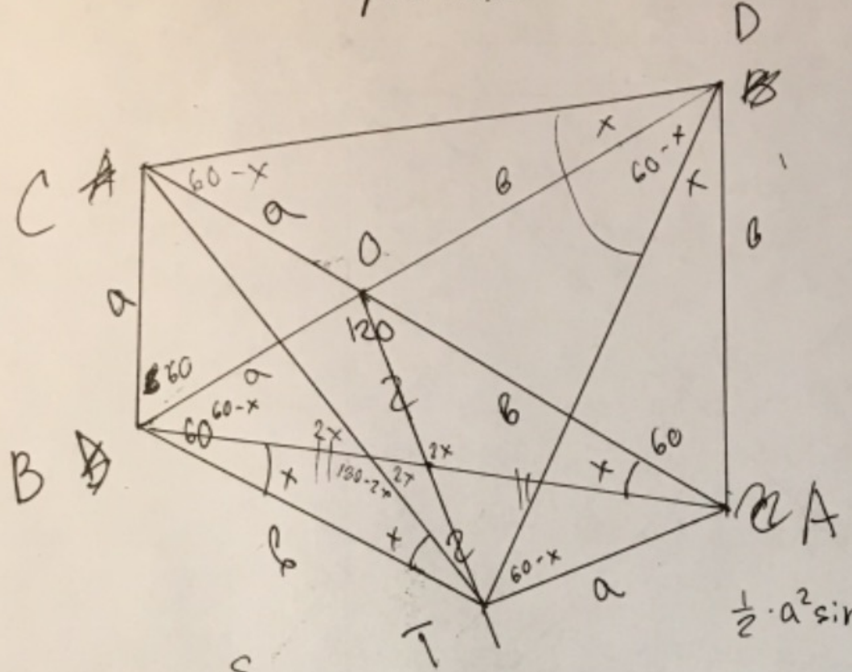
6

$CTDO$ – пар-м $\Rightarrow DT = b$, $TC = a$, $\angle DTC = 120^\circ$
$\angle ADB = 60^\circ$, $\angle CDT = 60^\circ - x \Rightarrow \angle ADT = 120^\circ$ (аналогично $\angle BCT = 120^\circ$)
$\Rightarrow \triangle ADT$ и $DCT$ равны, $\triangle ADT$ и $\triangle TBC$ также равны
$\Rightarrow \angle DTA = 60^\circ - x$, $\angle BTC = x$
$\angle ATB = \angle DTC - \angle DTA - \angle BTC = 120^\circ - 60^\circ + x - x = 60^\circ$
$AT = BT \Rightarrow \triangle ATB$ – р/б с углом $60^\circ \Rightarrow \triangle ATB$ – р/с чтд.

$$S_{ABT} = \frac{1}{2} AT \cdot BT \cdot \sin 60^\circ = \frac{\sqrt{3}}{4} \cdot (a^2 + b^2 - 2ab\cos 120^\circ) = \frac{\sqrt{3}}{4} \cdot (a^2 + b^2 + ab) =$$
$$= \frac{\sqrt{3}}{4} \cdot (25 + 4 + 10) = \frac{39\sqrt{3}}{4}$$
$$S_{ABCD} = 2 \cdot \frac{1}{2} ab \sin 120^\circ + \frac{1}{2} a^2 \sin 60^\circ + \frac{1}{2} b^2 \sin 60^\circ = \frac{\sqrt{3}}{2}\left(ab + \frac{1}{2}a^2 + \frac{1}{2}b^2\right) = \frac{\sqrt{3}}{2}(10 + 2 + 12{,}5) =$$
$$= \frac{\sqrt{3}}{2} \cdot 24{,}5 = \frac{\sqrt{3}}{4} \cdot 49$$
$$\frac{S_{ABT}}{S_{ABCD}} = \frac{\sqrt{3}}{4} \cdot 39 \cdot \frac{4}{\sqrt{3}} \cdot \frac{1}{49} = \frac{39}{49}$$

**Ответ:** б) $\dfrac{39}{49}$

Черновик



~~$b=2$~~

~~$a=5$~~

$b=5$

$a=2$

$\dfrac{S_{ABT}}{S_{ABCD}}$ — ?

$\frac{1}{2}\cdot a^2\sin 60 = \frac{1}{2}a^2\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}a^2$

$\dfrac{S}{2S+\frac{\sqrt{3}}{4}a^2+\frac{\sqrt{3}}{4}b^2}$

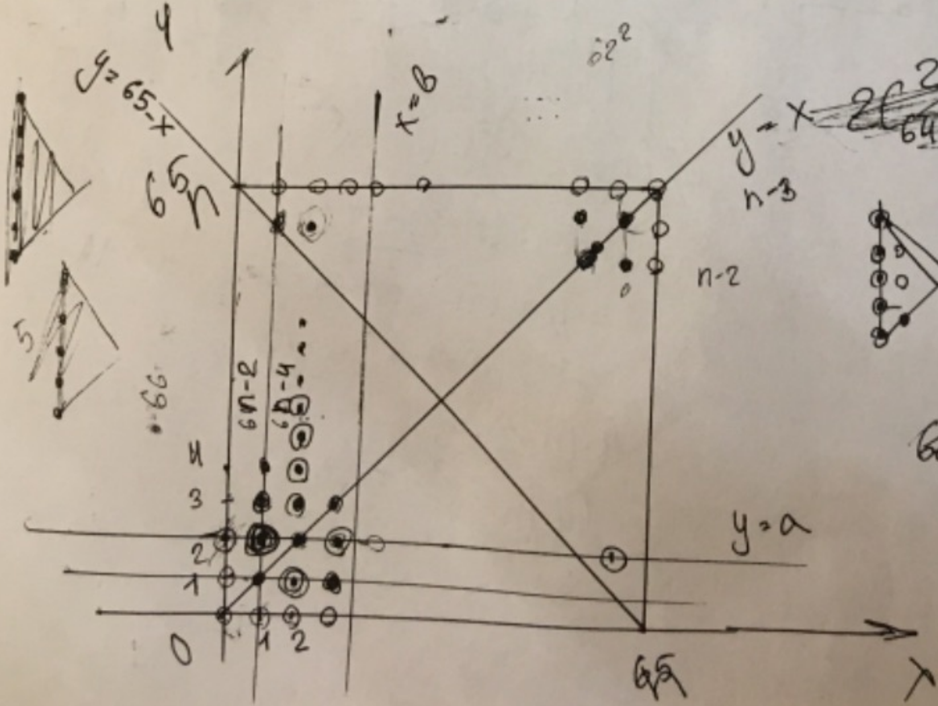
$S = \frac{1}{2}ab\sin 120° = \frac{1}{2}ab\sin 60°$

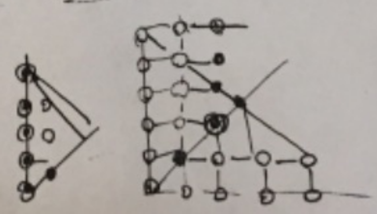
$S_{ABCD} = ab\sin 60° + \frac{1}{2}a^2\sin 60° + \frac{1}{2}b^2\sin 60° =$

$= \sin 60°(ab+\frac{1}{2}a^2+\frac{1}{2}b^2$

$\frac{\sqrt{3}}{2}$

$\dfrac{S}{S_{ABCD}} = \dfrac{\frac{1}{2}ab}{ab+\frac{1}{2}a^2+\frac{1}{2}b^2} = \dfrac{\frac{1}{2}\cdot 2\cdot 5}{10+\frac{1}{2}\cdot 4+\frac{1}{2}\cdot 25} = \dfrac{5}{12+12{,}5} = \dfrac{5}{24{,}5} = \dfrac{50}{245} = \dfrac{10}{49}$





$y = x$ ~~$+ 2C_{64}^{2}$~~ $+ 64$

$\frac{245}{10} = \frac{49}{2}$

$4\cdot(n-4)^2+41$

24,5

Черновик

$$\begin{cases} \frac{4}{x^2+y^2} + x^2y^2 = 5 \\ x^4 + y^4 + 3x^2y^2 = 20 \end{cases} \quad \begin{cases} x \neq 0 \\ y \neq 0 \end{cases}$$

$a = x^2$, $b = y^2$

$\frac{4}{a+b} + ab = 5 \Leftrightarrow ab = 5 - \frac{4}{a+b}$

$a^2 + b^2 + 3ab - 20 = 0$

$(a+b)^2 + ab = 20$

$$\begin{cases} a+b = t \\ ab = 5 - \frac{4}{t} \end{cases}$$

$t^2 + 5 - \frac{4}{t} = 20$

$t^2 - \frac{4}{t} - 15 = 0$

$\cancel{t} \; t^3 - 15t - 4 = 0$

$$\begin{array}{l|l} t^3 - 15t - 4 & t - 4 \\ \hline -\; t^3 - 4t^2 & t^2 + 4t + 1 \\ \quad 4t^2 - 15t & \\ \quad -\; 4t^2 - 16t & \\ \quad\quad t - 4 & \end{array}$$

$\frac{-4 \pm \sqrt{16-4}}{2} = -2 \pm \sqrt{3}$

$$\left[\begin{array}{l} t = 4 \\ \cancel{t = ...} \end{array}\right.$$

$t = 4$

$$\begin{cases} \cancel{x^2 + y^2 = 4} \Leftrightarrow \cancel{y^2 = 4 - x^2} \\ \cancel{x^2 y^2 = 4} \end{cases}$$

$$\begin{cases} a + b = 4 \Leftrightarrow b = 4 - a \\ ab = 4 \end{cases}$$

$4a - a^2 = 4$

$0 = a^2 - 4a + 4$

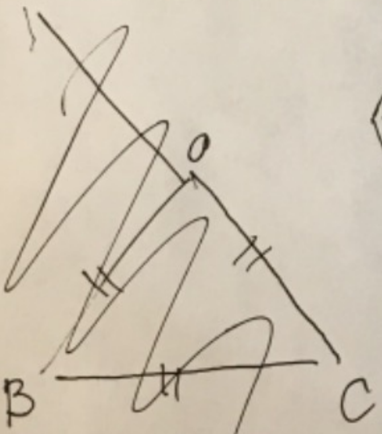
$a = 2$, $b = 2$

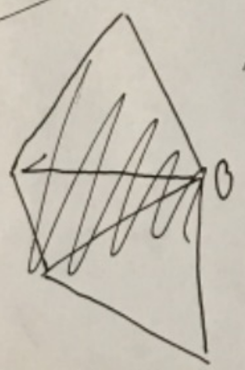
$x^2 = 2$

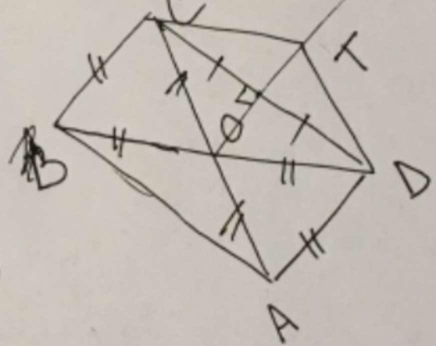
$y^2 = 2$

$x = \pm\sqrt{2}$

$y = \pm\sqrt{2}$




$BC = 2 = a$

$AD = 5$