Часть 1

Олимпиада: **Математика, 10 класс (1 часть)**

Шифр: **211005399**

ID профиля: **87096**

Вариант 11

Чистовик

Вариант 11

Математика, 10 кл.

1/2

N1

Дано:

$\triangle ABC$

$D \in AC$

$\omega \cap AB, BC$ в т. P и T

M - серед. AD

N - серед. CD

$PM \parallel TN$

б) $MP = \frac{1}{2}$, $NT = 2$

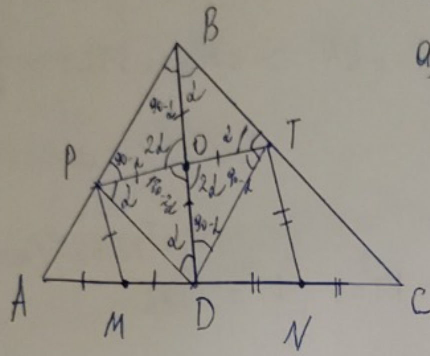
$BD = \sqrt{3}$

Найти:

а) $\angle ABC$

б) S_{ABC}

Решение:



а) 1) Пусть $\angle DPT = \alpha$, тогда т.к. OT, PO и OD – R, то $PO = OD = OT$, т.е. $\triangle POD$ - р/б, $\triangle DOT$ - р/б, тогда $\angle ODP = \angle DPO = \alpha$, $\angle DOP = 180^\circ - 2\alpha \Rightarrow \angle POB = 2\alpha$, тогда $\angle OPB = \angle OBP = 90 - \alpha$, тогда $\angle DPB = 90^\circ$

2) Т.к. $\angle DTB$ опирается на диаметр, то $\angle DTB = 90^\circ$

Тогда $\triangle APD$ и $\triangle DTC$ - прям., тогда $NT = DN = NC$ и $AM = PM = MD$

3) Т.к. $MP \parallel NT$, тогда PT - диаметр, т.е. $\angle DOT = 180^\circ - 180^\circ + 2\alpha = 2\alpha \Rightarrow \angle ODT = \angle OTD = 90 - \alpha \Rightarrow \angle OTB = \alpha = \angle TBO \Rightarrow$

$\angle ABC = \angle ABD + \angle DBC = 90 - \alpha + \alpha = 90^\circ$

б) 1) По условию $MP = \frac{1}{2}$ и $NT = 2 \Rightarrow MP = AM = MD = \frac{1}{2}$, $NT = DN = NC = 2 \Rightarrow$

$DC = 2NT = 4$, $AD = 2MP = 1$ т.е. $AC = 5$

2) По т. Пифа $\Rightarrow AB^2 + BC^2 = AC^2$; $PB^2 + BT^2 = PT^2 = BD^2 = 3$; $\begin{cases} AP^2 + PD^2 = AD^2 = 1 \\ PT^2 + TC^2 = DC^2 = 16 \end{cases}$

$\Downarrow$

$AP^2 + PD^2 + DT^2 + TC^2 = 17$

$AP^2 + TC^2 + 3 = 17$

$AP^2 + TC^2 = 14$

$AB = AP + PB \Rightarrow AB^2 = AP^2 + PB^2 + 2AP \cdot PB$

$BC = BT + TC \Rightarrow BC^2 = BT^2 + TC^2 + 2BT \cdot TC$ $\Rightarrow$

$\Rightarrow AC^2 = AP^2 + PB^2 + 2AP \cdot PB + BT^2 + TC^2 + 2BT \cdot TC.$

$AC^2 = AP^2 + 2AP \cdot PB + TC^2 + 2BT \cdot TC + 3$

$AC^2 = 14+3+2AP\cdot PB+2BT\cdot TC$

$25 = 17+2(AP\cdot PB+BT\cdot TC)$

$4 = AP\cdot PB + BT\cdot TC.$

3) Т.к. $MP \parallel NT$, то $\angle AMP = \angle DNT \Rightarrow \triangle APD \sim \triangle DTC \Rightarrow \frac{AP}{DT} = \frac{PD}{TC} = \frac{AD}{DC} = \frac{1}{4} \Rightarrow$

$$\begin{cases} DT = 4AP \\ TC = 4PD \end{cases}$$

4) По т. Пифа $DT^2+PD^2=PT^2=3$; $DT^2+TC^2=16$

$$\begin{cases} DT^2 + \frac{TC^2}{16} = 3 \\ DT^2+TC^2=16 \end{cases}$$

$\frac{15}{16}TC^2=13$

$TC^2 = \frac{13\cdot 16}{15} \Rightarrow \frac{4\sqrt{13}}{\sqrt{15}} = TC \Rightarrow DT^2 = 16 - \frac{13\cdot 16}{15} = 16\cdot\frac{2}{15} \Rightarrow DT = \frac{4\sqrt{2}}{\sqrt{15}}$

$BT^2+DT^2 = BD^2 = 3$

$BT^2 = 3 - DT^2 = 3 - \frac{16\cdot 2}{15} = \frac{45-32}{15} = \frac{13}{15} \Rightarrow BT = \frac{\sqrt{13}}{\sqrt{15}} \Rightarrow$

$BC = BT + TC = \frac{5\sqrt{13}}{\sqrt{15}}$

4) $AP = \frac{DT}{4} = \sqrt{\frac{2}{15}}$; $PD = \frac{TC}{4} = \sqrt{\frac{13}{15}}$

$PD^2+PB^2=BD^2=3$

$PB^2 = 3 - \frac{13}{15} = \frac{45-13}{15} = \frac{32}{15} \Rightarrow PB = \sqrt{\frac{32}{15}} = \frac{4\sqrt{2}}{\sqrt{15}} \Rightarrow AB = PB + AP = \frac{5\sqrt{2}}{\sqrt{15}}$, тогда

$S_{ABC} = \frac{1}{2}AB\cdot BC = \frac{1}{2}\cdot\frac{5\sqrt{2}}{\sqrt{15}}\cdot\frac{5\sqrt{13}}{\sqrt{15}} = \frac{25\sqrt{26}}{30} = \frac{5\sqrt{26}}{6}$

Ответ: а) $\angle ABC = 90^\circ$ б) $S_{ABC} = \frac{5\sqrt{26}}{6}$

Математика сокл.
2/2

Чистовик
Вариант 11

№2

$$\sqrt{x+2}-\sqrt{3-x}+3=2\sqrt{6+x-x^2}$$

1) ОДЗ: $x+2\geq 0$ $\quad$ $3-x\geq 0$ $\quad$ $6+x-x^2\geq 0$

$\underline{x\geq -2}$ $\quad$ $\underline{3\geq x}$ $\quad$ $\underline{x^2-x-6\leq 0}$

$D=1+24=25=5^2$

$x_1=\frac{1+5}{2}=3;\ x_2=\frac{1-5}{2}=-2$

$+\quad -2\quad -\quad 3\quad +$ $\Rightarrow x\in[-2;3]$

$\sqrt{x+2}-\sqrt{3-x}+3\geq 0$

Т.е. $x^2-x-2\leq 0$

$D=1+8=9=3^2$

$x_1=\frac{1+3}{2}=2;\ x_2=\frac{1-3}{2}=-1$

$+\quad -1\quad -\quad 2\quad +$ $\Rightarrow x\in[-1;2]$

2) $6+x-x^2=(x+2)(3-x)$

$$\sqrt{x+2}-\sqrt{3-x}=2\sqrt{6+x-x^2}-3$$

$$x+2-2\sqrt{(x+2)(3-x)}+3-x=4(6+x-x^2)-12\sqrt{6+x-x^2}+9$$

$$4(6+x-x^2)-10\sqrt{6+x-x^2}+4=0$$

Пусть $\sqrt{6+x-x^2}=t$

$4t^2-10t+4=0$

$2t^2-5t+2=0$

$D=25-4\cdot 4=9=3^2$

$t_1=\frac{5+3}{4}=2;\ t_2=\frac{5-3}{4}=0{,}5$

1) $\sqrt{6+x-x^2}=2$

$6+x-x^2=4$

$x^2-x-2=0$

$D=1+8=9=3^2$

$x_1=\frac{1+3}{2}=2;\ x_2=\frac{1-3}{2}=-1$

2) $\sqrt{6+x-x^2}=0{,}5$

$6+x-x^2=0{,}25$

$x^2-x-5{,}75=0$

$D=1+4\cdot 5{,}75=24$

$x_1=\frac{1+2\sqrt{6}}{2}=\sqrt{6}+0{,}5$ – нет

$x_2=\frac{1-2\sqrt{6}}{2}=0{,}5-\sqrt{6}$ – нет

~~Т.к. $\sqrt{6}<2{,}5$, то корни не подходят~~

Ответ: $x=2;\ x=-1$, ~~$x=2{,}5+\sqrt{6}$~~

№3

$ax^2 - 2a^2x - ay + a^3 + 4 = 0$

$y = x^2 - 2ax + a^2 + \frac{4}{a}$; $x_в = \frac{2a}{2} = a$; $y_в = \frac{4}{a}$ т.е. $B(a; \frac{4}{a})$

$5a^2 + 12ax + 4ay + 8x^2 + 8xy + 4y^2 = 0$

$D = (8x+4a)^2 - 16(8x^2 + 12ax + 5a^2) = -x^2 - 2ax - a^2 \geq 0 \Rightarrow x^2 + 2ax + a^2 \leq 0$

$(x+a)^2 \leq 0$ т.е. $x = -a$

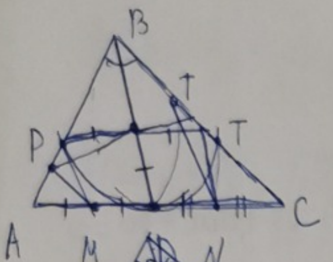
$y = 0,5a$

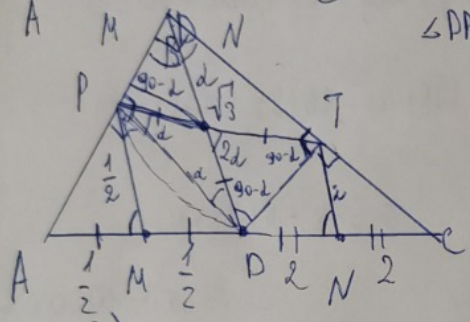
$\Rightarrow A(-a; 0,5a)$

$x+2-2\sqrt{(x+2)(3-x)}+3-x=4(6+x-x^2)-12\sqrt{6+x-x^2}$

№1

Черновик





$S=\frac{1}{2}\cdot AB\cdot BC=\frac{1}{2}\cdot 5\sqrt{\frac{13}{15}}\cdot 5\cdot\sqrt{\frac{2}{15}}=$

$90>\alpha+\beta$

$=\frac{\overset{5}{\cancel{25}}\cdot\sqrt{26}}{2\cdot\cancel{15}_3}=\frac{5\cdot\sqrt{26}}{6}$

$180^\circ>2\alpha+2\beta$

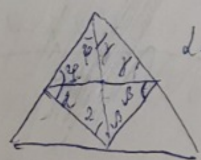
$\angle DPT=\angle DBT=\angle$ тк. опир на $\cup PT$

$180-2\alpha+180^\circ-2\beta>180^\circ$

$180^\circ-90+\alpha-\alpha=90^\circ$

$\angle POD+\angle DOT>180^\circ$

$\angle ABC=90^\circ$



$\alpha+\varphi+\beta+\gamma=180^\circ$

$DT^2+BT^2=BD^2=3$

$PD^2+PB^2=3$

~~AD~~ AB и BC

~~$\alpha+\varphi=\beta+\gamma$~~

$S=\frac{1}{2}AB\cdot BC$

$16AP^2+BT^2=3$ ~~$\varphi+\gamma=\alpha+\beta$~~

$AB=PB+AP=\sqrt{\frac{2}{15}}+\sqrt{\frac{32}{15}}=$

$AD^2=PD^2+AP^2$

$\frac{TC^2}{16}+PB^2=3$

$BC=5\cdot\sqrt{\frac{13}{15}}$

$=\sqrt{\frac{2}{15}}(1+\sqrt{16})=$

$DC^2=DT^2+TC^2$

$=\sqrt{\frac{2}{15}}\cdot 5$

$AP=\frac{DT}{4}=\sqrt{\frac{2}{15}}$

$16AP^2+BT^2+\frac{TC^2}{16}+PB^2=6=$

$PD=\sqrt{\frac{13}{15}}$

~~$AC^2=AD^2+DC^2+2AD\cdot DC=PD^2+AP^2+$~~

$\Rightarrow 16AP^2+\frac{TC^2}{16}=3$

$PB=\sqrt{3-\frac{13}{15}}=$

$AD=1+4=5$

$25=\sqrt{AB^2+BC^2}$

$=\sqrt{\frac{45-13}{15}}=\sqrt{\frac{32}{15}}=\sqrt{\frac{32}{15}}$

$\begin{cases} DT^2+\frac{TC^2}{16}=3 \\ DT^2+TC^2=16 \end{cases}$

~~PD~~ $BT=\sqrt{3-\frac{16\cdot 2}{15}}=\sqrt{\frac{45-32}{15}}=\sqrt{\frac{13}{15}}$

$AB^2+BC^2=25$

$\triangle APD\sim\triangle DTC$

$\frac{DT}{AP}=\frac{TC}{PD}=\frac{DC}{AD}=\frac{4}{1}=4$

$TC^2-\frac{TC^2}{16}=13$

$DT^2+\frac{13\cdot 16}{15}=16$

$DT^2=16\left(1-\frac{13}{15}\right)=16\cdot\frac{2}{15}\Rightarrow DT=4\sqrt{\frac{2}{15}}$

$BT=4AP$

~~$TC=4PD$~~

$\frac{15TC^2}{16}=13$

$PD=\frac{TC}{4}$

$TC=4\sqrt{\frac{13}{15}}$

$TC^2=\frac{13\cdot 16}{15}$

Черновик

№3

$5a^2+12ax+4ay+8x^2+8xy+4y^2=0$

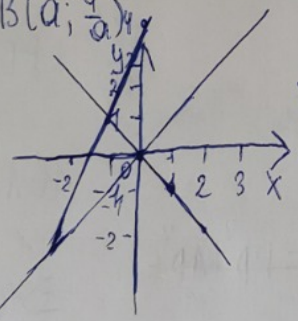
$ax^2-2a^2x-ay+a^3+4=0$

$ax^2-2a^2x+a^3+4=ay$

$y=\frac{ax^2-2a^2x+a^3+4}{a}=x^2-2ax+a^2+\frac{4}{a}$

$x_в=\frac{2a}{2}=a;\ y_в=a^2-2a^2+a^2+\frac{4}{a}=\frac{4}{a}$

$B(a;\frac{4}{a})$



$y-3x=4$

~~$y-4=3x$~~

$y=3x+4$

x	0	-1
y	4	1

$0=3x+4$

$-4=3x$

$x=-\frac{4}{3}$

$A(-a;0,5a)$

$B(a;\frac{4}{a})$

$16\cdot4=64$, $64\cdot2=128$

$16\cdot4=64$, $64\cdot3=192$

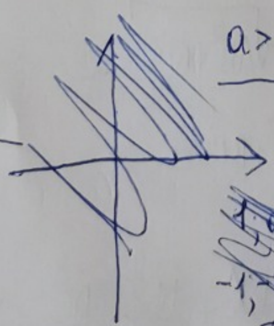
$16\cdot5=80$

$192-64=128$

$a>-\frac{4}{3}$

$y=-a$

$y=a$



~~1,05~~

~~$0,5a>-3a+4$~~

$3,5a>4$

~~$a>\frac{4}{3,5}=\frac{8}{7}\Rightarrow a>\frac{8}{7}$~~

$4y^2+8xy+4ay+8x^2+12ax+5a^2=0$

$D=(8x+4a)^2-16(8x^2+12ax+5a^2)=$

$=64x^2+64ax+16a^2-$

$-128x^2-192ax-$

$-80a^2$

$=-64x^2-128ax-64a^2\geq0$

$-x^2-2ax-a^2\geq0$

$x^2+2ax+a^2\leq0$

$(x+a)^2\leq0$

$x=-a$

$y_1=\frac{-(8x+4a)}{8}=\frac{-(-8a+4a)}{8}=\frac{4a}{8}=0,5a.$

~~$a<\frac{32}{a}+4$~~

$x+2-2\sqrt{(x+2)(3-x)}+3-x=4(6+x-x^2)-12\sqrt{6+x-x^2}+9$

$4(6+x-x^2)-12\sqrt{6+x-x^2}+2\sqrt{3x-x^2+6-2x}+9-5$

$$\begin{array}{r} \times\ 0{,}75 \\ 4 \\ \hline 3{,}00 \end{array}$$

$4(6+x-x^2)-10\sqrt{6+x-x^2}+4=0$

$2(6+x-x^2)-5\sqrt{6+x-x^2}+2=0$

$D=25-4\cdot 4=9=3^2$

$t_1=\frac{5+3}{4}=2;\ t_2=\frac{5-3}{4}=\frac{2}{4}=0{,}5$

$\sqrt{6+x-x^2}=2$

$6+x-x^2=4$

$x^2-x+4-6=x^2-x-2=0$

$D=1+8=9=3^2$

$x_1=\frac{1+3}{2}=2;\ x_2=\frac{1-3}{2}=-1$

$\sqrt{6}=1{,}4\cdot 1{,}7$

$6+x-x^2=0{,}25$

$x^2-x-5{,}75=0$

$D=1+4\cdot 5{,}75=24$

$$\begin{array}{r} \times\ 1{,}4 \\ 1{,}7 \\ \hline +\ 9\ 8 \\ 14\ \ \\ \hline 2{,}3\ 8 \end{array}$$

$\sqrt{6+x-x^2}=0{,}5$

$6+x-x^2=\frac{1}{4}$

$x^2-x+\frac{1}{4}-6=0$

$x^2-x+\frac{1}{4}-\frac{24}{4}=x^2-x-\frac{23}{4}=0$

$D=1+\frac{\cancel{4}\cdot 23}{\cancel{4}}=24=$

$x_1=\frac{1+2\sqrt{6}}{2}=0{,}5+\sqrt{6}\approx 2{,}9$

$x_2=\frac{1-2\sqrt{6}}{2}=0{,}5-\sqrt{6}\approx -1{,}9$

211005399 (U87096 M1274916)

Черновик

N2

$$\sqrt{x+2}-\sqrt{3-x}+3=2\sqrt{6+x-x^2}$$

ОДЗ: $x+2\geq 0$ $\quad 3-x\geq 0$ $\quad 6+x-x^2\geq 0$

$x\geq -2$ $\quad 3\geq x$ $\quad x^2-x-6\leq 0$

$D=1+24=25=5^2$

$x\in[-2;3]$

$x_1=\frac{1+5}{2}=3;\ x_2=\frac{1-5}{2}=-2$

$6+x-x^2=t$

$\sqrt{4}-\sqrt{1}+3=2\sqrt{6+2-4}$

$4=2\cdot 2=4$

$\sqrt{3}-\sqrt{2}+3=2\sqrt{6+1-1}=2\sqrt{6}$

$x\in[-2;3]$

$\sqrt{x+2}$

$-\sqrt{3-x}+3=\quad 2\sqrt{(x+2)(x-3)}$

$\sqrt{3}-\sqrt{2}+3=2\sqrt{6}$

$3{,}3=2\cdot\sqrt{6}$

~~$\sqrt{x+2}-2\sqrt{(x+2)(x-3)}-\sqrt{3-x}+3=0$~~

$\sqrt{2{,}5+\sqrt{6}}+\sqrt{2{,}5-\sqrt{6}}+3=2\sqrt{6+0{,}5\cdot\sqrt{6}-6-\sqrt{6}-0{,}25}=2\sqrt{-0{,}5\sqrt{6}-2{,}5}$

~~$\sqrt{x+2}\,(1-2\sqrt{x-3})-\sqrt{3-x}+3=0$~~

$\sqrt{6}+0{,}5;\quad \sqrt{6+\sqrt{6}+0{,}5-6-\sqrt{6}-0{,}25}=\cancel{...}-2=\sqrt{2{,}5+\sqrt{6}}-\sqrt{2{,}5-\sqrt{6}}$

~~$\sqrt{x+2}-2\sqrt{(x+2)(x-3)}-\sqrt{3-x}+3=0$~~

$x+2-2\sqrt{(x+2)(3-x)}+x-3=4(x+2)(x-3)-12\sqrt{(x+2)(x-3)}+9$

$2x-10+10\sqrt{(x+2)(3-x)}=4(x^2-x-6)$

$x-5+5\sqrt{(x+2)(3-x)}=4(x^2-x-6)$

~~$5\,\sqrt[5]{(x+2)(3-x)}=4(x+2)(3-x)+9$~~

~~$4(6+x-x^2)-5\sqrt{6+x-x^2}+4=0$~~

~~$D=25-64$~~

$\sqrt{x+2}-\sqrt{3-x}+3\geq 0$

$x+2+6\sqrt{x+2}+9\geq 3-x$

$6\sqrt{x+2}\geq -8-2x$

$36(x+2)\geq 64+32x+4x^2$

$4x^2-4x-8\leq 0$

$x^2-x-2\leq 0$

$D=1+8=9=3^2$

$x_1=\frac{1+3}{2}=2$

$x_2=\frac{1-3}{2}=-1$

~~$(x+2)(x-3)=x^2-3x+2x-6=x^2-x-6$~~

$(x+2)(3-x)=3x-x^2+6-2x=6+x-x^2$

$\sqrt{(x+2)}-\sqrt{3-x}+3=2\sqrt{(x+2)(3-x)}$

$x+2-2\sqrt{(x+2)(3-x)}+3-x=$

$=4(x+2)(3-x)-12\sqrt{(x+2)(3-x)}+9$

$5\sqrt{(x+2)(3-x)}$

$x-5+5\sqrt{-(6+x-x^2)}=4(x^2-x-6)$

N3

$5a^2+17$

ax^2-2

ax^2-

211005399 (U87096 M1274916)

Часть 2

Олимпиада: **Математика, 10 класс (2 часть)**

Шифр: **211005399**

ID профиля: **87096**

Вариант 11

Математика гоки.

1/2

Чистовик

11 вариант

№4

$\frac{4}{x^2+y^2}+x^2y^2=5$; $x^2y^2=5-\frac{4}{x^2+y^2}$

$x^4+y^4+3x^2y^2=20$

$x^4+y^4+2x^2y^2+x^2y^2=20$

$(x^2+y^2)^2+5-\frac{4}{x^2+y^2}=20$

Пусть $x^2+y^2=t$

$t^2+5-\frac{4}{t}=20$

$t^3+5t-4=20t$

$t^3-15t-4=0$

Т.е. $4=t^3-15t$. Тогда $t=4$ т.к. при $t>4$ разница между числами увеличивается, а если $t<4$, то $t^3-15t<0$, тогда $x^2+y^2=4 \Rightarrow x^2=4-y^2$

$x^2y^2=5-\frac{4}{4}=5-1=4$

$(4-y^2)y^2=\cancel{5}4$

$4y^2-y^4=\cancel{5}^4$

$y^4-4y^2+\overset{4}{\cancel{5}}=0$. Пусть $y^2=a$, тогда $a^2-4a+4=0$

$D=16-16=0$; $a_1=\frac{4}{2}=2 \Rightarrow y^2=a=2$ т.е. $y=\sqrt{2}$, тогда $x=\sqrt{4-2}=\sqrt{2}$

Ответ: $x=y=\sqrt{2}$

№5

По условию дана плоскость с квадратом, тогда ~~(y=x)~~ узлов по вертикали и горизонтали 64. Заметим, что сразу два узла могут лежать $y=x$, тогда вариантов выбора $C_{64}^2=\frac{64!}{62!\,2!}=\frac{63\cdot \cancel{64}^{32}}{\cancel{2}_1}=2016$. Аналогично, если оба узла на $65-x$ т.е. 4032 варианта. ~~Если прямые~~ Если две точки на разных прямых, то вариантов $63\cdot 64=4032$. Т.е. всего 8064 варианта. Допустим, одна из точек не лежит на прямой, а другая лежит. Тогда вариантов ~~3840·2~~ $64\cdot 60\cdot 2=3840\cdot 2=7680$.

В сумме будет $8064+7680=15744$ вариантов.

Ответ: всего 15744 варианта.

№6

Дано:

ABCD - вписан. четырехуг.

$AC\cap BD=O$

ΔBOC и ΔAOD - р/с

M - серед CD

T сим. O

б) $BC=2$, $AD=5$

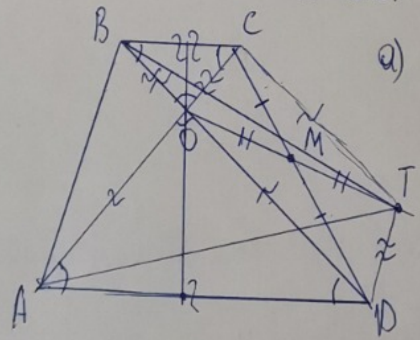
Док-ть:

а) ΔABT - р/с

Найти:

б) $\frac{S_{ABT}}{S_{ABCD}}$

Решение:



а) 1) По условию ΔBOC и ΔAOD - р/с $\Rightarrow$ $AO=OD=AD$ и $BO=BC=OC$. Все $\angle=60°$, тогда $\angle BCA=\angle CAD\Rightarrow BC\parallel AD$ т.е. ABCD - трапеция

2) Заметим, что $\Delta CMT=\Delta OMD$ (т.к. $OM=MT$ и $MD=CM$, $\angle OMD=\angle CMT$), $\Delta MDT=\Delta MOC$ ($OM=MT$, $MD=CM$, $\angle OMC=\angle DMT$)

Тогда $CT=OD=AO$, ~~Т.к. CT=AO и BO=BC~~ Т.к. $\Delta CMT=\Delta OMD$, то $\angle MOD=\angle MTC$ и $\angle MDO=\angle MCT$ т.е. $OD\parallel CT$, тогда $\angle OCT=180°-\angle COD=180°-120°=60°\Rightarrow\angle BCT=120°=$ $=\angle BOA$ и $BO=BC$, $AO=CT\Rightarrow\Delta BCT=\Delta AOB\Rightarrow AB=BT$. Аналогично $\Delta ADT=\Delta AOB$ т.е. $AB=AT=BT\Rightarrow$ ΔABT - р/с ■

б) 1) По а) ΔABT - р/с, тогда $S_{ABT}=\frac{\sqrt{3}a^2}{4}=\frac{\sqrt{3}AB^2}{4}$. Т.к. ABCD - трапеция, то $S_{ABCD}=\frac{1}{2}h(BC+AD)$

2) По т. cos $\Rightarrow AB^2=BO^2+AO^2-2BO\cdot AO\cdot\cos\angle AOB=4+25-2\cdot 2\cdot 5\cdot\cos 120°=29-20\cdot\left(-\frac{1}{2}\right)=$ $=39\Rightarrow S_{ABT}=\frac{\sqrt{3}\cdot 39}{4}=\frac{39\sqrt{3}}{4}$; $S_{ABCD}=\frac{1}{2}\cdot h(2+5)=\frac{7h}{2}$; $h=h_{AOD}+h_{BOC}$

$h_{AOD}=\sqrt{AO^2-\frac{1}{4}AD^2}=\sqrt{AD^2-\frac{1}{4}AD^2}=\frac{AD\sqrt{3}}{2}=\frac{5\sqrt{3}}{2}$; $h_{BOC}=\sqrt{BO^2-\frac{1}{4}BC^2}=\sqrt{\frac{3}{4}BC^2}=\frac{BC\sqrt{3}}{2}=\frac{2\sqrt{3}}{2}$

№5

Читовеи

11вариант.

Математика гок

2/2

№6

б) 3) Тогда $h = h_{AOD} + h_{BOC} = \frac{5\sqrt{3}}{2} + \frac{2\sqrt{3}}{2} = \frac{7\sqrt{3}}{2}$; тогда $S_{ABCD} = \frac{4 \cdot 7\sqrt{3}}{4} = \frac{49\sqrt{3}}{4}$

$\frac{S_{ABT}}{S_{ABCD}} = \frac{39\sqrt{3}}{\cancel{4}} \cdot \frac{\cancel{4}}{49\sqrt{3}} = \frac{39}{49}$

Ответ: б) $\frac{S_{ABT}}{S_{ABCD}} = \frac{39}{49}$

211005399 (U87096 M1274917)

Черновик

№4

$$\begin{cases} \frac{4}{x^2+y^2} + x^2y^2 = 5 \\ x^4+y^4+3x^2y^2 = 20 \end{cases}$$

$x^2y^2 = 5 - \frac{4}{x^2+y^2}$

$(x^2+y^2)^2 + x^2y^2 = 20$

$(x^2+y^2)^2 + 5 - \frac{4}{x^2+y^2} = 20$

$x^2+y^2 = t$ $\quad (x^2+y^2)^3 - 15(x^2+y^2) - 4 = 0$

$t^3 - 15t - 4 = 0$

$t^2 - \frac{4}{t} = 15; \ t \neq 0$ $\quad t=4$

$t^3 - 15t - 4 = 0$

$t(t^2-15) = 4$

$t^2 - 15 - \frac{4}{t} = 0$

$t^3 - 15t - 4 = 0$ $\quad t(t^2-15) = 4$

$t = 4$ $\quad 4 \cdot (16-15) = 4$

$x^2+y^2 = 4$

$x^2 = 4 - y^2$

$(4-y^2)y^2 = 5 - 1 = 4$

$4y^2 - y^4 = 4$

$y^4 - 4y^2 + 4 = 0$

$D = 16 - 16 = 0$

$\ell_1 = \frac{4}{2} = 2; \ y=2; \ x = 0$

$y^2 = 2$

$\boxed{y = \sqrt{2}, \ x = \sqrt{2}}$

$t^3 - 15t - 4 = 0$

$t(t^2-15) \cdot 4$

$t^2 - 15 < t$

$$\begin{array}{r|l} t^3 - 0t^2 - 15t - 4 & t-4 \\ \hline -\ t^3 - 4t^2 & t^2 + 4t \\ 4t^2 - 15t & \\ -\ 4t^2 - 8t & \\ -7t - 4 & \\ \end{array}$$

$-7t - 4$

$-8t + 24$

-28

$-t$

$$\begin{cases} t^2 - 15 = 1 \\ t = 4 \end{cases}$$

$7t - 4$

$t^2 - 15 = \frac{1}{2}$

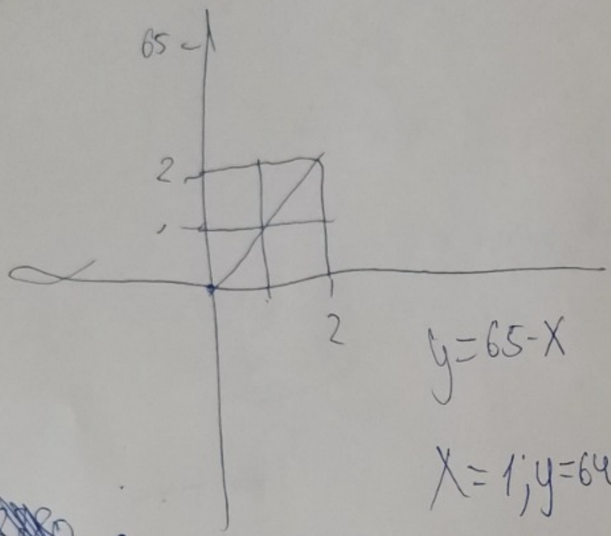
$t = \sqrt{\frac{31}{2}}$

$\sqrt{\frac{31}{2}} \cdot \frac{1}{2} = \sqrt{\frac{15,5}{2}}$

211005399 (U87096 M1274917)

N5

64 числа



$y=65-x$

$x=1, y=64; 2,63; 3,62$

$y=1,1; 2,2; 3,3$

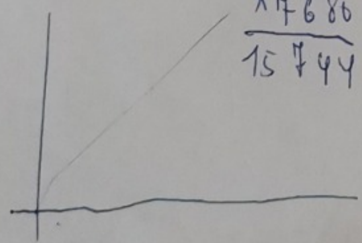
60·60 60·60

$C_{64}^2=\frac{64!}{62!\cdot 2!}=\frac{63\cdot 64}{2\cdot 1}=2016$

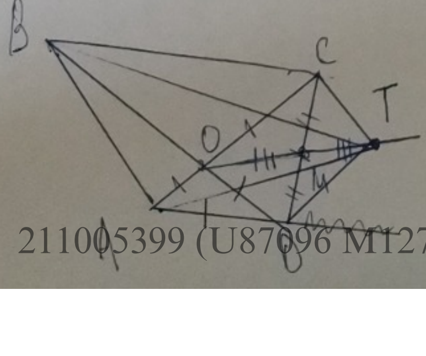
2016·2=4032, или обе на одной прямой

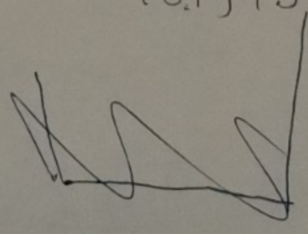
64-1-1-1-1=
=60

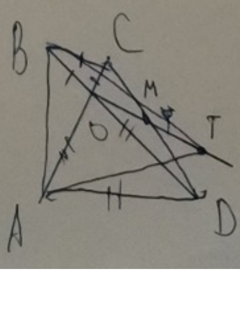
Всего - 8064 вар



N6

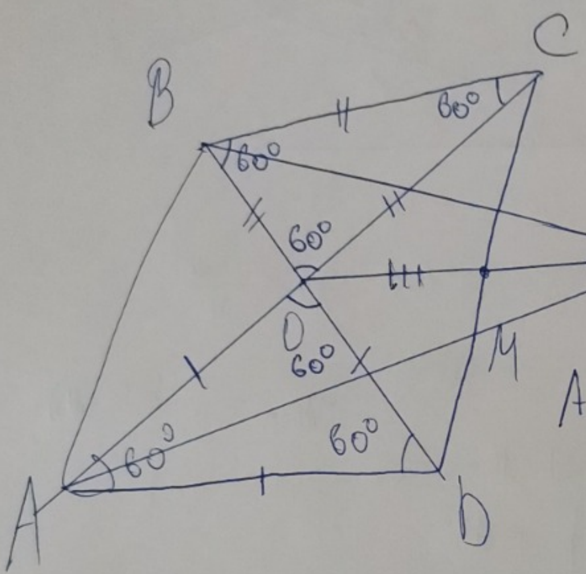


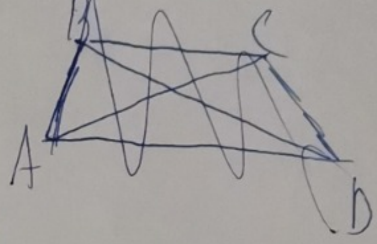




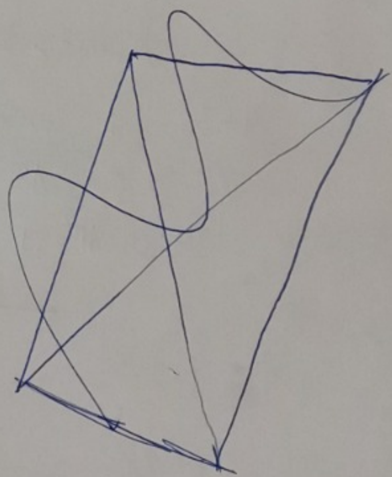
211005399 (U87696 M1274917)

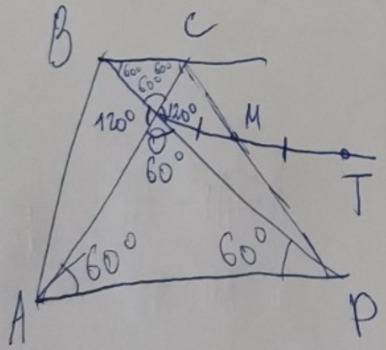
Черновик





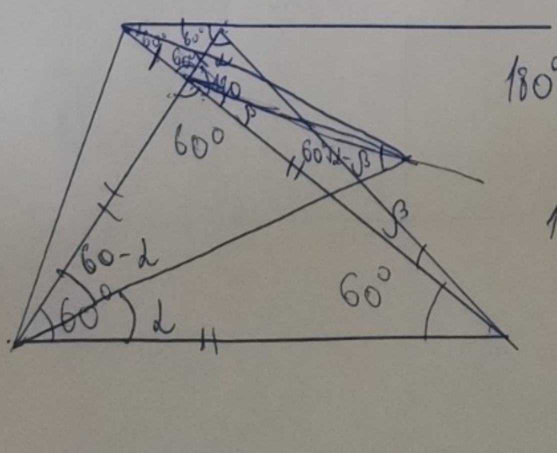
$AD \parallel BC$





$180-\beta$

$180-\beta$



$180^\circ-60-\alpha=120-\alpha=60+\beta$

$180^\circ-60+\alpha-60^\circ-\beta=$

$=60^\circ+\alpha-\beta$

$$\begin{cases} \dfrac{4}{x^2+y^2} + x^2y^2 = 5 \\ x^4+y^4+3x^2y^2=20 \end{cases}$$

$$x^2y^2 = 5 - \frac{4}{x^2+y^2}$$

$$(x^2+y^2)^2 + 5 - \frac{4}{x^2+y^2} = 20$$

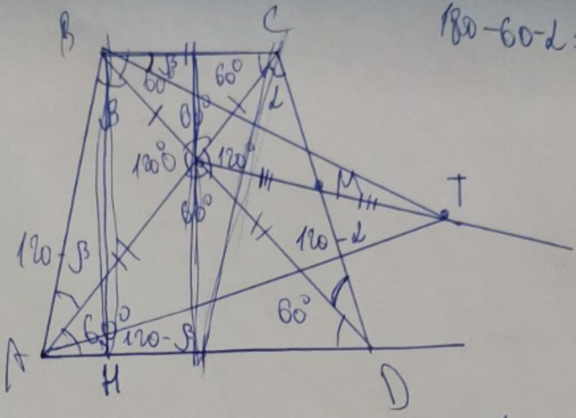
$$(x^2+y^2)^3 - 15(x^2+y^2) - 4 = 0$$

$$t^3 - 15t - 4 = 0$$

$$t(t^2-15) = 4$$

$$t^2 - 15 = \frac{4}{t}$$

$$t^2 - \frac{4}{t} = 15$$



$180 - 60 - \alpha = 120 - \alpha$

$y = x^3 - 5x = 1-5$

$-1+5$

$S_{ABT} = \frac{\sqrt{3}a^2}{4}$

$S_{ABCD} = \frac{1}{2} \cdot h(BC+AD) =$

$= 3{,}5 \cdot h$

Т. cos => $BO^2 + AO^2 - 2BO \cdot AO \cdot \cos 120° =$

$= AB^2$

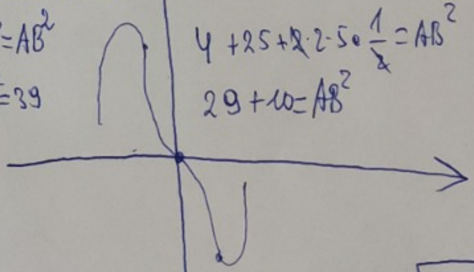
$4+25-2\cdot2\cdot5\cdot\cos120° = AB^2$

$29 - 20\cdot\cos120° = AB^2$

$29+10 = AB^2 = 39$

$4+25+2\cdot2\cdot5\cdot\frac{1}{2} = AB^2$

$29+10 = AB^2$



$y = t^3 - 15t$

$y = t^3 - 15t$

$-8+30=22$

$-27+45=18$

$-64+60=-4$

$-343+7\cdot15-4$

$\frac{S_{ABT}}{S_{ABCD}} = \frac{\sqrt{3}a^2}{4} \cdot \frac{2}{7h} =$

$= \frac{\sqrt{3}AB^2}{14h} =$

$= \frac{\sqrt{3}\cdot39}{14h} = \frac{39\sqrt{3}}{14h}$

$h = \sqrt{AB^2 - AH^2}$

$h^2 + AH^2 = AB^2$

$h_1 = \sqrt{AO^2 - \frac{1}{4}AO^2}$

$= \sqrt{\frac{3AO^2}{4}} = \frac{5\sqrt{3}}{2}$

$h_2 = \sqrt{BC^2 - \frac{1}{4}BC^2} = \sqrt{\frac{3}{4}BC^2}$

$= \sqrt{\frac{3\cdot4}{4}} = \sqrt{3}$

$h = \frac{5\sqrt{3}}{2} + \sqrt{3} = \frac{7\sqrt{3}}{2}$

$$\frac{S_{ABT}}{S_{ABCD}} = \frac{39\sqrt{3}}{14\cdot\frac{7\sqrt{3}}{2}} = \frac{39\sqrt{3}\cdot2}{14\cdot7\sqrt{3}} = \frac{39}{49}$$