Часть 1

Олимпиада: **Математика, 10 класс (1 часть)**

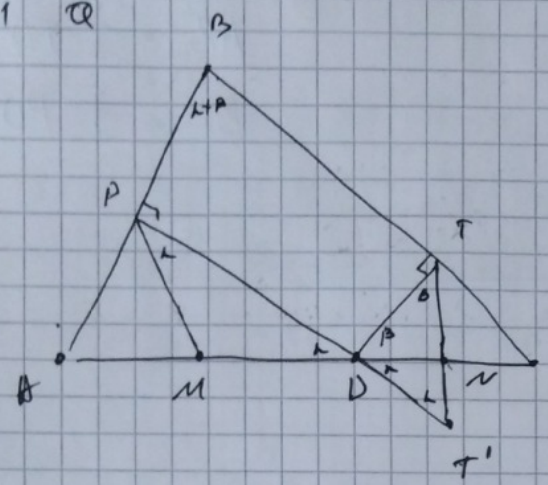
Шифр: **211007715**

ID профиля: **383306**

Вариант 10

Чистовик

№1



T' - пересечение PD и TN

$BTDP$ - вписанный.

$\Rightarrow \angle PDT = 180 - \angle PBT$

$\triangle APD$ - прямоуг.

PM - медиана

$\rightarrow PM = AM = MD$

Аналогично $TN = ND = NC$

Пусть $\angle MPD = \alpha$

$\Rightarrow \angle MBD = \angle MDP = \angle NDT' = \angle DT'N$

($\triangle PMD$ - р/б; верт. углы; $MP \parallel TT'$)

$\angle NTD = \angle NDT$ ($\triangle NTD$ - р/б)

Пусть $\angle NTD = \beta$.

$\angle ABC = 180 - \angle PDT = \alpha + \beta$.

В $\triangle DTT'$: $180 = 2 \cdot (\alpha + \beta)$

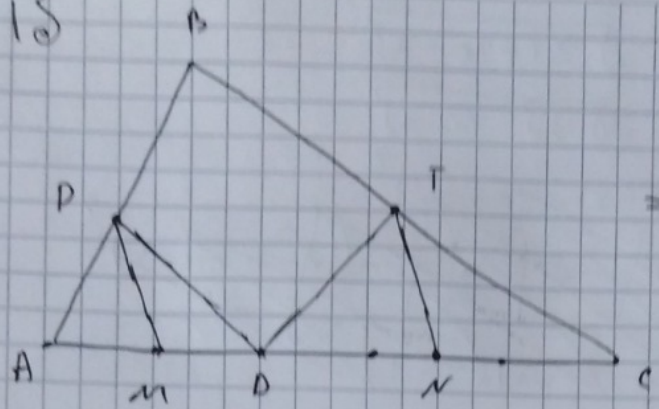
$\Rightarrow \alpha + \beta = 90$

$\Rightarrow \angle ABC = 90$

Ответ: $\angle ABC = 90^\circ$

Чистовик

15



$\angle ABC = \angle DPB = \angle DTB = 90^\circ$

$\Rightarrow$ BTDP - прямоугольник

$\Rightarrow BP = TD$

$BD = \sqrt{5}$

$\Rightarrow TD^2 + PD^2 = 5$

$PM = 1 \Rightarrow AD = 2$

$TN = \frac{3}{2} \Rightarrow DC = 3$

$\frac{PD}{BC} = \frac{2}{5} \quad \frac{TD}{AB} = \frac{3}{5} \Big| \Rightarrow BC = \frac{5}{2} PD; \quad AB = \frac{5}{3} TD$

$\triangle ABC \sim \triangle APD \qquad \triangle ABC \sim \triangle CTD$

PDTB - прямоугольник, $\Rightarrow PD \parallel BC, TD \parallel AB$

$PD^2 + TD^2 = 5$

$AB^2 + BC^2 = AC^2 = 25$

$PD = \sqrt{5 - TD^2}$

$\frac{25}{9}\left(TD^2\right) + \frac{25}{4} \cdot \left(5 - TD^2\right) = 25$

$\frac{TD^2}{9} + \frac{5}{4} - \frac{TD^2}{4} = 1$

$4TD^2 + 45 - 9TD^2 = 36$

$9 = 5TD^2$

$TD = \frac{3}{\sqrt{5}} \qquad TD^2 + PD^2 = 5$

$PD^2 = 5 - \frac{9}{5} = \frac{16}{5}$

$PD = \frac{4}{\sqrt{5}}$

$S = \frac{AB \cdot BC}{2} = \frac{\frac{5}{\sqrt{5}} \cdot \frac{10}{\sqrt{5}}}{2} = \frac{50}{5 \cdot 2} = 5$

Ответ: $S = 5$

(2)

Чистовик

$t = (x-3)\cdot(7-x)$

$4t^2 + 37t + 9 = 0$

$D = 37^2 - 36\cdot 4 = 1225$

$t_1, t_2 = \frac{-37 \pm \sqrt{1225}}{8}$

$\frac{-37 \pm 35}{8} = -\frac{1}{4}$

$-\frac{72}{8} = -9$

$(x+3)\cdot(x-7) = \frac{1}{4}$

$(x+3)\cdot(x-7) = 9$

$x^2 + 4x + 21 = 9$

$x^2 - 4x + 12 = 0$

$D = 16 - 12\cdot 4 < 0$

Корней нет

$x^2 - 4x + 21 = \frac{1}{4}$

$x^2 - 4x + 20\frac{3}{4} = 0$

$D = 16 - 4\cdot 20\frac{3}{4} < 0$

Корней нет.

Ответ: корней нет

№2 Чистовик

$$\sqrt{x+3} - \sqrt{7-x} + 4 = 2 \cdot \sqrt{21+4x-x^2}$$

$$\sqrt{x+3} - \sqrt{7-x} + 4 = 2 \cdot \sqrt{(x+3)\cdot(7-x)}$$

$$\sqrt{x+3} - \sqrt{7-x} + 4 = 2 \cdot \sqrt{(x+3)\cdot(7-x)}$$

$$\sqrt{x+3} - \sqrt{7-x} - 6 = -\left(\sqrt{x+3} - \sqrt{7-x}\right)^2$$

$$\sqrt{x+3} - \sqrt{7-x} + 6 = \left(\sqrt{x+3} - \sqrt{7-x}\right)^2 = \left(\sqrt{7-x} - \sqrt{x+3}\right)^2$$

$$6 = \sqrt{7-x} - \sqrt{x+3} \cdot \left(\sqrt{7-x} - \sqrt{x+3} - 1\right)$$

$$t = (x+3)$$

$$\sqrt{x+3} - \sqrt{7-x} = 2 \cdot \left(\sqrt{(x+3)\cdot(7-x)} - 2\right)$$

$$x+3 + 7 - x - 2\sqrt{(x+3)\cdot(7-x)} = 4 \cdot \left((x+3)\cdot(7-x) - 4 \cdot \sqrt{(x+3)\cdot(7-x)} + 4\right)$$

$$5 - \sqrt{(x+3)\cdot(7-x)} = 2 \cdot (x+3)\cdot(7-x) - 8 \cdot \sqrt{(x+3)\cdot(7-x)} + 8$$

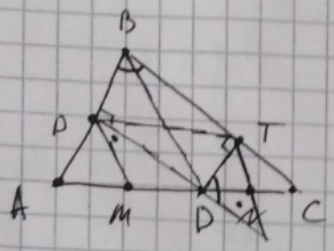
$$7 \cdot \sqrt{(x+3)\cdot(7-x)} = \left(2 \cdot (x+3)\cdot(7-x) + 3\right)$$

$$49 \cdot (x+3)\cdot(7-x) = \left(2 \cdot (x+3)(7-x)\right)^2 + 9 + 12\,(x+3)\cdot(7-x)$$

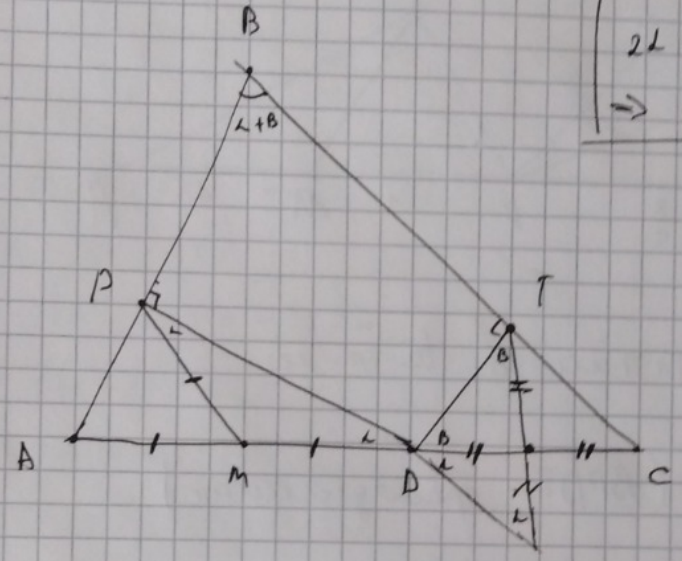
$$\left(2 \cdot (x+3)\cdot(7-x)\right)^2 + 37\left((x+3)\cdot(7-x)\right) + 9 = 0$$

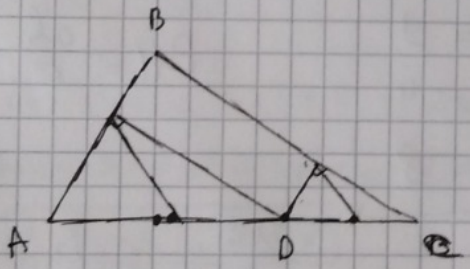
$$4 \cdot (x+3)\cdot(7-x) \cdot \left(4 \cdot (x+3)\cdot(7-x) + 1\right)$$

Чернови к



$$\boxed{\begin{array}{l} 2\alpha + 2\beta = 180 \\ \Rightarrow \alpha + \beta = 90 \end{array}}$$





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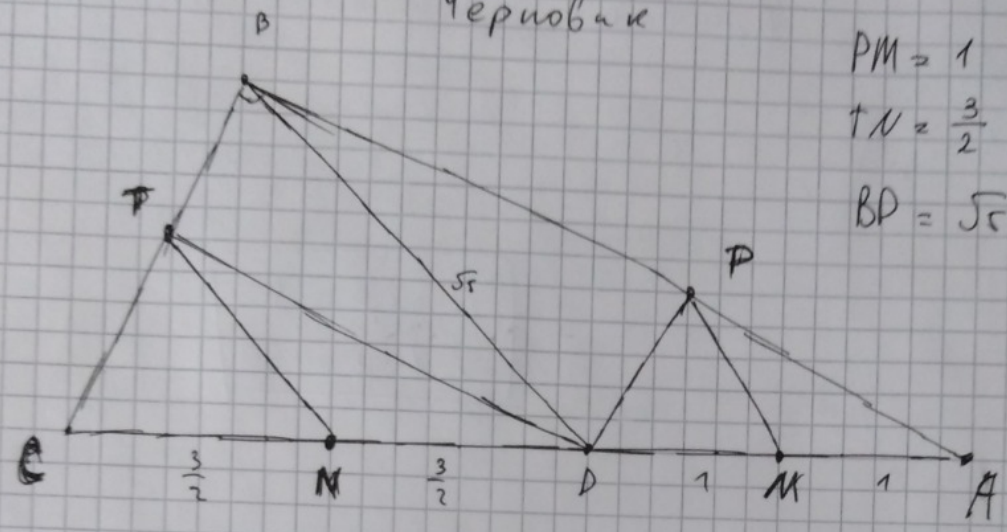
Черновик



$PM = 1$

$TN = \frac{3}{2}$

$BP = \sqrt{5}$

$\boxed{AC = 5}$ - медиана в $\triangle$ тр-ке

$TD^2 + PD^2 = 5$ (BPDT - прямоугольник)

$\frac{PD}{BC} = \frac{2}{5}$

$\frac{TD}{AB} = \frac{3}{5}$

$\boxed{\begin{aligned} TD^2 + TC^2 &= 9 \\ PD^2 + PA^2 &= 4 \\ TC^2 + PA^2 &= 8 \end{aligned}}$

$BH = \frac{ab}{c}$

$\frac{TD}{TD + AP} = \frac{3}{5}$

$\frac{ab}{2} = \frac{c \cdot h}{2}$

$h = \frac{ab}{c}$

$5TD = 3TD + 3AP$

$2TD = 3AP$

$TD = 1,5\ AP$

(2)

Черновик

$\sqrt{x+3} + \sqrt{7-x} + 4 = 2 \cdot \sqrt{21+4x-x^2}$

$25 - (x-2)^2 = 5 - (x-2) \cdot 5 + (x-4)$

$= (3-x) \cdot (x+1) =$

$= 3x - x^2 + 3 =$

$-x^2 + 4x + 21 =$

$-(x^2 - 4x - 21) =$

$= -(x^2 - 4x + 4 - 25) =$

$= -((x-2)^2 - 25) =$

$= 25 - (x-2)^2 =$

$= (5 - (x-2)) \cdot (5 + (x-2)) =$

$= (7-x) \cdot (3+x)$

$\sqrt{x+3} - \sqrt{7-x} + 4 = 2\sqrt{(x+3) \cdot (7-x)}$

~~$\sqrt{x+3} + \sqrt{7-x} + 4 = (x+3) + (7-x) = 100$~~

$\boxed{x \geq -3 \; ; \; x \leq 7}$

$(x+3) + (7-x) + 16 - 2\sqrt{(x+3) \cdot (7-x)} + 8\sqrt{x+3} - 8\sqrt{7-x} = 4 \cdot (x+3) \cdot (7-x)$

$\sqrt{x+3} - \sqrt{7-x} + 14 = 100$

$\sqrt{x+3} - \sqrt{7-x} = 86$

$\sqrt{10}$ $\quad$ 0

$\Rightarrow$ Решений нет.

(3)

Черновик

$\sqrt{x+3} - \sqrt{7-x} + 4 = 2\sqrt{21+4x-x^2}$

$\sqrt{x+3} - \sqrt{7-x} + 4 = 2 \cdot \sqrt{25-(x-2)^2}$

$\sqrt{x+3} - \sqrt{7-x} + 4 = 2 \cdot \sqrt{(5-(x-2)) \cdot (5+(x-2))} = 2 \cdot \sqrt{(x+3) \cdot (7-x)}$

$((x+3)+(7-x)) + \sqrt{x+3} - \sqrt{7-x} + 4 = (x+3) + 2\sqrt{(x+3)\cdot(7-x)} + (7-x)$

$10 + \sqrt{x+3} - \sqrt{7-x} + 4 = (\sqrt{x+3} + \sqrt{7-x})^2$

$-((x+3)+(7-x)) + \sqrt{x+3} - \sqrt{7-x} + 4 = -(x+3) + 2\sqrt{(x+3)\cdot(7-x)} - (7-x)$

$\sqrt{x+3} - \sqrt{7-x} - 6 = -(\sqrt{x+3} - \sqrt{7-x})^2$

$-6 = -1 \cdot (\sqrt{x+3} - \sqrt{7-x}) \cdot (\sqrt{x+3} - \sqrt{7-x} + 1)$

$6 = (\sqrt{x+3} - \sqrt{7-x}) \cdot (\sqrt{x+3} - \sqrt{7-x} + 1)$

$6 = (x+3) + (7-x) - 2\sqrt{(x+3)\cdot(7-x)} + (\sqrt{x+3} - \sqrt{7-x})$

$-4 =$

$\underbrace{(x+3)+(7-x)}_{26} + 16 - 2\sqrt{(x+3)\cdot(7-x)} + 8\sqrt{x+3} - 8\sqrt{7-x} = 4 \cdot (x+3) \cdot 7 - x$

$13 - \sqrt{(x+3)\cdot(7-x)} + 4\sqrt{x+3} - 4\sqrt{7-x} = 2 \cdot (x+3) \cdot (7-x)$

$43 - 10 + (x+3) + x - 7$

$26 - 10 + (x+3) + (7-x) - 2\sqrt{(x+3)\cdot(7-x)} + 8\sqrt{x+3} - 8\sqrt{7-x} = 4 \cdot (x+3) \cdot (7-x)$

$16 + (\sqrt{x+3} - \sqrt{7-x})^2$

4

Черновик

$A = 5a^2 - 4ay + 8x^2 - 4xy + y^2 + 12ax = 0$

$D = (4y + 12x)^2 - 4 \cdot 5 \cdot (8x^2 - 4xy + y^2) =$

$= 16y^2 + 144x^2 + 96xy - (160x^2 - 20y^2 - 80xy) =$

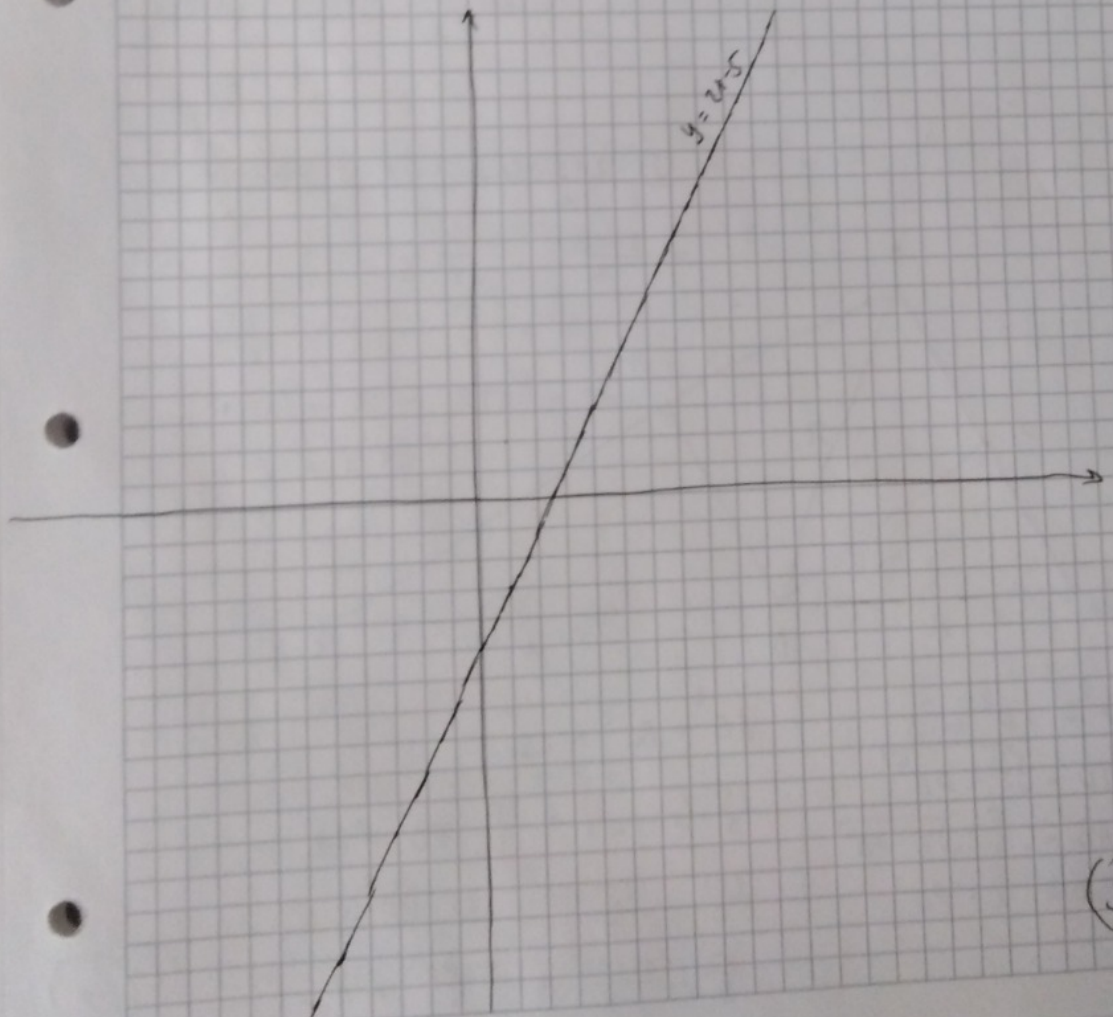
$= -16x^2 + 16xy - 4y^2 = -4 \cdot (4x^2 - 4xy + y^2) = -4 \cdot (2x - y)^2$

$D < 0$

$B = ax^2 - 2a^2x + ay + a^3 + 3 = 0$

$ay = -ax^2 + 2a^2x - a^3 - 3$

$y = -x^2 + 2ax - a^2 - \frac{3}{a}$



(5)

Черновик

$y = 2x - 5$

$A \in 5a^2 - 4ay + 8x^2 - 4xy + y^2 + 12ax = 0$

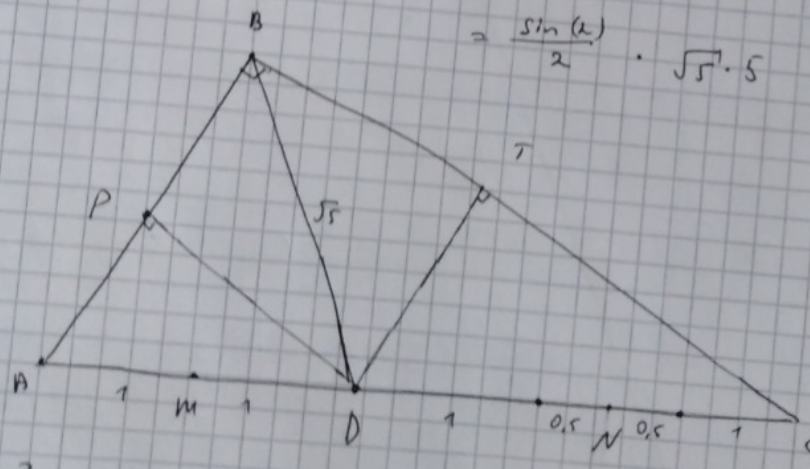
$D = (12x - 4y)^2 - 4 \cdot 5 \cdot (8x^2 - 4yx + y^2)$

$144x^2 + 16y^2 - 96xy - (160x^2 - 80xy + 20y^2) =$

$= -16x^2 + 16xy - 4y^2 = -4 \cdot (2x^2 + y)^2 \leq 0$

$B \in ax^2 - 2a^2x - ay + a^3 + 3 = 0$

$a^3 - 2x \cdot a^2 + (x^2 - y) \cdot a + 3 = 0$



$\frac{\sqrt{5} \cdot 2 \cdot \sin(\alpha)}{2} + \frac{\sqrt{5} \cdot 3 \cdot \sin(\alpha)}{2} =$

$= \frac{\sin(\alpha)}{2} \cdot \sqrt{5} \cdot 5$

$PD^2 + TD^2 = 5$

$TD = \sqrt{5 - PD^2}$

$AB = \frac{5}{3} \cdot \sqrt{5 - PD^2}$

$BC = \frac{5}{2} \cdot PD$

$AB^2 + BC^2 = 25$

$\frac{25}{9} \cdot (5 - PD^2) + \frac{25}{4} \cdot PD^2 = 25$

$\frac{125}{9} - \frac{25 \cdot PD^2}{9} + \frac{25 \cdot PD^2}{4} = 25$

$\frac{5}{9} - \frac{5 \cdot PD^2}{9} + \frac{5 \cdot PD^2}{4} = 1$

(6)

Черновик

$20 - 20PD^2 + 45PD^2 = 36$

$25PD^2 = 16$

$PD = \frac{4}{5}$

$BC = \frac{4}{5} \cdot \frac{5}{2} = 2$

$AB = \frac{5}{3} TD$

$BC = \frac{5}{2} PD$

$PD^2 + TD^2 = 5$

$\frac{16}{25} + TD^2 = 5$

$TD^2 = 4\frac{9}{25} = \frac{109}{25}$

~~$TD = \frac{\sqrt{109}}{5} = \sqrt{109}$~~

$PD^2 = 5 - TD^2$

$AB^2 + BC^2 = 25 = \frac{25}{9} TD^2 + \frac{25}{4} PD^2 = 25$

$\frac{25}{9} \cdot TD^2 + \frac{25}{4} \cdot (5 - TD^2) = 25$

$\frac{TD^2}{9} + \frac{5}{4} - \frac{TD^2}{4} = 1$

$4TD^2 + 45 - 9TD^2 = 36$

$9 = 5TD^2$

$\boxed{\frac{3}{\sqrt{5}} = TD}$

$\frac{25}{9} \cdot (5 - PD^2) + \frac{25}{4} \cdot PD^2 = 25$

$\frac{5}{9} - \frac{PD^2}{9} + \frac{PD^2}{4} = 1$

$20 - 4PD^2 + 9PD^2 = 36$

$5PD^2 = 16$

$PD = \frac{4}{\sqrt{5}}$

$TD^2 + PD^2 = \frac{9}{5} + \frac{16}{5} = \frac{25}{5} = 5$

~~№ 8~~

$AB = \frac{5}{\sqrt{5}}$ $BC = \frac{10}{\sqrt{5}}$

$S = \frac{50}{5} : 2 = 5$

(7)

Часть 2

Олимпиада: **Математика, 10 класс (2 часть)**

Шифр: **211007715**

ID профиля: **383306**

Вариант 10

Чистовик Вариант - 10

№1

$$\begin{cases} \frac{6}{x^2+y^2} + x^2y^2 = 10 \quad / \cdot 5 \\ x^4+y^4+7x^2y^2 = 81 \end{cases}$$

$$\begin{cases} \frac{30}{x^2+y^2} + 5x^2y^2 = 50 \\ x^4+2x^2y^2+y^4+5x^2y^2 = 81 \end{cases}$$

$$(x^2+y^2)^2 + 5x^2y^2 - \left(\frac{30}{x^2+y^2} + 5x^2y^2\right) = 81-50$$

$$(x^2+y^2)^2 - \frac{30}{x^2+y^2} = 31$$

$$(x^2+y^2)^3 - 31(x^2+y^2) - 30 = 0$$

Пусть $t = x^2+y^2$

$$t^3 - 31t - 30 = 0$$

$$\begin{array}{l} t^3 - 31t - 30 \;\big|\; (t+1) \\ \underline{t^3 + t^2} \qquad\qquad\quad t^2 - t - 30 \\ -t^2 - 31t - 30 \\ \underline{-t^2 - t} \\ -30t - 30 \\ \underline{-30t - 30} \\ 0 \end{array}$$

$$t^3 - 31t - 30 = (t+1)\cdot(t^2 - t - 30) = 0$$

$$t^2 - t - 30 = 0$$

$$D = 1 + 120 = 121$$

$$t_1, t_2 = \frac{1 \pm 11}{2} = 6; -5$$

$$t^3 - 31t - 30 = (t+1)\cdot(t+5)\cdot(t-6)$$

$t = -1$; $t = -5$; $t = 6$

(1)

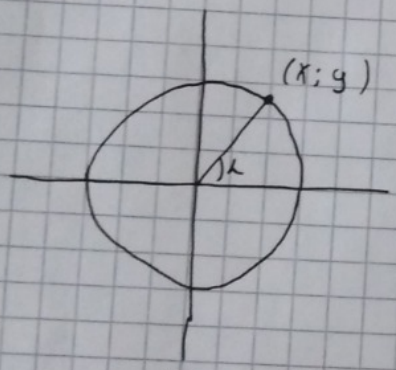
Чистовик

$x^2+y^2=-1$; $x^2+y^2=-5$; $x^2+y^2=6$

Не имеют решений ~~#~~ ~~[illegible]~~

$x^2+y^2=6$ - окружность с центром в $(0;0)$ радиусом $\sqrt{6}$

$$\begin{cases} \frac{6}{x^2+y^2}+x^2y^2=10 \\ x^4+y^4+7x^2y^2=81 \end{cases} \Leftrightarrow \begin{cases} x^2y^2=9 \\ 5x^2y^2=81-6^2=45 \end{cases} \Leftrightarrow |xy|=3$$



$x=\cos(\alpha)\cdot\sqrt{6}$

$y=\sin(\alpha)\cdot\sqrt{6}$

$|xy|=|\cos(\alpha)\cdot\sin(\alpha)\cdot 6|=3$

$|\cos(\alpha)\cdot\sin(\alpha)|=\frac{1}{2}$

$|\sin(2\alpha)|=2\cdot|\cos(\alpha)\cdot\sin(\alpha)|=1$

$2\alpha=\frac{\pi}{2}$; $\frac{3\pi}{2}$; $\frac{5\pi}{2}$; $\frac{7\pi}{2}$

$\Rightarrow \alpha=\frac{\pi}{4}$; $\frac{3\pi}{4}$; $\frac{5\pi}{4}$; $\frac{7\pi}{4}$

$\Rightarrow \sin(\alpha)=\pm\frac{\sqrt{2}}{2}$ $\cos(\alpha)=\pm\frac{\sqrt{2}}{2}$

$\Rightarrow$ 4 решения Ответ:

$x=\frac{\sqrt{2}}{2}\cdot\sqrt{6}=\sqrt{3}$; $y=\frac{\sqrt{2}}{2}\cdot\sqrt{6}=\sqrt{3}$

$x=-\frac{\sqrt{2}}{2}\cdot\sqrt{6}=-\sqrt{3}$; $y=\frac{\sqrt{2}}{2}\cdot\sqrt{6}=\sqrt{3}$

$x=-\frac{\sqrt{2}}{2}\cdot\sqrt{6}=-\sqrt{3}$; $y=-\frac{\sqrt{2}}{2}\cdot\sqrt{6}=-\sqrt{3}$

$x=\frac{\sqrt{2}}{2}\cdot\sqrt{6}=\sqrt{3}$; $y=-\frac{\sqrt{2}}{2}\cdot\sqrt{6}=\sqrt{3}$

Чистовик

№5

Сначала выберем узел на главных диагоналях ($y = x$ или $y = 69 - x$). Вариантов $68 \cdot 2$

Теперь посчитаем, сколько возможных позиций у 2 узла:

Всего $68 \cdot 68$

Нельзя выбрать тот же: $68 \times 68 - 1$

Нельзя выбрать в той же строке или столбце.

Вариантов в строке - 67

в столбце - 67

$\Rightarrow$ Нельзя выбрать $(67 \times 2 + 1)$ узел

$\Rightarrow$ Можно выбрать $68 \times 68 - (67 \times 2 + 1)$

$\Rightarrow$ Всего способов:

$$(68 \times 2) \times (68 \times 68 - (67 \times 2 + 1)) =$$

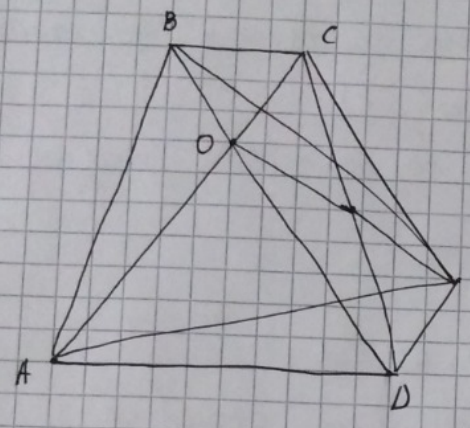
$$= 136 \times (4624 - 135) =$$

$$= 136 \times 4489 = 610504$$

Ответ: 610504

Чистовик

6.



Заметим, что $BC \parallel AD$

$\angle OAD = \angle OCB = 60$

$OCTD$ - пар-мм

M - середина CD

T $CM = MD$, $OM = MT$

$\Rightarrow BC = OC = TD$

$\triangle BCO$ - р/с, $OCTD$ - пар-мм

$\Rightarrow BCTD$ - равнобокая трапеция

$\Rightarrow BCTD$ - вписанный четырехугольник

$\Rightarrow \angle CBT = \angle CDT$

$\angle CDT = \angle DCO$ ($TD \parallel CO$)

$ABCD$ - вписанный ($\angle BCA = \angle BDA = 60$)

$\Rightarrow \angle DCO = \angle DBA \Rightarrow \angle TBC = \angle ABD$

$\angle TBC + \angle OBT = \angle ABO + \angle OBT = 60$

$AD - OD = CT$

$\Rightarrow ACTD$ - равнобокая трапеция ($TD \parallel CO$)

$\Rightarrow ACTD$ - вписанный четырехугольник

$\angle TAD = \angle TCD = \angle CDO = \angle CAB$

$ACTD$-вписанный, $CT \parallel OD$, $ABCD$ - вписанный

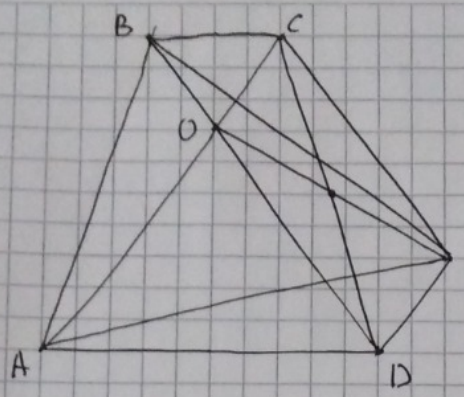
$\Rightarrow \angle TAD + \angle OAT = \angle BAC + \angle OAT = 60$

$\Rightarrow \triangle ABT$ имеет 2 угла по 60 градусов

$\Rightarrow \triangle ABT$ - правильный

4

Чистовик



$BC = BO = OC = TD = 2$

$AO = AD = OD = CT = 7$

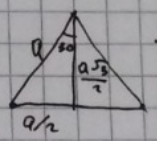
Запишем т. Косинусов для $\triangle ABD$

$BD = 2 + 7 = 9$

$AB^2 = BD^2 + AD^2 - 2 \cdot AB \cdot BD \cdot \cos(60)$

$AB^2 = 81 + 49 - 63 = 67$

$AB = \sqrt{67}$



$S = \frac{a \cdot a \cdot \sqrt{3}}{2 \cdot 2}$

$\triangle ABT$ – р/с $\Rightarrow S_{ABT} = \frac{AB^2 \cdot \sqrt{3}}{4} = \frac{67\sqrt{3}}{4}$

$S_{\triangle BCD} = S_{\triangle BTD}$ ($BC = TD$, BD – общая, $\angle CBD = \angle TDO = 60$)

$\angle COD = 180 - 60$, $CO \parallel TD$

$S_{ABCD} = S_{ABD} + S_{BCD} = S_{ABD} + S_{ATD} = S_{ABTD}$

$\frac{S_{ABT}}{S_{ABCD}} = \frac{S_{ABT}}{S_{ABTD}} = \frac{S_{ABT}}{S_{ABT} + S_{ATD}}$

$S_{ATD} = \frac{AD \cdot DT \cdot \sin(\angle BDA + \angle BDT)}{2} = \frac{AD \cdot DT \cdot \sin(120)}{2} =$

$= \frac{AD \cdot DT \cdot \sqrt{3}}{4} = \frac{14 \cdot \sqrt{3}}{4}$

$S_{ABTD} = \frac{67\sqrt{3}}{4} + \frac{14\sqrt{3}}{4} = \frac{81\sqrt{3}}{4}$

$\frac{S_{ABT}}{S_{ABCD}} = \frac{S_{ABT}}{S_{ABTD}} = \frac{67 \cdot \sqrt{3}}{4} : \frac{81\sqrt{3}}{4} = \frac{67}{81}$

Ответ: $\frac{67}{81}$

5

Вариант - 10

Черновик

$67 \nearrow \cdot 67 \searrow - 67 \cdot 2$

~~$(67 \cdot 2) \cdot 67 \cdot 67 - 67 \cdot 2 + 1$~~

68

$(68 \cdot 2) \cdot (68 \cdot 68 - 68 \cdot 2)$

$(68 \cdot 2) \cdot ((68 \cdot 68 - 1) - 67 \cdot 2)$

$136 \cdot (4624 - 1 - 134)$

$136 \cdot (4489) = 610504$

67×65

335

402

4356

68×2

16

12

136

68×68

544

408

4624

4489×136

26934

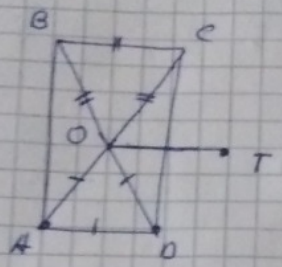
13467

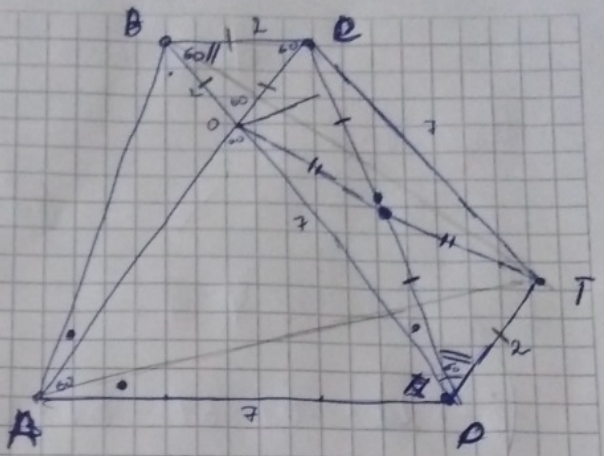
4489

610504

(1)

Черновик





$\angle BAO = \angle ODC$

$\angle DAC = \angle TCA$

$\angle TCD = \alpha = DAT$

$\angle DCA = 60 - \alpha$

$\angle ADC = 60 + \alpha$

$\angle ODA = \alpha = \angle BAC$

$CAT + \alpha = 60$

$\Rightarrow \angle BAT = 60$

$\angle ABD = \angle ACD = 60 - \alpha$

$\angle AOD + \angle OAD + \angle ADO = 180 = 120 + \alpha + \angle ADO$

$\angle ADO = 60 - \alpha$

$\Rightarrow \angle ABT = 60 - (60 - \alpha) + (60 - \alpha) = 60$

$\Rightarrow ABT - p/c$

2

Черновик

$S_{ABCD} = S_{ABTD}$ (CT // BD $\Rightarrow S_{BCD} = S_{BTD}$

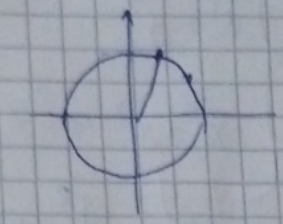
$BD = 2 + 7 = 9$

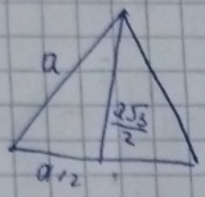
$AB^2 = AD^2 + BD^2 - 2 \cdot AB \cdot BD \cdot \cos(60) =$

$= 49 + 81 - 63 = 67$

$AB = \sqrt{67}$

$S_{ABT} = \frac{67\sqrt{3}}{4}$





$S_{ADT} = \frac{7 \cdot 2 \cdot \sin(120)}{2} = \frac{7\sqrt{3}}{2}$

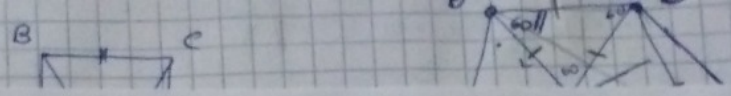
$S = \frac{a^2\sqrt{3}}{4}$

$\frac{a \cdot b \cdot \sin(\alpha)}{2}$

$\frac{a \cdot a \cdot \frac{\sqrt{3}}{2}}{2} = \frac{a^2\sqrt{3}}{4}$

$\frac{S_{ABT}}{S_{ABTD}} = \frac{S_{ABT}}{S_{ABT} + S_{ATD}} = \frac{\frac{67\sqrt{3}}{4}}{\frac{67\sqrt{3}}{4} + \frac{14\sqrt{3}}{4}} =$

$= \frac{67\sqrt{3}}{4} : \frac{81\sqrt{3}}{4} = \frac{67}{81}$



Черновик

$$\begin{cases} \frac{6}{x^2+y^2} + x^2y^2 = 10 \\ x^4+y^4+7x^2y^2 = 81 \end{cases}$$

$$(x^2+y^2)^2 + 5x^2y^2 = 81$$

$$(x^2+y^2)^2 - \frac{6}{x^2+y^2} = 81-50=31$$

$$\frac{(x^2+y^2)^3-6}{x^2+y^2} = 31$$

$$(x^2+y^2+2xy)^2 = x^4+y^4+4x^2y^2+4x^3y+4xy^3+2x^2y^2$$

$$x^4+y^4+6x^2y^2+4xy\cdot(x^2+y^2)$$

$$(x+y)^4 - 4xy\cdot(x^2+y^2)+x^2y^2 = 81$$

$$x^4+2x^2y^2+y^4+5x^2y^2 = 81$$

$$\begin{cases} (x^2+y^2)^2+5x^2y^2 = 81 \\ 6+x^2y^2\cdot(x^2+y^2) = 10(x^2+y^2) \end{cases}$$

$$(x^2+y^2)^3+5x^2y^2\cdot(x^2+y^2) = 81(x^2+y^2)$$

$$(x^2+y^2)^2+5x^2y^2-\left(\frac{30}{x^2+y^2}+5x^2y^2\right) = 81-50$$

$$(x^2+y^2)^3-30 = 31\cdot(x^2+y^2)$$

$t = x^2+y^2$

Черновик

$t^3 - 31t - 30 = 0$ $t = -1$

$-1 + 31 - 30 = 0$

$$\begin{array}{l} t^3 - 31t - 30 \;\big|\; t+1 \\ t^3 + t^2 \quad\quad\;\; t^2 - t - 30 \\ \hline -t^2 - 31t - 30 \\ -t^2 - t \\ \hline -30t - 30 \\ -30t - 30 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 68 \\ \times\; 68 \\ \hline 544 \\ 408 \\ \hline 4624 \\ 135 \\ \hline 4489 \end{array}$$

$$\begin{array}{r} 4489 \\ \times\; 136 \\ \hline 26934 \\ 13467 \\ 4489 \\ \hline 610504 \end{array}$$

$(t+1)\cdot(t^2 - t - 30) = 0$

$t^2 - t - 30 = 0$

$D = 1 + 120 = 121$

$t_1, t_2 \; \frac{1 \pm 11}{2} = 6, -5$

$(t+1)\cdot(t-6)\cdot(t+5) = 0$

$x^2 + y^2 = -1$, $x^2 + y^2 = -5$, $x^2 + y^2 = 6$

Не имеют решения в натуральных числах.

$\rightarrow x^2 + y^2 = 6$

$$\begin{cases} \frac{6}{x^2+y^2} + x^2y^2 = 10 \\ x^4 + y^4 + 7x^2y^2 = 81 \end{cases} \Leftrightarrow$$

$1 + x^2y^2 = 10$ $x^2y^2 = 9$

$36 + 5x^2y^2 = 81$

5

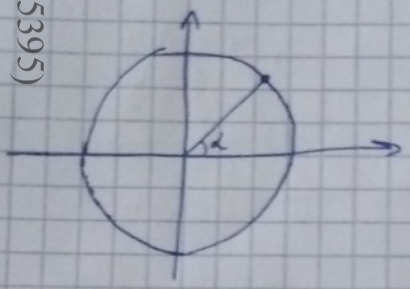
211007715 (U383306 M1275395)

Черновик

$x^2 + y^2 = 6$ – окружность с центром $O(0;0)$ радиуса $\sqrt{6}$

$x^2 y^2 = 9$

$|xy| = 3$



~~$\sin(\alpha) \cdot \cos(\alpha) \cdot \sqrt{6}$~~

$(|\sin(\alpha)| \cdot \sqrt{6}) \cdot (|\cos(\alpha)| \cdot \sqrt{6}) = 3$

$|\sin(\alpha) \cdot \cos(\alpha)| = \frac{1}{2}$

$|\sin(2\alpha)| = 2\,|\sin(\alpha) \cdot \cos(\alpha)|$

$|\sin(2\alpha)| = 1, \quad 2\alpha = \frac{\pi}{2}, \quad 2\alpha = \frac{3\pi}{2}$

~~$\sin(\alpha)$~~ $\alpha = \frac{\pi}{4}, \frac{3}{4}\pi$

$\sin(\alpha) = \frac{\sqrt{2}}{2}, \quad \sin(\alpha) = \frac{\sqrt{2}}{2}$

$\cos(\alpha) = \frac{\sqrt{2}}{2}, \quad \cos(\alpha) = -\frac{\sqrt{2}}{2}$

$x = \frac{\sqrt{2}}{2} \cdot \sqrt{6}, \quad y = \frac{\sqrt{2}}{2} \cdot \sqrt{6}$

$x = \frac{\sqrt{2}}{2} \cdot \sqrt{6}, \quad y = -\frac{\sqrt{2}}{2} \cdot \sqrt{6}$

$x = -\frac{\sqrt{2}}{2} \cdot \sqrt{6}, \quad y = -\frac{\sqrt{2}}{2} \cdot \sqrt{6}$

$x = -\frac{\sqrt{2}}{2} \cdot \sqrt{6}, \quad y = \frac{\sqrt{2}}{2} \cdot \sqrt{6}$

$\frac{\sqrt{2}}{2} \cdot \sqrt{6} = \frac{2 \cdot \sqrt{3}}{2} = \sqrt{3}$

6