Часть 1

Олимпиада: **Математика, 10 класс (1 часть)**

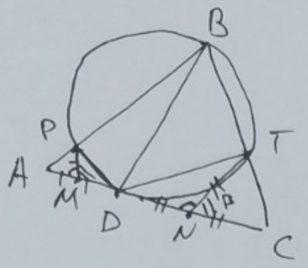
Шифр: **211006972**

ID профиля: **825447**

Вариант 10

№1

Чистовик



$PM \parallel TN$

$\angle BTD = \angle BPD = 90^\circ$ т.к. опираются на диаметр

$\angle APD = \angle NTC = 90^\circ$

т.к. $PM \parallel TN$

$\angle PMA = \angle TND = \alpha$

т.к. TN и PM медианы исходящие из прямого угла

$AM = MP = PM \quad DN = NT = NC$

$\angle PMD = \angle TNC = \beta$

$\angle DTC = 90 = \angle DTN + \angle NTC = \frac{180-\alpha}{2} + \frac{180-\beta}{2}$

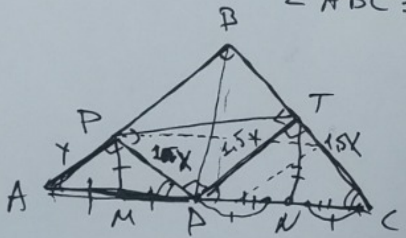
$\angle ADD = 90 = \angle APM + \angle MPD = \frac{180-\alpha}{2} + \frac{180-\beta}{2}$

$\angle PDA = \angle TCN = \frac{180-\beta}{2}$

$PD \parallel BC \quad \angle PDT + \angle BTD = 180$

$\angle PDT = 90$

$\angle ABC = 90^\circ$



$DB = \sqrt{5}$

$PM = 1 \quad TN = 1,5$

$AC = 2PM + 2TN = 5$

$\triangle APD \sim \triangle DTC$ по углам

$\frac{PM}{TN} = \frac{1}{K} \Rightarrow K = 1,5$

$AP = \frac{PT}{1,5} \quad \frac{TC}{1,5} = PD$

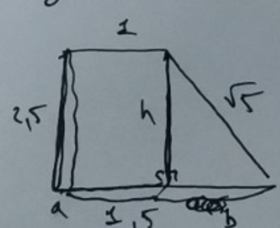
$AB = AP + PB = AP + DT = 2,5y$

$BC = 2,5x$

$\triangle ABC \sim \triangle APD$ по 2 сторонам и углу между ними.

Заметим что $S_{PMNT} = \frac{S_{ABC}}{2} \quad PT = BD = \sqrt{5}$

$MN = \frac{AC}{2} = \frac{5}{2} = 2,5$



$6,25 = h^2 + a^2 \quad 5 = h^2 + b^2 \quad a + b = 0,5$

$1,25 = (a-b)(a+b) \quad a - b = 2,5 \quad a = 1,5 \quad b = -1$

$h = 2 \quad S = 2 \cdot \frac{1+1,5}{2} \Rightarrow S_{ABC} = 5$ см

1

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№2 Чистовик

$\sqrt{x+3}-\sqrt{7-x}+4=2\sqrt{(x+3)(7-x)}$

$x+3=b$

$\sqrt{b}-\sqrt{10-b}+4=2\sqrt{b(10-b)}$

ОДЗ: $b\geq 0$ $b\leq 10$

$\sqrt{b}-\sqrt{10-b}=2\sqrt{b(10-b)}-4$

пусть $\sqrt{b}-\sqrt{10-b}\geq 0$ это при $b>5$
возведем 2 части в квадрат

$10-2\sqrt{b(10-b)}=4b(10-b)-16\sqrt{b(10-b)}+16$

$4\underbrace{b(10-b)}_{y^2}-14\underbrace{\sqrt{b(10-b)}}_{y}+6=0$

$D=196-4\cdot4\cdot6=100$

$y=\frac{14\pm10}{8}=0{,}5$

$=3$

$\sqrt{b(10-b)}=0{,}5$

$b(10-b)=0{,}25$

$-b^2+10b=0{,}25$

$b^2-10b+0{,}25=0$

$D=100-1=99$

$b_{1,2}=\frac{10\pm\sqrt{99}}{2}$ но $b<5$ $b_2=\frac{10+\sqrt{99}}{2}$

$\sqrt{b(10-b)}=3$

$b^2-10b+9=0$

$D=64$

$b_{3,4}=\frac{10\pm8}{2}=9$

$=1$

$b_1=9$ для $b>5$

$x=6$

пусть $b<5$

$\sqrt{10-b}-\sqrt{b}=4-2\sqrt{b(10-b)}$

$10-2\sqrt{(10-b)b}=16+4\sqrt{b(10-b)}-16\sqrt{b(10-b)}$

в этом случае $b_1=\frac{10-\sqrt{99}}{2}$ $b_4=$ ~~$\frac{10+\sqrt{99}}{2}$~~

и для $b=5$ нету, подставим все b

для $b=9$ все верно, для $b=1$ $\varnothing$

$x=6$

для $b=\frac{10+\sqrt{99}}{2}$ $\varnothing$ и для $b=\frac{10-\sqrt{99}}{2}$

$x=2-\frac{\sqrt{99}}{2}$

2

$1{,}25=(a-b)(a+b)$ $a-b=2{,}5$ $a=1{,}5$ $b=-1$

$h=2$ $S=2\cdot\frac{1+1{,}5}{2}$ => $S_{ABC}=5$ см

№2

$\sqrt{x+3} - \sqrt{7-x} + 4 = 2\sqrt{(x+3)(7-x)}$

$\sqrt{a} - \sqrt{b} + 4 = 2\sqrt{ab}$

$\sqrt{a} + 4 = 2\sqrt{ab} + \sqrt{b}$

$\sqrt{a} + 4 = \sqrt{b}\,(2\sqrt{a} + 1)$

$a + 8\sqrt{a} + 16 = b\,(4a + 4\sqrt{a} + 1)$

$a + \underline{16 - 4ab} - b = 4\sqrt{a}\,(b-2)$

$a(1-4b) \qquad = 4\sqrt{a}\,(b-2)$

$\sqrt{a} + \frac{16}{\sqrt{a}} - 4\sqrt{a}b - \frac{b}{\sqrt{a}} = 4(b-2)$

$\sqrt{a}\,(1-4b) + \frac{1}{\sqrt{a}}(16-b) = 4(b-2)$

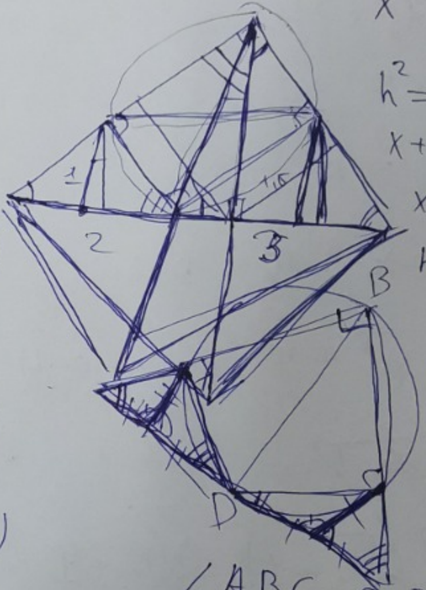
$\frac{h}{x} = \frac{y}{h}$

$h^2 = xy$

$x + y = 5$

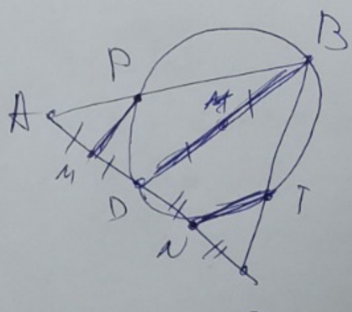
$x = 5 - y$

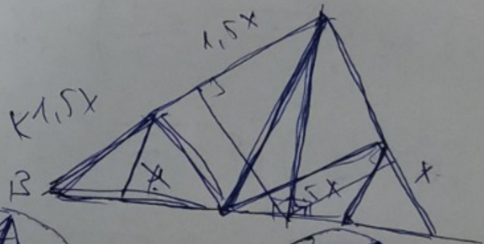
$h^2 = 5y - y^2$

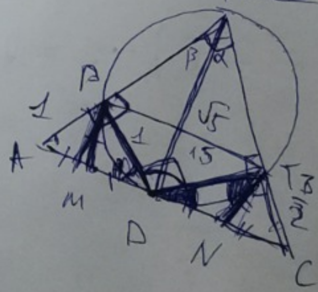


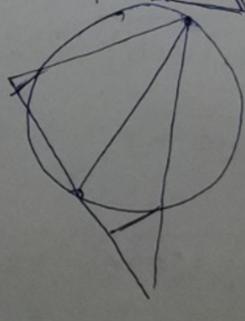
$\angle ABC = 90°$

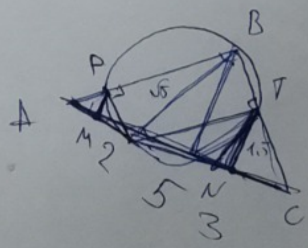
№1

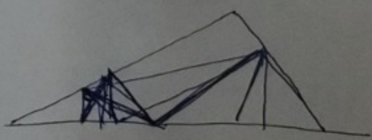












№2

№2 Черновик

$\sqrt{x+3} - \sqrt{7-x} + 4 = 2\sqrt{21+4x-x^2}$

$\boxed{\sqrt{x+3}} - \boxed{\sqrt{7-x}} + 4 = 2\sqrt{(7-x)(3+x)}$

$x+3 \geq 0 \quad 7-x \geq 0$

$x \geq -3 \quad x \leq 7$

т.к. $x+3 = b$

$7-x = -x-3+10 = 10-b$

$\sqrt{b} - \sqrt{10-b} + 4 = 2\sqrt{b(10-b)}$

$\cancel{2\sqrt{b}\,(\sqrt{10-b}-1)}$

$\sqrt{10-b}\,(2\sqrt{b}-1) = \frac{2\sqrt{b}+1}{2} + 3,5$

$(2\sqrt{b}+1)\left(\sqrt{10-b} - \frac{1}{2}\right) = 3,5$

$(2\sqrt{b}+1)(2\sqrt{10-b}-1) = 7$

$4\sqrt{b(10-b)} + 2(\sqrt{10-b} - \sqrt{b}) - 1 = 7$

$2\sqrt{b(10-b)} + (\sqrt{10-b} - \sqrt{b}) = 4$

$2\sqrt{(c+5)(5-c)} + (\sqrt{5-c} - \sqrt{c+5}) = 4$

ООО

$\sqrt{100-4c^2} + (\sqrt{5-c} - \sqrt{c+5})$

№2

$\sqrt{x+3} - \sqrt{7-x} \mp 4 = 2\sqrt{21+4x-x^2}$

$\sqrt{x+3} - \sqrt{7-x} + 4 = 2\sqrt{(7-x)(x+3)}$

$4 = \sqrt{x+3}\left(-1+\sqrt{7-x}\right) + \sqrt{7-x}\left(1+\sqrt{x+3}\right)$

$b = x-2$

$4 = \sqrt{b+5}\left(-1+\sqrt{5-b}\right) + \sqrt{5-b}\left(1+\sqrt{b+5}\right)$

$4 = 2\sqrt{25-b^2} - \sqrt{b+5} + \sqrt{5-b}$

$4 + \sqrt{b+5} = 2\sqrt{25-b^2} + \sqrt{5-b}$

$16 + 8\sqrt{b+5} + b + \cancel{5} = 100 - b^2 + 4(5-b)\sqrt{5+b} + \cancel{5} - b$

$b^2 + 2b + b\sqrt{5+b} - 12\sqrt{5+b} - 84 = 0$

$(b+1)^2 + b\sqrt{5+b} - 12\sqrt{5+b} - 85 = 0$

$\sqrt{x+3} - \sqrt{7-x} \mp 4 = 2\sqrt{21+4x-x^2}$

$x =$

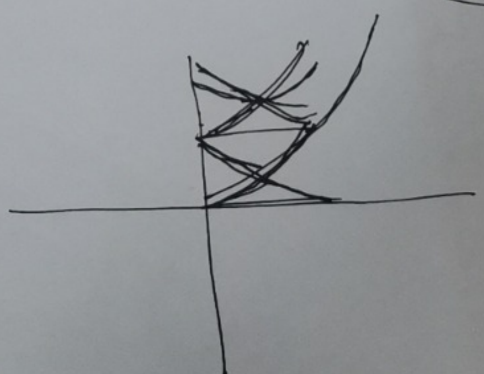
$\sqrt{x+3} - \sqrt{7-x} + 4 = 2\sqrt{(7-x)(x+3)}$

$\boxed{x = 6}$

$3 - 1 + 4 = 2\sqrt{9}$

$6 = 6$

$\sqrt{b} - \sqrt{10-b} \mp 4 = 2\sqrt{b(10-b)}$



x

$2b^2$

$2b \mp 10 + \sqrt{\cdot}$

Часть 2

Олимпиада: **Математика, 10 класс (2 часть)**

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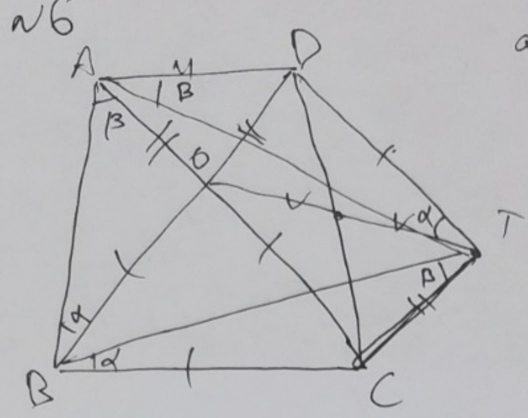
Вариант 10

Чистовик

№6



а) DC и OT точкой пересечения поделились пополам т.е. DOCT параллелепипед.

$DT \parallel OC$ $\quad DO \parallel TC$

$DT = OC$ $\quad DO = TC$

$\angle DOC = 180 - 60 = 120$

$\angle DTC = \angle DOC$

$\angle TBC = \alpha$ $\quad \angle DOC + \angle OCT = 180$

$\angle OCT = 60$

$\angle BCT = \angle OCB + \angle TCO = 120$

$\angle TBC + \angle BTC = 60$

$\angle TBC = \alpha$ $\quad \angle BTC = \beta$

$\angle ODT = 60$

$\triangle ADT = \triangle BCT$ $\quad \angle DAT = \angle BAO = \beta$

$\angle AOB = 180 - \angle BOC = 120°$

$\triangle AOB = \triangle BCT$.

$\angle ABO = \angle TBC$

т.к. $\angle OBC = 60$ то $\angle ABT = 60°$

$\angle OBC = \angle OBT + \alpha$ $\quad \angle ABT = \angle OBT + \alpha$

$\angle BAO = \beta = \angle BCT$

$\angle BAO = 60 = \angle TAC + \beta$ $\quad \angle TAB = \angle TAC + \beta = 60$

$\triangle ABT$ – правильный

б) $BC = 2$ $\quad AD = 7$ $\quad \dfrac{S_{ABT}}{S_{ABCD}}$ – ?

$AB^2 = AO^2 + OB^2 - 2\cos 120\ AO \cdot OB$ $\quad AO = AD$ $\quad OB = BC$

$AB^2 = 4^2 + 7^2 + 4 \cdot 7 = 93$

$S_{ABT} = \dfrac{\sin 60\ AB \cdot AB}{2} = \dfrac{93}{2} \sin 60$

$S_{ABCD} = BD \sin 60 \cdot \dfrac{AD + BC}{2}$ $\quad BD = BO + OD = AB + BC = 9$

$S_{ABCD} = \dfrac{9^2}{2} \sin 60$

$\dfrac{S_{ABT}}{S_{ABCD}} = \dfrac{93 \frac{\sin 60}{2}}{81 \frac{\sin 60}{2}} = \dfrac{93}{81} = 1 \dfrac{12}{81} = 1 \dfrac{4}{27}$

3

№4 чистовик

$$\begin{cases} \frac{6}{x^2+y^2}+x^2y^2=10 \\ x^4+y^4+2x^2y^2+5x^2y^2=81 \end{cases} \Longrightarrow x^2y^2=10-\frac{6}{\underbrace{x^2+y^2}_{\geq 0}} \Rightarrow x^2y^2\leq 10 \quad (1)$$

$$(x^2+y^2)^2=81-5x^2y^2$$

$$\frac{6}{\sqrt{81-5x^2y^2}}+x^2y^2=10$$

$$\frac{6}{\sqrt{81-5x^2y^2}}=10-x^2y^2$$

$$\frac{36}{81-5x^2y^2}=(10-x^2y^2)^2$$

$36=(81-5x^2y^2)(10-x^2y^2)^2$ при $x^2y^2\leq 10$ из (1)

при каждом увеличении x^2y^2 значение $\underbrace{(81-5x^2y^2)}_{\geq 0}\underbrace{(10-x^2y^2)^2}_{\geq 0}$ уменьшается

Это значит, что система имеет лишь 1 значение x^2y^2 при котором ее возможно решить

$x^2y^2=9$

$36=(9-5)(9)\cdot(10-9)^2=36$

$\boxed{x^2y^2=9}$

$$\frac{6}{x^2+y^2}=10-x^2y^2=1$$

4

$$\begin{cases} 6=x^2+y^2 \\ x^2y^2=9 \end{cases}$$

$x^2=6-y^2$

$(6-y^2)y^2=9$

$y^4-6y^2+9=0$

$(y^2-3)^2=0$

$y^2=3$ $\quad x^2=3$ $\qquad |x|=\sqrt{3}$ $\quad |y|=\sqrt{3}$

$\left|\begin{array}{l} x=-\sqrt{3} \\ y=-\sqrt{3} \end{array}\right.$ $\quad \left|\begin{array}{l} x=-\sqrt{3} \\ y=\sqrt{3} \end{array}\right.$ $\quad \left|\begin{array}{l} x=\sqrt{3} \\ y=-\sqrt{3} \end{array}\right.$ $\quad \left|\begin{array}{l} x=\sqrt{3} \\ y=+\sqrt{3} \end{array}\right.$

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№4

$$\begin{cases} \dfrac{6}{x^2+y^2} + x^2y^2 = 10 \\ x^4+y^4+7x^2y^2=81 \end{cases}$$

$$x^4+y^4+2x^2y^2+5x^2y^2=81$$

$$(x^2+y^2)^2=81-5x^2y^2$$

$$\frac{6}{\sqrt{81-5x^2y^2}}+x^2y^2=10$$

$$6+\sqrt{81-5x^2y^2}\,x^2y^2=10\sqrt{81-5x^2y^2}$$

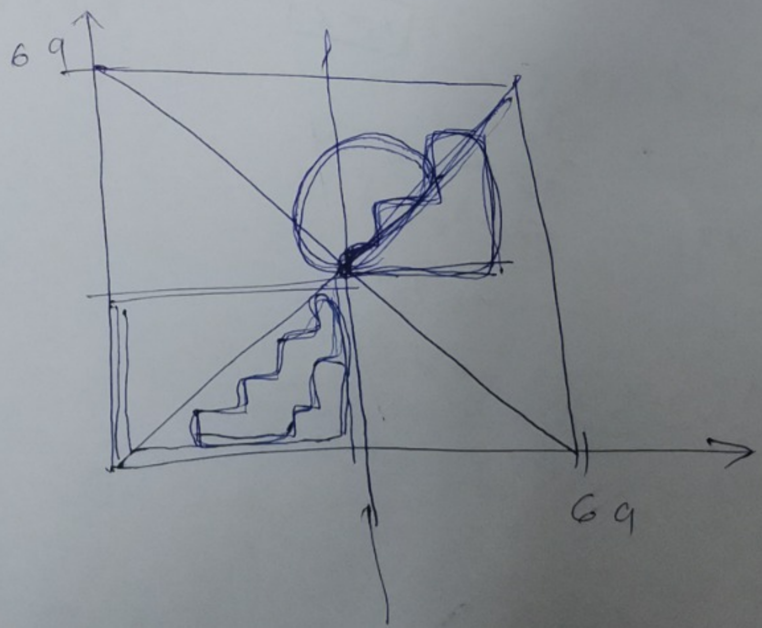
$$6=(10-x^2y^2)\sqrt{81-5x^2y^2}$$

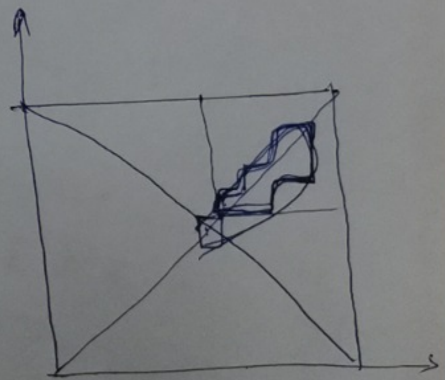
$$36=(10-x^2y^2)^2(81-5x^2y^2)$$ заметим, что это можно записать как

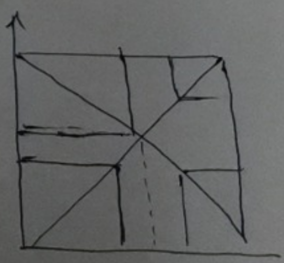
$$36=(10-a)^2(81-5a)$$ при $a \geq 0$

если $a\nearrow$ то $(10-a)^2(81-5a)\searrow$ т.к. каждый ее член уменьшится, пока $10-a$ не станет меньше 0

(ОДЗ: $\sqrt{81-5a}$ $81-5a>0$
$a<16.2$







№4

$$\begin{cases} \dfrac{6}{x^2+y^2} + x^2y^2 = 10 \\ x^4+y^4+7x^2y^2 = 81 \end{cases}$$

пусть $x^2 \geqslant y^2$ $\quad y^2 \to x^2$

$$\begin{cases} \dfrac{6}{a+b} + ab = 10 \\ a^2+b^2+7ab = 81 \end{cases}$$

$$\frac{6}{x^2+y^2} = 1$$

$x^2+y^2=6$

$x^4+y^4=18$

$\frac{6}{a+b} \leqslant 10$

$(x^2+y^2)^2 = 6^2$

$$\boxed{\begin{cases} x^2+y^2=6 \\ x^2y^2=9 \end{cases}}$$

$\frac{6}{10} \leqslant a+b$

$0{,}6 \leqslant a+b$

$x^2 = 6-y^2$

$(a+b)^2 \leqslant 81$

$(6-y^2)y^2 = 9$

$a+b \leqslant 9$

ab макс $= 4{,}5^2$

$6y^2 - y^4 = 9$

$y^4 - 6y^2 + 9 = 0$

$(y^2-3)^2 = 0$

$$\begin{cases} \dfrac{6}{a+b} + ab = 10 \\ a^2+b^2+7ab = 81 \end{cases}$$

$\boxed{y^2=3}$ $\boxed{x^2=3}$

$x^2y^2=9$

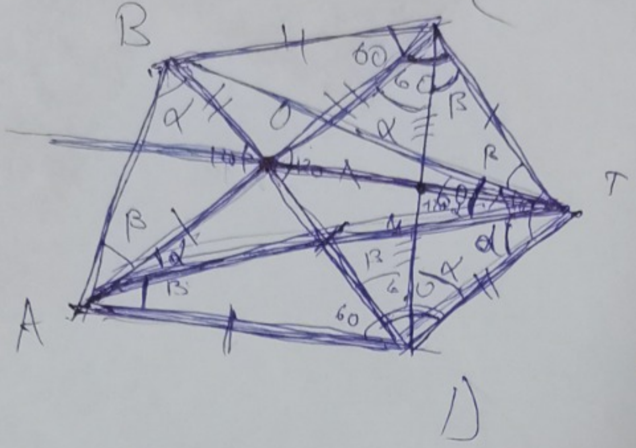
при $x^2=y^2$ $\boxed{ab=9}$

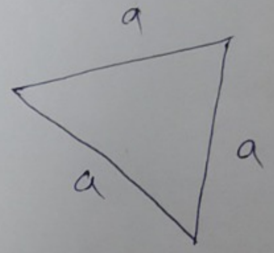
$$\frac{6}{\sqrt{81-5ab}} + ab = 10$$

$6 + ab\sqrt{81-5ab} = 10\sqrt{81-5ab}$

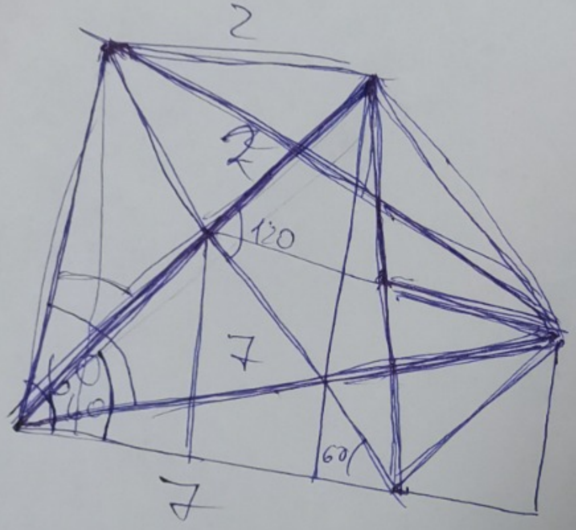
$36 = (10-ab)^2(81-5ab)$ т.к. ab

если $ab \nearrow$ то $(10-ab)^2(81-5ab) \searrow$ т.е. ab имеет лишь 1 значение (возможное)

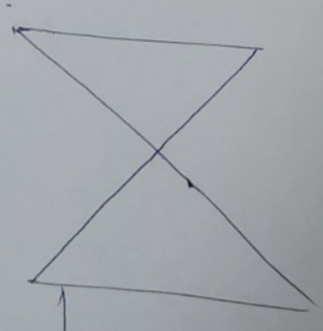


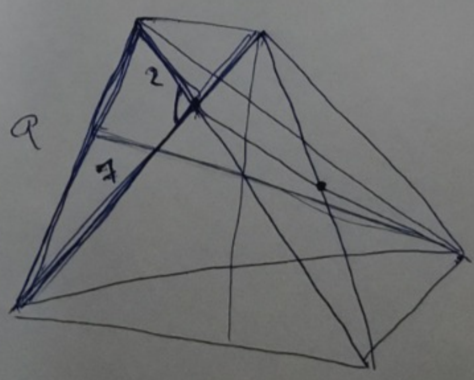


$a^2 = a^2 + a^2 - 2\cos 60\, a a$



$a^2 = b^2 + c^2 -$





$a^2 = 4^2 + 7^2 - 2\cdot 4\cdot 7\cdot \cos 120$

$a^2 = 4^2 + 7^2 + 4\cdot 7 = 16 + 49 + 28 = \sqrt{93}$

49 65 93

$S_{трап} = 9 \sin 60 \cdot \frac{9}{2}$

$S_{\Delta} = \frac{\sqrt{93} \cdot \sin 60 \cdot \sqrt{93}}{2}$

$\frac{S_{трап}}{S_{\Delta}} = \frac{81}{93}$

$$\frac{6}{x^2+y^2} + x^2y^2 = 10 \qquad x^2 = y^2$$

$$x^4 + y^4 + 7x^2y^2 = 81$$

$$\frac{6}{2x^2} + x^4 = 10 \qquad \frac{3}{x^2} + x^4$$

$$2x^4 + 7x^4 = 81$$

$$x^4 = 9$$

$$x^2 = 3$$

$$|x| = 3 \quad |y| = 3$$

БОО $\quad y^2 > x^2 \quad k > 0$

$$y^2 = x^2 + k$$

$$\frac{6}{2x^2+k} + x^4 + x^2k$$

$$11 = x^4 + y^4 - \frac{42}{x^2+y^2}$$

$$x^4 + y^4 + 7$$

$$11 = \frac{x^6 + y^6 + x^4y^2 + x^2y^4 - 42}{x^2+y^2}$$

$$x^6 + y^6 + x^4y^2 + x^2y^4 - 11x^2 - 11y^2 - 42 = 0$$

4

$$\begin{cases} \dfrac{6}{x^2+y^2} + x^2y^2 = 10 \\ x^4+y^4+7x^2y^2 = 81 \end{cases}$$

$$\begin{cases} \dfrac{6}{x+y} + xy = 10 \\ x^2+y^2 + 7xy = 81 \end{cases}$$

$(x+y)^2 + 5xy = 81$

$$\frac{6}{a} + b = 10$$

$$a^2 + 5b = 81$$

$$b = \frac{81-a^2}{5}$$

$$\frac{6}{a} + \frac{81-a^2}{5} = 81 \qquad a \neq 0$$

$$\frac{30+81a-a^3}{5} = 81a$$

$30+81a-a^3 = 405a$

a^3

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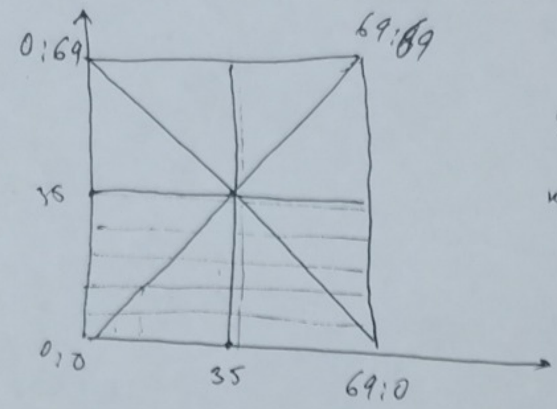
№5



За узлы буду считать квадраты

Один из узлов обязательно будет лежать

на одной из четвертей

найдем кол-во способов