# Часть 1

Олимпиада: **Математика, 10 класс (1 часть)**

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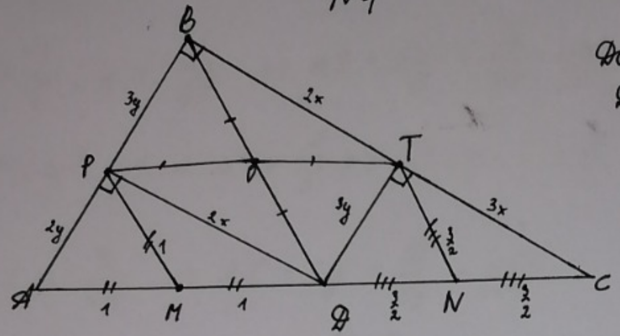
Вариант 10

# Чистовик

## N1




Дано: $\triangle ABC$

$D \in AC$; Окр $(O; OD) \cap AB = P$

$\cap BC = T$

$AM = MD$; $DN = NC$

$PM \parallel TN$

Найти: а) $\angle ABC$

б) $MP = 1$; $NT = \frac{3}{2}$; $BD = \sqrt{5}$

Найти: $S_{ABC}$

Решение:

а) 1) $B, T, D$ и $P$ лежат на окр. $(O; OD)$ => четырехуг. $BPDT$ - впис.;

$\angle BPD = \angle BTD = 90°$ (опир. на диам. $BD$) => $\angle DTC = \angle APD = 90°$

2) В п/у $\triangle APD$ и $\triangle DTC$ $PM$ и $TN$ - медианы => $PM = AM = MD$ и $DN = NT = NC$

3) $TN \parallel PM$ => $\angle PMD = \angle TNC$ (как соотв. при $PM \parallel TN$ и сек. $AC$)

$PM = MD$ => $\angle MPD = \angle MDP = \underline{\alpha} = \frac{180° - \angle PMD}{2} = \frac{180° - \angle TNC}{2} = \underline{\angle TCN} = \angle NTC$

($TN = NC$)

4) Аналогично $\angle AMP = \angle DNT$ и $\underline{\angle MPA = \angle PAM = \beta} = \frac{180° - \angle AMP}{2} =$

$= \frac{180° - \angle DNT}{2} = \angle NTD = \angle TND.$

Тогда: в $\triangle APD$: $\angle PAM = \beta$; $\angle PDM = \alpha$; $\angle APD = \underline{\alpha + \beta = 90°}$

В $\triangle ABC$: $\angle BAC = \beta$; $\angle BCA = \alpha$ => $\underline{\angle ABC} = 180° - (\alpha + \beta) \underline{= 90°}$

б) $MP = AM = MD = 1$; $DN = NT = NC = \frac{3}{2}$

$AC = 2 \cdot 1 + 2 \cdot \frac{3}{2} = 5$

$\triangle APD \sim \triangle ABC$ ($\angle A$ - общ.; $\angle B = \angle P = 90°$) => $k_1 = \frac{AD}{AC} = \frac{2}{5}$

Тогда: Пусть $PD = 2x$ => $BC = 5x$; $AP = 2y$ => $AB = 5y$; $BP = 5y - 2y = 3y$

Аналогично: $\triangle DTC \sim \triangle ABC$ ($\angle C$ - общ.; $\angle B = \angle T = 90°$) => $k_2 = \frac{DC}{AC} = \frac{3}{5}$

$TC = \frac{3}{5} \cdot 5x = 3x$; $TD = \frac{3}{5} \cdot 5y = 3y$; $BT = 5x - 3x = 2x$

Получили: $BT = PD = 2x$; $BP = TD = 3y$; $\angle BTD = \angle BPD = 90°$ => $BPDT$ - прямоуг.

Из п/у $\triangle APD$: $4y^2 + 4x^2 = 4$ => $x^2 + y^2 = 1$

Из п/у $\triangle BTD$: $4x^2 + 9y^2 = 5 = 4(x^2+y^2) + 5y^2 = 4 + 5y^2$ => $5y^2 = 1$

Чистовик

$5y^2 = 1$

$y^2 = \frac{1}{5}$; $y = \frac{1}{\sqrt{5}}$ => $\underline{AB} = 5y = \frac{5}{\sqrt{5}} \underline{= \sqrt{5}}$

$x^2 + y^2 = 1 = x^2 + \frac{1}{5}$ => $x^2 = 1 - \frac{1}{5} = \frac{4}{5}$

$x = \frac{2}{\sqrt{5}}$ => $\underline{BC} = 5y = \frac{10}{\sqrt{5}} = \underline{2\sqrt{5}}$

$S_{ABC} = \frac{1}{2} \cdot BC \cdot AB = \frac{1}{2} \cdot 2\sqrt{5} \cdot \sqrt{5} = 5.$

Ответ: а) $90°$

б) 5

Чистовик

№2

$\sqrt{x+3} - \sqrt{7-x} + 4 = 2\sqrt{21+4x-x^2}$

ОДЗ: $x+3>0$ $\quad$ $7-x>0$

$x>-3$ $\quad$ $x<7$

$-3<x<7$

Пусть: $\sqrt{x+3} = a, \overset{a>0}{\vee} \Rightarrow a^2 = x+3$; $a^2-10 = x-7$; $10-a^2 = 7-x$

Получим: $a - \sqrt{10-a^2} + 4 = 2 \cdot a \cdot \sqrt{10-a^2}$

$\underset{>0}{a+4} = \underset{>0}{(2a+1)} \cdot \underset{>0}{\sqrt{10-a^2}}$

Возведём в квадрат:

$a^2+8a+16 = (4a^2+4a+1)(10-a^2)$

$a^2+8a+16 = 40a^2+40a+10-4a^4-4a^3-a^2$

$4a^4+4a^3-38a^2-32a+6=0 \quad 1:2$

$2a^4+2a^3-19a^2-16a+3=0$

Заметим, что $a=-1$ - корень $(2-2-19+16+3=0)$, но $-1<0$

$$\begin{array}{r|l} -2a^4+2a^3-19a^2-16a+3 & a+1 \\ \underline{2a^4+2a^3} & 2a^3-19a+3 \\ -\,-19a^2-16a+3 & \\ \underline{-19a^2-19a} & \\ -\,3a+3 & \\ \underline{3a+3} & \\ 0 & \end{array}$$

$2a^3-19a+3=0$. Заметим, что $a=3$ - корень $(2\cdot27-19\cdot3+3=54-57+3=0)$

$3>0$ - подходит

$$\begin{array}{r|l} -2a^3-19a+3 & a-3 \\ \underline{2a^3-6a^2} & 2a^2+6a-1 \\ -6a^2-19a+3 & \\ \underline{6a^2-18a} & \\ -\,-a+3 & \\ \underline{-a+3} & \\ 0 & \end{array}$$

$2a^2+6a-1=0$

$D_1 = (-3)^2 - 2\cdot(-1) = 9+2 = 11$

$a = \frac{-3\pm\sqrt{11}}{2}$

$a_1 = \frac{-3-\sqrt{11}}{2} < 0 \qquad a_2 = \frac{-3+\sqrt{11}}{2} > 0$

Вернёмся к $x$

1) $\sqrt{x+3} = 3 \Rightarrow x+3=9$; $x=6$ - входит в ОДЗ

2) $\sqrt{x+3} = \frac{\sqrt{11}-3}{2}$; $\underset{>0}{2\sqrt{x+3}} = \underset{>0}{\sqrt{11}-3}$. $\quad 4x+12 = 11-9-6\sqrt{11}$

$4x = -10-6\sqrt{11}$

$x = \frac{-5-3\sqrt{11}}{2}$ - не входит в ОДЗ.

Ответ: 6

3

Чистовик

№3

1) $5a^2 - 4ay + 8x^2 - 4xy + y^2 + 12ax$

$(2a)^2 - 2\cdot 2a\cdot y + (2x)^2 - 2\cdot 2x\cdot y + y^2 + 2\cdot 2a\cdot 2x = (2a + 2x - y)^2$

$a^2 + 4x^2 + 4ax = (a + 2x)^2$

Тогда: $\underbrace{(2a + 2x - y)^2}_{\geqslant 0} + \underbrace{(a + 2x)^2}_{\geqslant 0} = 0$

$\Downarrow$

$2a + 2x - y = 0$ и $a + 2x = 0$

$2\cdot(-2x) + 2x = y$ ; $a = -2x$

$y = -2x$

2) $ax^2 - 2a^2x + a^3 + 3 = ay$

$\Downarrow$

$a \neq 0$. Тогда разделим на $a$:

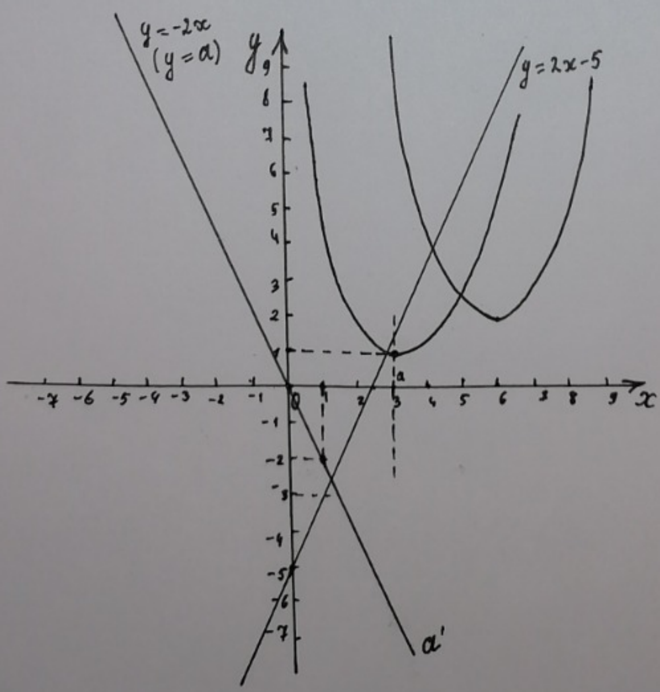
$x^2 - 2ax + a^2 + \frac{3}{a} = y$

$\left.\begin{array}{l} x_в = \frac{2a}{2} = a \\ y_в = a^2 - 2a^2 + a^2 + \frac{3}{a} = \frac{3}{a} \end{array}\right\} \Rightarrow B\left(a; \frac{3}{a}\right).$

3) $2x - y = 5$

$y = 2x - 5$

4)



# Черновик

№ 2 $\sqrt{x+3} - \sqrt{7-x} + 4 = 2\sqrt{21+4x-x^2}$

$21+28-49=0$

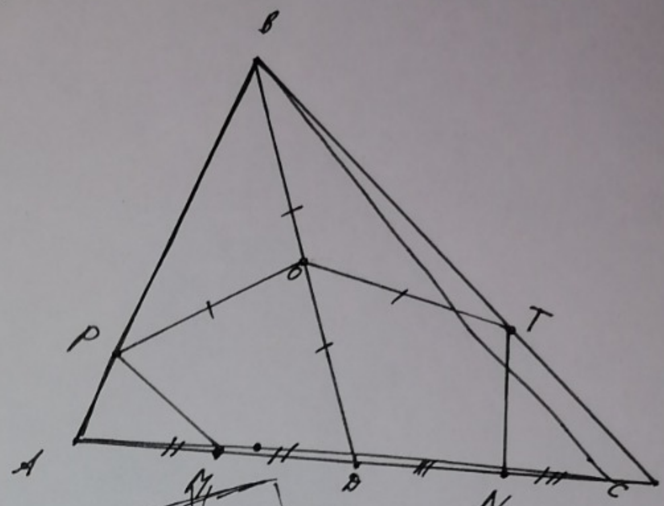
$\sqrt{(x+3)}\sqrt{(7-x)} = \sqrt{7x+21-x^2-3x} = \sqrt{21+4x-x^2}$

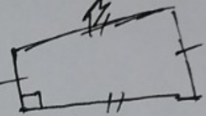
$a - b + 4 = 2ab$ $\quad 1 : b$

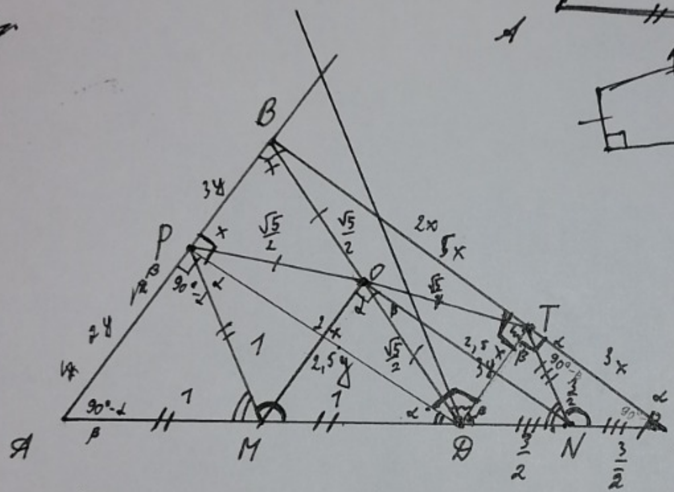
$a - 2ab - b + 4 = 0$

$\frac{a}{b}$ $\quad b=0$, $x=7$, $\sqrt{10}+4 = 2\sqrt{0}$ $\ominus$

$\frac{a}{b} - 1 + \frac{4}{b} = 2a$






1) $\angle BTD = \angle BPD = 90°$

$\angle DPA = \angle CTD = 90°$;

мед. $\Rightarrow PM = AM = MD$

и $TN = NC = DN$

а) $PM \parallel TN$ $\quad \angle ABC - ?$

$180° - (90° - \alpha + 90° - \beta) = \alpha + \beta$.

б) $MP = 1$; $NT = \frac{3}{2}$; $BD = \sqrt5$ $\quad S_{ABC} - ?$

$AC = 2 + 3 = 5$.

$$\begin{array}{r} 625 \\ \times\ \ \ 4 \\ \hline 2500 \end{array}$$

$4x^2 + 9y^2 = 5$ $\quad k_1 = \frac{2}{5}$ $\quad \frac{3}{5}$

~~$3x^2+$~~ $\quad \frac12 \quad BC \cdot AB$

$4x^2 + 4y^2 +$ $\quad 4y^2 + 4x^2 = 4$ $\quad$ ~~$25y^2 + 25$~~

$1 + \frac32 = \frac52$

$y^2 + x^2 = 1$ $\quad xy = y\sqrt{1-y^2}$

$x^2 = 1 - y^2$. $\quad x = \sqrt{1-y^2}$ $\quad$ ~~$6{,}25y^2 + 6{,}25x^2 = \frac{25}{4}$~~

~~$4y$~~ $\quad \frac12 \cdot 2y \cdot 2x = 2\cancel{4}x\cancel{2}y$ $\quad \frac12 \cdot 3y \cdot 3x = 4{,}5\, yx$.

$\frac12 \cdot 5y \cdot 2x = \frac{3y}{5} \cdot 2x = 6xy$ $\quad 5xy$

$S = 8xy + 4{,}5\, yx = 12{,}5\, xy$.

$\frac12 \cdot 5x \cdot 3y = 7{,}5\, xy$

Черновик

$a - b + 4 = 2 \cdot a \cdot b$

$3 - 1 + 4 = 2 \cdot 3 = 6.$

$\sqrt{a} - \sqrt{b} + 4 = 2\sqrt{ab} \quad (\ )^2$

$a + b + 16 - 2\sqrt{ab} + 8\sqrt{a} - 8\sqrt{b} = 4ab$

$a + b + 16 - \sqrt{a} + \sqrt{b} - 4 + 8\sqrt{a} - 8\sqrt{b} = 4ab$

$a + b + 12 + 7\sqrt{a} - 7\sqrt{b} = 4ab$

$a - 2\sqrt{ab} + b \quad + 16 + 8\sqrt{a} - 8\sqrt{b} = 4ab$

$(\sqrt{a} - \sqrt{b})^2 + 8(\sqrt{a} - \sqrt{b}) = 4ab - 16$

$(\sqrt{a} - \sqrt{b})(\sqrt{a} - \sqrt{b} + 8) = 4(ab - 4)$

$16 - 8a + 2a^2 = 40a^2 -$
$- 4a + 10 - 4a^4 + 4a^3$

$6 - 4a + 2a^2 = 40a^2 - 4a^4 + 4a^3$

$4a^4 - 4a^3 - 38a^2 - 4a + 6 = 0$

$2a^4 - 2a^3 - 19a^2 - 2a + 3 = 0$

$\underbrace{2 + 2}_{4} - 19 \underbrace{+ 2 + 3}_{5} = -10$

$2 \cdot 8 = 16$

$2^4 = 16$

$16 + 16 - 19 \cdot 4 + 4 + 3 =$

$x > -3 \qquad x < 7$

$-3 < x < 7.$

$a = x + 3$

$b = 7 - x$

$a + b = 10$

$2 \cdot 81 - 2 \cdot 27 - 19 \cdot 9 -$
$- 18 + 3$

$162 - 54 -$
$- 171 - 18 + 3 =$
$= 165 -$

$4 < \sqrt{x+3} + 4 < \sqrt{10} + 4$

$-\sqrt{7 - x}$

$\begin{array}{r} 8 \\ \times 19 \\ 9 \\ \hline 171 \end{array}$

$a = 7 - x$

$\cancel{a = x - 7}$

$-7 < -x < 3$

$< \sqrt{7-x} < 2$

$3 < 7 - x < 10$

$171$

$\sqrt{3} < \sqrt{7-x} < \sqrt{10}.$

$-\sqrt{10} < -\sqrt{\quad} < -\sqrt{3}$

$4 - \sqrt{10} < S < \sqrt{10} + 4 - \sqrt{3}.$

$a = \sqrt{7 - x}$

$a^2 = 7 - x$

$-a^2 = x - 7$

$10 - a^2 = x + 3$

$162 + 54 - 171 + 18 + 3$

$-5 - 3\sqrt{11} \vee -6$
$-3\sqrt{11} \vee -1$
$<$

$\frac{-5 - 3\sqrt{11}}{2} \vee -3$

$\sqrt{10 - a^2} - a + 4 = 2a\sqrt{10 - a^2}$

$4 - a = (2a - 1) \cdot \sqrt{10 - a^2}$

$16 - 8a + a^2 = (4a^2 - 4a + 1)(10 - a^2)$

$16 - 8a + a^2 = 40a^2 - 4a + 10 - 4a^4 + 4a^3 - a^2$

# Черновик

$\sqrt{x+3} = a$

$a^2 = x + 3$

$a^2 - 10 = x - 7$

$10 - a^2 = 7 - x$

$a - \sqrt{10-a^2} + 4 = 2 \cdot a \cdot \sqrt{10-a^2}$

$4 + a = (2a+1) \cdot \sqrt{10-a^2}$

$16 + 8a + a^2 = (4a^2 + 4a + 1)(10 - a^2)$

$16 + 8a + a^2 = 40a^2 + \underline{40a} + \underline{10} - 4a^4 - 4a^3 \underline{- a^2}$

$6 - 32a + 2a^2 = 40a^2 - 4a^4 - 4a^3$

$6 - 32a = 38a^2 - 4a^4 - 4a^3$

$3 - \cancel{32}\,16a = 19a^2 - 2a^4 - 2a^3$

$2a^4 + 2a^3 - 19a^2 + 16a + 3 = 0$

$2 + 2 - 19 - 16 + 3$

$2 - 2 - 19 + 16 + 3 = 0$ – верно! $a = -1$ – подходит

$$
\begin{array}{l|l}
2a^4 + 2a^3 - 19a^2 - 16a + 3 & a+1 \\
\underline{2a^4 + 2a^3} & 2a^3 - 19a + 3 \\
\quad -19a^2 - 16a + 3 & \\
\quad \underline{-19a^2 - 19a} & \\
\qquad 3a + 3 & \\
\qquad\quad 0 &
\end{array}
$$

$19 \times 3 = 57$

$D_1 = 9 +$ .

$2a + 2x - y = 0$ и $a + 2x = 0$

$a = -2x.$

$2 \cdot (-2x) + 2x = y$

$y = -2x.$

$5a^2 - 4ay + 8x^2 - 4xy + y^2 + 12ax = 0$

$ax^2 - 2a^2x - ay + a^3 + 3 = 0$

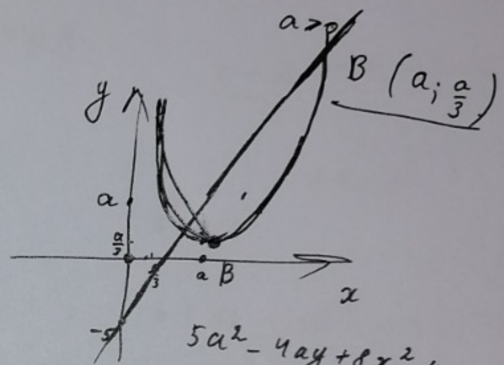
$ax^2 - 2a^2x + a^3 + 3 = ay$

$a \neq 0$

$y = x^2 - 2ax + a^2 + \frac{3}{a}$

$x_в = \frac{2a}{2} = a$

$y_в = \cancel{a^2 - 2a^2 + a^2} + \frac{3}{a} = \frac{3}{a}$




$5a^2 - 4ay + 8x^2 - 4xy + y^2 + 12ax$

$y = 2x - 5.$

$2a^3 - 19a + 3 = 0$

$2 \cdot 27 - 19 \cdot 3 + 3 = 54 - 54$

$2 \cdot 27 - 19 \cdot 3 + 3 = 54 - 57 + 3 = 0$ – !

$a = 3$ (+)

$2a^3 - 19a + 3 \,|\, a - 3$

$5a^2 + 8x^2 - 4xy + y^2 - 4ay + 12ax = 0$

$\underbrace{4x^2 - 4xy + y^2}$

$5a^2 + 4x^2 - 4ay + 12ax.$

$(2a)^2 + (2x)^2 - 2 \cdot (2x \cdot y) + y^2 - 2 \cdot 2a \cdot y + 2 \cdot 2a \cdot 2x.$

$a^2 + 4x^2 + 4ax.$

$= (2a + 2x - y)^2 + (a + 2x)^2 = 0$

# Часть 2

Олимпиада: **Математика, 10 класс (2 часть)**

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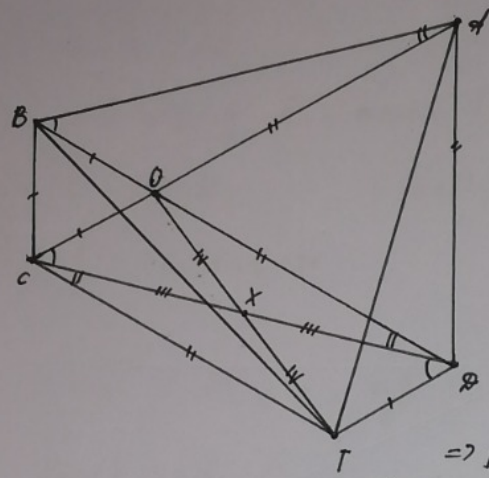
Вариант 10

Чистовик

№6



Дано: $ABCD$: $AC \cap BD = O$

$BO = OC = BC$; $AO = AD = OD$

$CX = XD$; $OX = XT$

а) Док-ть: $\triangle ABT$ - прав.

б) $BC = 2$; $AD = 7$

Найти: $S_{ABT} : S_{ABCD}$

Решение:

1) $\triangle OCX$ и $\triangle TDX$: $OX = XT$; $CX = XD$; $\angle OXC = \angle DXT$ (верт.) $\Rightarrow$

$\Rightarrow \triangle OCX = \triangle TDX \Rightarrow OC = DT$

Аналогично: $OD = CT$

Получим $OC = DT$; $OD = CT \Rightarrow ODTC$ - пар-мм. ($DT \parallel OC$; $OD \parallel CT$.

$\triangle BOA$ и $\triangle COD$: $BO = OC$; $OA = OD$; $\angle BOA = \angle COD = 180° - 60° = 120° \Rightarrow \triangle BOA = \triangle COD \Rightarrow$

$\Rightarrow CD = BA$

$\angle ABO = \angle OCD = \angle CDT = \alpha$; $\angle BAO = \angle ODC = \angle DCT = \beta$

2) $\triangle ADT$ и $\triangle TCB$: $AD = CT$; $TD = BC$; $\angle ADT = 60° + \alpha + \beta = \angle BCT \Rightarrow \triangle ADT = \triangle TCB \Rightarrow$

$\Rightarrow \underline{BT = AT}$

Из $\triangle BCD$: $\angle COD = 180° - \alpha - \beta = 120° \Rightarrow \alpha + \beta = 60° = \angle BDT = \angle OCT$

$\angle ADT = 60° + 60° = 120° = \angle BOA$

3) $\triangle BOA$ и $\triangle TDA$: $OA = AD$; $BO = DT$; $\angle BOA = \angle TDA = 120° \Rightarrow \triangle BOA = \triangle TDA \Rightarrow$

$\Rightarrow \underline{AT = AB}$

4) Получим: $BT = AT$ и $AT = AB$, т.е. $BT = AT = AB \Rightarrow \triangle ABT$ - правильн.

ч.т.д.

б) $S_{ABCD} = \frac{1}{2} \cdot d^2 \cdot \sin 60° = \frac{1}{2} \cdot 81 \cdot \frac{\sqrt{3}}{2} = \frac{81\sqrt{3}}{4}$.

Из $\triangle ABO$: по т. cos $AB^2 = BO^2 + OA^2 - 2 \cdot BO \cdot OA \cdot \cos 120° = 4 + 49 -$

$- 2 \cdot 2 \cdot 7 \cdot \left(-\frac{1}{2}\right) = 53 + 14 = 67 \Rightarrow AB = \sqrt{67}$

$\triangle ABT$ - правильн. $\Rightarrow S_{ABT} = \frac{AB \cdot \sqrt{3}}{4} = \frac{\sqrt{67} \cdot \sqrt{3}}{4} = \frac{\sqrt{201}}{4}$

$\frac{S_{ABT}}{S_{ABCD}} = \frac{\sqrt{201}}{4} : \frac{81\sqrt{3}}{4} = \frac{\sqrt{201}}{81\sqrt{3}} = \frac{\sqrt{67}}{81}$

Ответ: б) $\frac{\sqrt{67}}{81}$

1

211005173 (U128915 M1274269)

Чистовик

№ 5

$$\begin{cases} \dfrac{6}{x^2+y^2} + x^2y^2 = 10 \\ x^4+y^4+7x^2y^2 = 81 \end{cases}$$

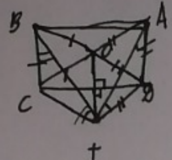
Пусть $x^2 = a$; $y^2 = b$ ; $a \geq 0$ и $b \geq 0$ ; $a+b \neq 0$ – ОДЗ.

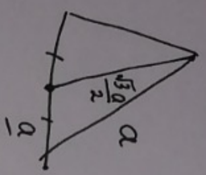
$$\begin{cases} \dfrac{6}{a+b} + ab = 10 \\ a^2+b^2+2ab+5ab = 81 \end{cases} \quad | \cdot (a+b) \quad \begin{cases} 6 + ab(a+b) = 10ab \\ (a+b)^2 + 5ab = 81 \end{cases} \Rightarrow 5ab = 3 + \frac{ab}{2}(a+b)$$

$$(a+b)^2 + \cancel{15} + 3 + \frac{ab}{2}(a+b) = 81$$

$$\left(a+b+\frac{ab}{2}\right)(a+b) = 78.$$

Черновик




$a^2 - \frac{a^2}{4} = \frac{3}{4}a^2$

$\frac{\sqrt{3}a}{2}$

$S = \frac{1}{2} \cdot \frac{\sqrt{3}a}{2} \cdot a =$

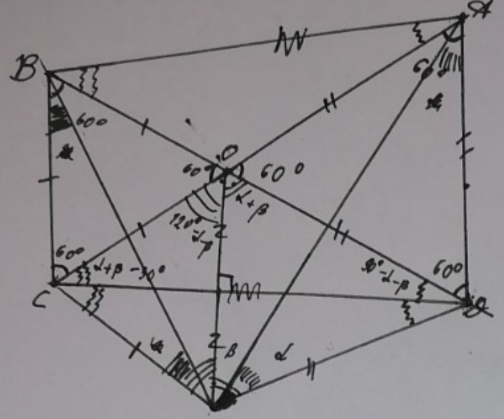
1) Д-ть ABT - прав.


$4a^2 - a^2 = 3a^2$

$\sqrt{3}a$

$S = \frac{1}{2} \cdot \sqrt{3}a \cdot 2a = \sqrt{3}a^2$.

$S_{ABT} : S_{ABCD}$.




$30° + \alpha + \beta$

$AD \parallel BC$.

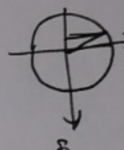
$OT \perp CD$.

$BC = CT = BC$

$OD = DT = AD$

$\frac{1}{2} \cdot \sqrt{3}a \cdot 2a = a^2\sqrt{3}$.

$\frac{1}{2}8 \quad \frac{8\sqrt{3}}{4}$




$180° - 2\alpha - 2\beta$

$120°$

$240 - 2\alpha - 2\beta$

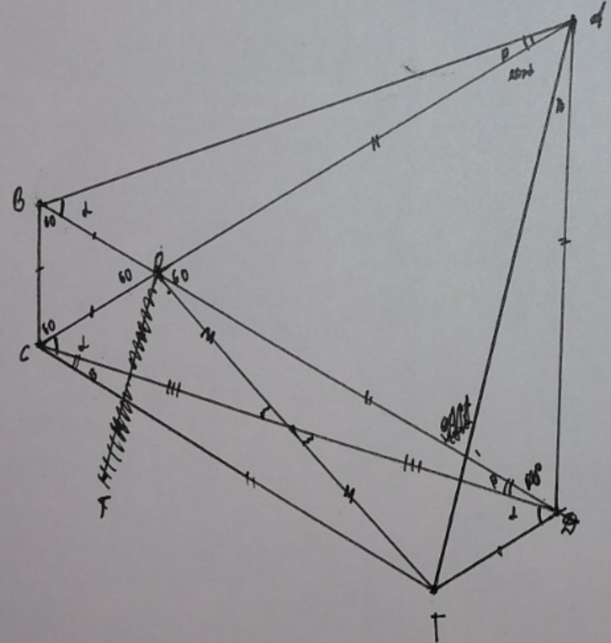
$240 - 180 = 60$

Д-ть BADT - впис.

$\angle DAT = \angle DBT$

$180° - 240° + 2\alpha + 2\beta = 2\alpha + 2\beta = 60°$

$\frac{67}{3}$
$201$




$BT = AT$.

$\triangle ADT = \triangle BCT$ (2 стор. и угол)

$120 = 180° - \alpha - \beta$

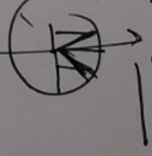
$\alpha + \beta = 60°$

$S_{ABCD} = \frac{1}{2} \cdot 9 \cdot 9 \cdot \sin 60° = \frac{81}{4}$

$AB^2 = 4 + 49 - 2 \cdot 2 \cdot 7 \cdot \cos 120° =$

$-\frac{1}{2}$

$= 4 + 49 + 14 = 49 + 18 =$



$49 + 18 = 67$



13
$67 | 13$

~~Чистовая~~ Черновик

N 5

$$\begin{cases} \frac{6}{x^2+y^2} + x^2y^2 = 10 \\ x^4+y^4+7x^2y^2 = 81 \end{cases}$$

Пусть $x^2 = a; y^2 = b \Rightarrow \begin{cases} \frac{6}{a+b} + ab = 10 \ (1) \\ a^2+b^2+2ab+5ab = 81 \ (2) \end{cases}$, при этом $a \geq 0$ и $b \geq 0$; $a$ и $b$ не могут быть равны 0 одноврем. – ОДЗ

$a+b \neq 0 \Rightarrow \overset{(1)}{6} + ab(a+b) = 10ab \Rightarrow 5ab = 3 + \frac{ab}{2}(a+b)$

$(2): (a+b)^2 + 3 + \frac{ab}{2}(a+b) = 81$

$(a+b)\left(a+b+\frac{ab}{2}\right) = 78$

$a \geq 0$ и $b \geq 0 \Rightarrow a+b \geq 0;\ a+b+\frac{ab}{2} > 0$

$78 = 1\cdot 78 = 2\cdot 39 = 3\cdot 26 = 6\cdot 13$

Т.к. $a+b < a+b+\frac{ab}{2}$, то:

① $a+b=1$ и $a+b+\frac{ab}{2} = 78$

$a = 1-b$ $\quad 1 + \frac{ab}{2} = 78 \Rightarrow \frac{ab}{2} = 77$

$ab = 154$

$(1-b)b - 154 = 0$

$-b^2+b-154=0$

$b^2-b+154=0$

$D = 1 - 4\cdot 154 < 0$

② $a+b=2$ и $a+b+\frac{ab}{2} = \cancel{81}\ 39$

$a = 2-b$

$\frac{ab}{2} = \cancel{79}\ 37$

$ab = \cancel{158}\ 74$

$(2-b)b = \cancel{158}\ 74$

$-b^2+2b-\cancel{158}^{74}=0$

$b^2-2b+\cancel{158}^{74}=0;\ D_1 = 1 - \cancel{158}^{74} < 0$ $\varnothing$

③ $a+b=3$ $\quad a+b+\frac{ab}{2} = \cancel{81}^{26} \Rightarrow \frac{ab}{2} = \cancel{78}^{23} \Rightarrow ab = \cancel{156}^{46}$

$b = 3-a$

$a(3-a) - \cancel{156}^{46} = 0$

$a^2 - 3a + \cancel{156}^{46} = 0;\ D = 9 - 4\cdot\cancel{156}^{46} < 0$ $\varnothing$

④ $a+b=6$ и $a+b+\frac{ab}{2} = 13 \Rightarrow \frac{ab}{2} = 7;\ ab = 14 \Rightarrow a(6-a) = 14$

$b = 6-a$

$a^2-6a+14=0$

$D_1 = 9-14$

$(x^2+y^2)\left(x^2+y^2+\frac{x^2y^2}{2}\right) = 78$

$a \geq (a +$

$x^4$

$\frac{6}{a+b} + ab = 10$

$\cancel{\frac{4z}{x^2+y^2}}$ $(a+b)\left(a+b+\frac{ab}{2}\right) = 78$

$\frac{6}{a+b} = 10 - ab$

$\varnothing$ $6a+6b+3ab = 780 - 78ab$

$6a+6b+81ab = 780$

$2a+2b+27ab = \cancel{260}\ 390$

$\cancel{3\frac{a}{b}+27}$

$2x^2+2y^2+27x^2y^2 = 390°$

$4a^2 - a^2 - 2a^2$

Черновик

$$\begin{cases} \frac{6}{x^2+y^2} + x^2y^2 = 10 \\ x^4+y^4+7x^2y^2=81 \end{cases}$$

ОДЗ: $x \neq 0$ и $y \neq 0$

$x^2y^2 = 10 - \frac{6}{x^2+y^2}$

$x^4+y^4+70-\frac{42}{x^2+y^2}=81$

$x^4+y^4-\frac{42}{x^2+y^2}=11$

$ab=4 \Rightarrow a=\frac{4}{b}$

$b^2+5a=81$ $\quad b=\frac{4}{a}$

$b^2+\frac{20}{b}=81$ $\quad |\cdot b$

$b^3+20-81b=0$

$b^3-81b+20=0$

$1000-810+20$

$-7$

$567+20-343$

$-8:$

$-512+648+20$

$6 + x^2y^2(x^2+y^2) = 10$

$x^2y^2(x^2+y^2)=4$

$x^4+y^4+2x^2y^2+5x^2y^2=81$

$(x^2+y^2)^2+5x^2y^2=81$

$a^2 \cdot b = 4 \quad a=\frac{4}{b}$

$b+5a=81$

$b=81-5a = 81-\frac{20}{b}$

$b \neq 0$

$b^2 = 81b-20$

$b^2-81b+20=0$

$\frac{16}{a^2}+5a=81$

$16+5a^3=81a^2$

$5a^3-81a^2+16=0$

$512-81\cdot 8+20=$

$4^3=64$

$9^3=729$

$5\cdot 6^3$

$81 \times 8 = 648$; $64 \times 8 = 512$; $81 \times 7 = 567$; $49 \times 7 = 343$; $81 \times 9 = 729$; $64 \times 5 = 320$; $36 \times 6 = 216$; $216 \times 5 = 1080$; $729 \times 5 = 3645$; $81 \times 16 = 1296$; $81 \times 81 = 6561$; $81 \times 36 = 2916$

$6 = 10ab - ab(a+b)$

$6 = ab(10-a-b)$

$ab = \frac{6}{10-a-b}$

$\times 79$, $2$, $158$

$(a+b)^2 + \frac{30}{10-(a+b)} = 81$

$a+b = u$

$10(a+b)^2$

$u^2 + \frac{30}{10-u} = 81$ $\quad u \neq 10$

$u^2(10-u) + 30 = 810 - 81u$

$10u^2 - u^3 - 780 + 81u = 0$

$u^3 - 10u^2 - 81u + 780 = 0$

$10ab - 6 = ab(a+b)$

$10 - \frac{6}{ab} = (a+b)$

$\frac{10ab-6}{a+b} = ab$

$10ab - ab(a+b) = 6$

$ab(10-a-b) = 6$

$ab = \frac{10-a-b}{6}$

$ab = \frac{6}{10-a-b}$

$(a+b)^2 + \frac{30}{10-a-b} = 81$

$6^3 = 36$, $\times 6$, $216$

$+\,216$, $780$, $996$

$+\,360$, $486$, $846$

$\times 81$, $6$, $486$

$ab(a+b) = 10ab - 6$

$(a+b)^2 = 81 - 5ab$

$\frac{ab}{a+b} = \frac{10ab-6}{81-5ab}$

~~$81ab - 5a^2b^2 = 10a^2b - 6$~~

$(ab + a + b)(a+b) = 75 + 5ab.$

$5ab = 81 - (a+b)^2 = (9-a-b)(9+a+b)$

$10ab = 2(9-a-b)(9+a+b) = 6 + ab(a+b)$

$2(81 - a^2 - 2ab - b^2) = 6 + a^2b + ab^2$

$162 - 2a^2 - 4ab - 2b^2 = 6 + a^2b + ab^2.$

$456$

Черновик

$\begin{cases} \frac{6}{a+b} + ab = 10 \\ a^2 + b^2 + 7ab = 81 \end{cases}$

$+\,125$, $780$, $905$

$8\,|\,3$, $26$

$250$, $655$

$\times 81$, $5$, $405$

$a \neq 0$ $\quad b \neq 0$

ОДЗ: $a$ и $b$ вместе $\neq 0$

$6 + a^2b + ab^2 = 10ab \Rightarrow 5ab = 3 + \frac{ab(a+b)}{2}$

$a^2 + b^2 + 2ab + 5ab = 81$

$(a+b)^2 + 3 + \frac{ab(a+b)}{2} = 81$

$(a+b)^2 + \frac{ab(a+b)}{2} = 78$

$(a+b)\left(a + b + \frac{ab}{2}\right) = 78$

$78 < 80$

$78 = 2 \cdot 39 = 2 \cdot 3 \cdot 13.$

$6 \cdot 13$

$6 \cdot 6 + 7$

$ab = 14$