Часть 1

Олимпиада: **Математика, 9 класс (1 часть)**

Шифр: **211006800**

ID профиля: **317521**

Вариант 15

№3?

Чистовик

ω: $a^2x^2+a^2y^2-6a^2x-2a^3y+4ay+a^4+4=0$

$a^2x^2-6a^2x=a^2(x^2-2\cdot3\cdot x+9-9)=$

$=a^2(x-3)^2-9a^2$

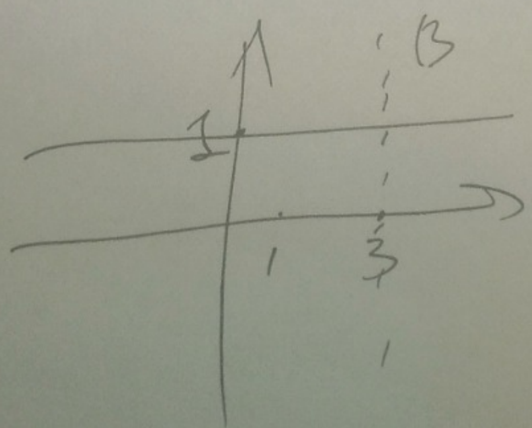
$a^2y^2-2a^3y+4ay=a^2(y^2-2\cdot a\cdot y+\frac{4y}{a})=$

$=a^2(y-(a-\frac{2}{a}))^2-4-a^4+4a^2$

ω: $a^2((x-3)^2+(y-(a-\frac{2}{a}))^2-5)=0 \;|:a^2\neq0$

$(x-3)^2+(y-(a-\frac{2}{a}))^2-5=0$

$B(3;a-\frac{2}{a})$.



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$\cancel{32\cdot 16}\cdot 543+\sum_{i=2}^{n-1}a_i=581$ ~~Чистовик~~

$$\sum_{i=2}^{n-1}a_i=38.$$

$a_2\geq 17.$

$a_{n-1}\leq 30.$

1) $\exists\, n\geq 5$:

$\cancel{\min\left(\sum_{i=2}^{n-1}a_i\right)=a_2+a_3+a_4=38}.$

$\min(a_2+a_3+a_4+\ldots+a_{n-1})=16+17+18+\min(a_5+\ldots)=$

$=38+13+\ldots>38.$ Противор.

2) $n\leq 3$:

$a_2=38>a_3.$ Прот.

3) $n\neq 2$; т.к. $\sum_{i=2}^{n-1}a_i\neq 0.$

Из 1), 2) и 3) $\Rightarrow n=4$;

$a_2+a_3=38$: $\Leftrightarrow$

$$\left[\begin{array}{l}\begin{cases}a_2=17\\ a_3=21\end{cases}\\ \begin{cases}a_2=18.\\ a_3=20.\end{cases}\end{array}\right.$$

(Если $a_2=19$, то $a_3=19\Rightarrow a_2=a_3$, но $a_2<a_3$)

Ответ: $(16;17;21;31)$ и $(16;18;20;31)$.

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№2 Чистовик

$\exists\, a_1; a_2; \ldots; a_n$ — набор написанных чисел

$(a_1 < a_2 < \ldots < a_n)$:

$$\begin{cases} 32a_1 + \sum\limits_{i=2}^{n} a_i = 581 \\ \sum\limits_{i=1}^{n-1} a_i + 17a_n = 581 \end{cases} \quad (=)$$

$$32a_1 + \sum_{i=2}^{n} a_i = \cancel{581} \sum_{i=1}^{n-1} a_i + 17a_n \quad \Big| - (a_2 + a_3 + \ldots + a_{n-1})$$

$\Leftrightarrow$

$$32a_1 + a_n = a_1 + 17a_n \Leftrightarrow$$

$$31a_1 = 16a_n$$

$a_1 = \frac{16a_n}{31} \in \mathbb{N} \Rightarrow a_n \vdots 31$ $(a_n = 31k, k \in \mathbb{N})$, т.к. НОД$(16; 31) = 1$.

$a_1 = 16k \quad a_n = 31k.$

$*\; 32 \cdot 16 = 2^5 \cdot 2^4 = 2^9 = 512$

$512 + 31 = 543$

$$32 \cdot 16k + \sum_{i=2}^{n-1} a_i + 31k = 581 \Leftrightarrow$$

$$543k + \sum_{i=2}^{n-1} a_i = 581.$$

Если $k > 1$:

$543k \geq 543 \cdot 2 > 581 \Rightarrow k = 1 \Rightarrow$

$\Rightarrow a_1 = 16;\ a_n = 31.$

2

№2 Чистовик

$\exists a_1; a_2; \ldots; a_n$ – набор написанных чисел

$(a_1 < a_2 < \ldots < a_n)$:

$$\begin{cases} 32a_1 + \sum_{i=2}^{n} a_i = 581 \\ \sum_{i=1}^{n-1} a_i + 17a_n = 581 \end{cases} \Leftrightarrow$$

$$32a_1 + \sum_{i=2}^{n} a_i = \sum_{i=1}^{n-1} a_i + 17a_n \quad \Big| -(a_2+a_3+\ldots+a_{n-1})$$

$\Leftrightarrow$

$$32a_1 + a_n = a_1 + 17a_n \Leftrightarrow$$

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Если $k > 1$:

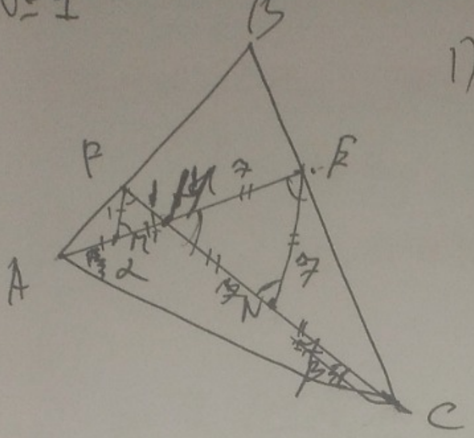
$543k \geq 543 \cdot 2 > 581 \Rightarrow k = 1 \Rightarrow$

$\Rightarrow a_1 = 16; \ a_n = 31.$

2

№1

Черновик



1) $\triangle AFC$ – прямоуг. =>

=> $MM = AM = MH = 1$ (FM-медиана)

Аналогично:

$EN = NM = NC = 7$.

2) Т.к. $FM \parallel EN$:

$\angle HME = \angle MEN = \angle HFM = \angle EHN$

($\triangle FMH$ и $\triangle ENM$ – р/б) | =>

$\angle FHM = \angle FME$ (верт.)

=> $\triangle FMH$ и $\triangle EHN$ – равностор.

3) $\angle EHN = 60°$ => $\angle AHC = 120°$

$\angle BAE = \angle BCA = 90° - 60° = 30°$ | =>

J' $\angle EAC = \alpha$, $\angle FCA = \beta$; $\alpha + \beta = 60°$

=> $\angle B = 180° - 60° - 60° = 60°$.

4) $S = \frac{1}{2} \sin(60°) \cdot 10\sqrt{3} \cdot 6\sqrt{3} =$

$= 45\sqrt{3}$.

$BC = \frac{15}{\cos(60°)} = 10\sqrt{3}$

$AB = \frac{9 \cdot 2}{\sqrt{3}} = 6\sqrt{3}$

Отв: $60°$; $45\sqrt{3}$.

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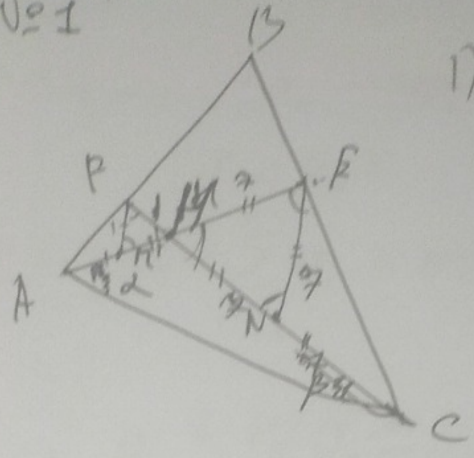
Черновик

№1



1) $\triangle AFC$ – прямоуг. $\Rightarrow$

$\Rightarrow FM = AM = MH = 1$ (FM–медиана)

Аналогично:

$EN = NM = NC = 7$.

2) Т.к. $FM \parallel EN$:

$\angle FME = \angle MEN = \angle \cancel{FHM}\ HFM = \angle EHN$

($\triangle FMH$ и $\triangle ENM$ – р/б)

$\angle FHM = \angle \cancel{FHE}$ (верт.)

$\Rightarrow$

$\Rightarrow \triangle FMH$ и $\triangle EHN$ – равностор.

3) $\angle EHN = 60^\circ \Rightarrow \angle AHC = 120^\circ$

$\angle BAE = \angle BCA = 90^\circ - 60^\circ = 30^\circ$

Г $\angle EAC = \alpha$, $\angle FCA = \beta$; $\alpha + \beta = 60^\circ$

$\Rightarrow$

$\Rightarrow \angle B = 180^\circ - 60^\circ - 60^\circ = 60^\circ$.

4) $S = \frac{1}{2} \sin(60^\circ) \cdot 10\sqrt{3} \cdot 6\sqrt{3} =$

$= 45\sqrt{3}$.

$BC = \frac{15}{\cos(60^\circ)} = 10\sqrt{3}$

$AB = \frac{9 \cdot 2}{\sqrt{3}} = 6\sqrt{3}$

Отв: 60°; $45\sqrt{3}$.

1

2 $a_1 < a_2 < a_3 < \ldots < a_n$ черновик

$$\begin{cases} 32a_1 + \sum_{i=2}^{n} a_i = 581 \\ 17a_n + \sum_{i=1}^{n-1} a_i = 581. \end{cases}$$

$$32a_1 + \sum_{i=2}^{n} a_i = 17a_n + \sum_{i=1}^{n-1} a_i \quad \Big| -(a_2 + a_3 + \ldots + a_{n-1})$$

$\Leftrightarrow$

$$32a_1 + a_n = 17a_n + a_1$$

$$31a_1 = 16a_n$$

НОД$(31;16) = 1 \Rightarrow$

$\Rightarrow a_1 \vdots 16$

$a_n \vdots 31$

$a_1 = \frac{16 \cdot a_n}{31} \in N \Rightarrow$

$\Rightarrow a_n = 31k$, где $k \in N$, т.к. 31 - простое.

$a_1 = 16k$. $a_n = 31k$.

$32 \cdot 16 = 2^4 \cdot 2^5 = 2^9 = 512$.

$$32 \cdot 16k + \sum_{i=2}^{n} a_i = 581.$$

$\Leftrightarrow$

$$512 + \sum_{i=2}^{n} a_i = 581.$$

$$\begin{array}{r} 581 \\ -512 \\ \hline 69 \end{array}$$

$$\sum_{i=2}^{n} a_i = 69.$$

Заметим, что $k = 1$, т.к. если $k > 1$, то $32a_i > 581$.

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$= a^2y^2 - 2a^3y + 4ay =$ Черновик $2 \cdot \frac{2}{\cancel{a}} \cdot \cancel{a} =$

$= a^2\left(y^2 + 2 \cdot y \cdot \left(\frac{2}{a} - a\right) + \left(\frac{2}{a} - a\right)^2\right) - a^2\left(\frac{2}{a} - a\right)^2 =$

$= a^2\left(y + \frac{2}{a} - a\right)^2 - a^2\left(\frac{4}{a^2} + a^2 - 4\right) =$

$= a^2\left(y - \left(-\frac{2}{a} + a\right)\right)^2 - 4 - a^4 + 4a^2$

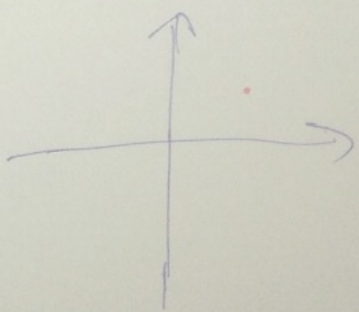
ω: $F + G + a^4 + 4 = a^2(x-3)^2 + a^2\left(y - \left(a - \frac{2}{a}\right)\right)^2 -$

$- 9a^2 - \cancel{a^4} - \cancel{4} + 4a^2 + \cancel{a^4} + \cancel{4} =$

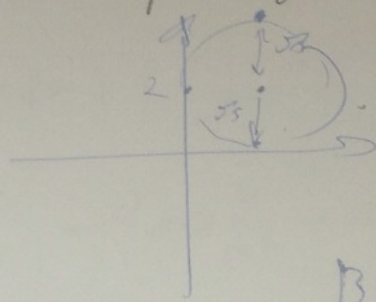
$= a^2\left((x-3)^2 + \left(y - \left(a - \frac{2}{a}\right)\right)^2 - 5\right) = 0 \quad | : a^2 \neq 0$

$(x-3)^2 + \left(y - \left(a - \frac{2}{a}\right)\right)^2 - 5 = 0.$

$B = \left(3; \left(a - \frac{2}{a}\right)\right)$



черновик



$(y-2)^2-5=0.$

~~$y^2-2yb+b^2-5=0$~~

$y-2=\pm\sqrt{5}$

$y=2\pm\sqrt{5}.$

$B(3;\ \cancel{a-\frac{2}{a}})$.

$B(3;\ a-\frac{2}{a})$.

$5a^2-6ax-4ay+2x^2+2xy+y^2=0$

$2x^2-6ax+2xy-4ay+y^2+5a^2=0 \qquad (1)$

1) $x\neq 0$: $(1) \Leftrightarrow -2-\frac{6a \pm 2y}{x}-\frac{4ay+y^2+5a^2}{x^2}=0.$

$y^2+4ay+5a^2=y^2+2\cdot y\cdot 2a+(2a)^2-4a^2+5a^2=$

$=(y+2a)^2+a^2.$

$(1) \Leftrightarrow \underbrace{2x^2-6ax+2xy}_{=}+(y+2a)^2+a^2$

~~$=2\left(x^2-2\cdot 3\cdot x\cdot \frac{a\cdot 3}{2}+\right.$~~

$=2\left(x^2-x\cdot(y-3a)\right)=2\left(x^2-\right.$

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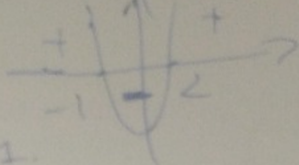
1) $a - \frac{2}{a} > 1 \Leftrightarrow$ черновик $a^2 - a - 2$.

$D = 1 + 8 = 9$.

$a_1 = \frac{1+3}{2} = 2$

$a_2 = \frac{1-3}{2} = -1$.



$\cancel{\frac{a^2-2}{a} > 1}$

$$\left[\begin{array}{l} \begin{cases} a > 0 \\ a^2 - 2 > a \end{cases} \\ \begin{cases} a < 0 \\ a^2 - 2 < a \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} a > 2 \\ 2 > a > -1 \end{array}\right. \Leftrightarrow a \in (-1; 0) \cup (2; +\infty).$$

2) $a - \frac{2}{a} < 1 \Leftrightarrow$

$$\left[\begin{array}{l} \begin{cases} a > 0 \\ a^2 - 2 < a \end{cases} \\ \begin{cases} a < 0 \\ a^2 - 2 > a \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} a \in (0; 2) \\ a \in (-\infty; -1) \end{array}\right. \Leftrightarrow a \in (-\infty; -1; 0; 2).$$

$$\begin{array}{r} + \, 512 \\ 31 \\ \hline 5443 \end{array}$$

$a_1 = 16$

$a_n = 31$.

$17 \cdot 31 = 527$.

$$\begin{array}{r} \times\, 31 \\ 17 \\ \hline +217 \\ 31 \\ \hline 527 \end{array}$$

Черновик

$$\begin{array}{r} -581 \\ 512 \\ \hline -69 \\ 31 \\ \hline 38 \end{array} \qquad \begin{array}{r} -527 \\ 512 \\ \hline 15 \end{array}$$

$$\begin{cases} 512 + \sum_{i=2}^{n-1} a_i + 31 = 581 \\ 527 + \sum_{i=2}^{n-1} a_i + 16 = 581. \end{cases}$$

$\cancel{\sum}\ a_2 + a_3 + \ldots + a_{n-1} = 38$.

$$\begin{array}{r} +18 \\ 17 \\ \hline 35 \end{array}$$

$\min(a_2) = 17$

$\min(a_{n-1}) = 30$.

17

$\min(a_2 + a_3) = 17 + 18 = 35$.

$a_4 \geq 19$

$a_4 + 35 \geq 35 + 19 > 38$. => n не больше 4. =>

~~$a_1 + a_n$~~. Если $n = 2$, то $\underbrace{512 + 31 = 581}_{\text{неверно}}$, т.к. $\sum_{i=2}^{n-1} a_i = 0$.

$$\begin{array}{r} -38 \\ 17 \\ \hline 21 \end{array}$$

=> $n = \{3; 4\}$.

1) $n = 3$:

$\cancel{16 + 31} + a_2 \cancel{+ a_n} = 581$

$a_2 = 38$ => $n = 4$.

2) $n = 4$:

$$\begin{cases} a_2 + a_3 = 38 \\ a_2 \geq 17 \\ a_3 \leq 30 \end{cases}$$

$$\Leftrightarrow \left[\begin{array}{l} \begin{cases} a_2 = 17 \\ a_3 = 21 \end{cases} \\ \begin{cases} a_2 = 18 \\ a_3 = 20 \end{cases} \end{array} \right. \quad (a_2 \neq 19, \text{ т.к. иначе } a_3 = a_2 = 19, \text{ то } a_3 > a_2).$$

~~$a_2 \neq 19$, т.к.~~

Ответ: $(16; 17; 21; 31)$ или $(16; 18; 20; 31)$.

~~$W = (ax-3a)^2+(ay-a^2+2)^2+ \ldots -5a^2=0$~~ $\mid : a$

Черновик

$F = a^2x^2-6a^2x = a^2(x^2-2\cdot 3\cdot x+9-9) =$

$= a^2(x-3)^2-9a^2.$

$G = a^2y^2-2a^3y+4ay = a^2\left(y^2-2\cdot a\cdot y+\frac{4y}{a}\right) =$

$= a^2\left(y^2-2\cdot y\cdot\left(a-\frac{2}{a}\right)+\left(a-\frac{2}{a}\right)^2\right)-a^2\left(a-\frac{2}{a}\right)^2 =$

$= a^2\left(y-a+\frac{2}{a}\right)^2-a^2\left(a^2-\frac{4}{a^2}+4\right) =$

$= a^2\left(y-a-\frac{2}{a}\right)^2-\left(a^4+4+4a^2\right) =$

~~$= a^2\left(y-a-\frac{2}{a}\right)^2-\left(a^2-2\cdot 2a^2+2^2\right) =$~~

~~$= a^2\left(y-a-\frac{2}{a}\right)^2-(a-2)^2$~~

$\cancel{F+G} = F+G+a^4+4 = a^2(x-3)^2-9a^2+a^2\left(y-a-\frac{2}{a}\right)^2 -$

$- \cancel{a^4}-\cancel{4}-4a^2+\cancel{a^4}+\cancel{4} =$

$-9a^2-4a^2 = -(9+4)a^2 = -13a^2$

$= a^2\left((x-3)^2+\left(y-\left(a+\frac{2}{a}\right)\right)^2-13\right) = 0.$

$\Leftrightarrow$

$G =$

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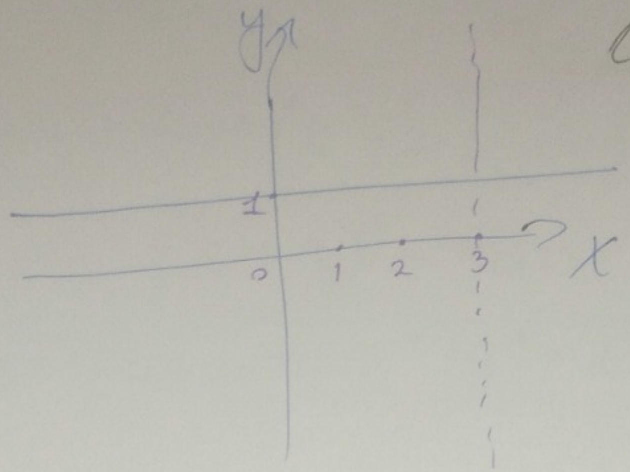
F

(3)



Черновик

G=

$a^2 \neq 0$.

ω: $a^2x^2+a^2y^2-6a^2x-2a^3y+4ay+a^4+4=0$. $\Leftrightarrow$

$(x-b_x)^2+(y-b_y)^2+R=0$.

$C_1+(ax-b_x)^2=a^2x^2-6a^2x=$, где C const.

$=(ax)^2-2\cdot ax\cdot 3a+(3a)^2-(3a)^2=$

$=(ax-3a)^2-9a^2$

$C_2+(ay-b_y)^2=(ay)^2-2a^3y+4ay=$

$=(ay)^2-2\cdot ay\cdot(a^2+2)+(a^2+2)^2-(a^2+2)^2=$

$=(ay-a^2+2)^2-a^4\cancel{+4a^2+4}$.

$C_1+C_2+(x-b_x)^2+(x-b_y)^2=\cancel{(ax-3a)^2+(ay-a^2+2)^2-a^4+4-}$

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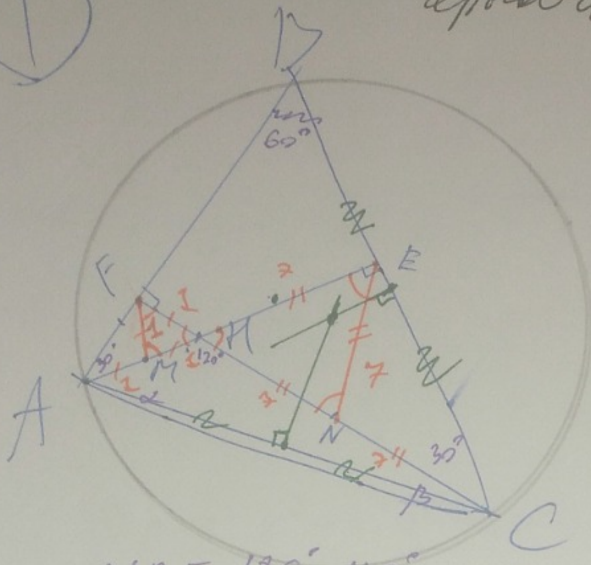
1

черновик $\angle ABC - ? = 60°$

$S - ?$ $45\sqrt{3}$

$R - ?$

FM || EN



$AH = 2$

$CH = 14$

$\alpha + \beta = 180° - 120° = 60°$

$\angle B = 180° - 2 \cdot 30° - (\alpha + \beta) = 180° - 60° - 60° =$

$= 60°$

$CF = 15$

$AE = 9$

$BC = \frac{15}{\cos(30°)} = \frac{15 \cdot 2}{\sqrt{3}} = \frac{30}{\sqrt{3}} = 10\sqrt{3}$

$AB = \frac{9 \cdot 2}{\sqrt{3}} = 3 \cdot 2\sqrt{3} = 6\sqrt{3}$

$S = \frac{1}{2}\sin(60°) \cdot 10\sqrt{3} \cdot 6\sqrt{3} = \frac{1}{2} \cdot \frac{\sqrt{3}}{2} \cdot 3 \cdot 10 \cdot 6 =$

$= \sqrt{3} \cdot 3 \cdot 5 \cdot 3 = \boxed{45\sqrt{3}}$

33 + 18

$581 - 543 = 38$

$33 + 18 = 51$

$38 - 17 = 21$

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Часть 2

Олимпиада: **Математика, 9 класс (2 часть)**

Шифр: **211006800**

ID профиля: **317521**

Вариант 15

② 1) Нам без разницы, какую из карточек он вытянет первой: с числом n

Э (Сначала он вытянет карточку-дубль (всего 20 вариантов);

затем он вытащит любую карту, на кот. нет числа n

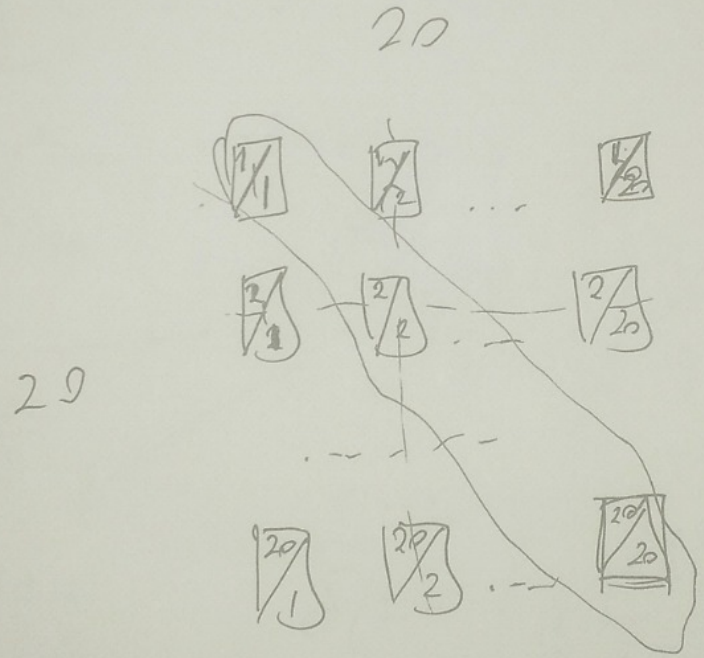
(всего $20 \cdot 19$) (дубли такие он может вытянуть во II раз).

Всего способов:

$$20 + 20 \cdot 19 = 20(1+19) = 20^2 = 400.$$

Ответ: 400.

③ .



$$20 + (20^2 - 20 - 19) = 20 + (20(20-1) - 19) = 20 + (20 \cdot 19 - 19) =$$

$$= 20 + (19(20-1)) = 20 + 19^2 = 381.$$

$$\begin{array}{r} \times\, 19 \\ 19 \\ \hline +\, 171 \\ 19 \\ \hline 361 \end{array}$$

② $\Leftrightarrow \left[\begin{array}{l} \begin{cases} b=3a-3 \\ a=11 \end{cases} \\ \begin{cases} b=3a-3 \\ a=-2 \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} b=30 \\ a=11 \end{cases} \\ \begin{cases} b=-9 \\ a=-2 \end{cases} \end{array}\right.$

т.е.

① $\Leftrightarrow \left[\begin{array}{l} \begin{cases} x^2+y^2=11 \\ x^2y^2=30 \end{cases} \\ \begin{cases} x^2+y^2=-2 \\ x^2y^2=-9 \end{cases} \end{array}\right.$ (посторонние корни) $\Leftrightarrow$

2.5.3.

④ $\begin{cases} x^2+y^2=11 \\ x^2y^2=30 \end{cases}$. Пусть $f=x^2,\ g=y^2$:

④ $\Leftrightarrow \begin{cases} f+g=11 \\ fg=30 \end{cases}$

$f=\frac{30}{g}$.

$\frac{30}{g}+g=11$

$30+g^2=11g$.

$g^2-11+30=0$.

$D=121-120=1$

$g_1=\frac{11+1}{2}=6$

$g_2=5$.

$f_1=11-g$

$f_1=5$

$f_2=6$.

$\left[\begin{array}{l} \begin{cases} x^2=5 \\ y^2=6 \end{cases} \\ \begin{cases} x^2=6 \\ y^2=5 \end{cases} \end{array}\right. \Leftrightarrow$

$\left[\begin{array}{l} \begin{cases} x=\pm\sqrt{5} \\ y=\pm\sqrt{6} \end{cases} \\ \begin{cases} x^2=\pm\sqrt{6} \\ y^2=\pm\sqrt{5} \end{cases} \end{array}\right.$

Ответ: $(x;y)=\{(\pm\sqrt{5};\pm\sqrt{6});\ (\pm\sqrt{6};\pm\sqrt{5})\}$.

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$$(3) \Leftrightarrow \begin{cases} a^4 + b^2 = 31 & (4) \\ 3a - 6b - b^2 = 3. & (5) \end{cases}$$

$D = 36 - 4(3-3a) =$
$= 36 - 12 - 12a =$
$= 24 - 12a = 4(6-3a) =$
$= 2^2 \cdot 3(2-a).$

$(5) \Leftrightarrow -b^2 + 6b + (-3a+3) = 0. \Leftrightarrow$

$$\left[\begin{array}{l} b = \frac{-6 + 2\sqrt{3(2-a)}}{2} = -3 + \sqrt{3(2-a)} \\ b = -3 - \sqrt{3(2-a)}. \end{array}\right.$$

$(5): \Leftrightarrow$

$$\left[\begin{array}{l} a^4 + 3(2-a) + 9 - 6\sqrt{3(2-a)} = 31 \\ a^4 + 3(2-a) + 9 + 6\sqrt{3(2-a)} = 31. \end{array}\right. \Leftrightarrow$$

$$\begin{cases} 3(x^2+y^2) - x^2y^2 = 3 \\ (x^2+y^2)^2 + x^2y^2 = 31 \end{cases} \Leftrightarrow$$

$\exists\, a = x^2+y^2,\ b = x^2y^2:$

$$\begin{cases} 3a - b = 3 & (1) \\ a^2 - b = 31 & (2) \end{cases} \Leftrightarrow$$

$\begin{array}{r} \overset{3}{28} \\ \times\ 4 \\ \hline 112 \end{array}$ $\quad$ $112 : 4 = 28$; $\ -8$, $\ 32$

$D = 9 + 112 = 121.$

$(1) \Leftrightarrow \cancel{a = \frac{3+b}{3} = 1 + \frac{1}{3}b}.$

$-b = 3 - 3a.$

$(2) \Leftrightarrow$

$a^2 + 3 - 3a = 31.$

$a^2 - 3a - 28 = 0 \Leftrightarrow$

$$\Leftrightarrow \left[\begin{array}{l} a = \frac{3+11}{2} = 7 \\ a = \frac{3-11}{2} = -4 \end{array}\right.$$

(4) $\begin{cases} 3x^2+3y^2-x^2y^2=3 & (1) \\ x^4+y^4-x^2y^2=31. & (2) \end{cases}$

$(1) \Leftrightarrow$

$3y^2(3-x^2) = 3-3x^2$

1) $x^2 \neq 3$:

$(1) \Leftrightarrow$

$y^2 = \dfrac{3-3x^2}{3-x^2} \Leftrightarrow$

$y = \pm\sqrt{\dfrac{3-3x^2}{3-x^2}} =$

$\cancel{= \pm\sqrt{1-\dfrac{2x^2}{3-x^2}}} =$

$= \pm\sqrt{\dfrac{3(3-x^2)-6}{3-x^2}} = \pm\sqrt{3-\dfrac{6}{3-x^2}}.$

2) $x^2 = 3$:

$0 = 3-3\cdot 3$, но $9 \neq 3. \Rightarrow$

$\Rightarrow x^2 \neq 3.$

Аналог. $y^2 \neq 3$.

(т.к. (1) и (2) симметрично относит. x и y.)

$(2) \Leftrightarrow$

$x^4 - 2x^2y^2$:

$\exists\, a = x^2,\ b = y^2$:

$\begin{cases} 3a+3b-ab=3 \\ a^2+b^2-ab=31 \end{cases}$

$\exists\, x+y = a$

$xy = b$

$(x^2+y^2)^2 = x^4+y^4+2x^2y^2$

$(2) \Leftrightarrow$

$\Leftrightarrow \quad (x+y)^4 + x^2y^2 = 31.$

$(1) \Leftrightarrow$

$3(\ldots) = 3x^2+3y^2+6xy.$

$3(x+y) - 6xy - x^2y^2 = 3.$

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$\{ a^4 + b^2 = 31$ (4)

Проверка:

$3 \cdot 5 + 3 \cdot 6 - 30 = 3$

$15 + 18 = 33$

$33 = 33$

$25 + 36 - 30 = 31$

$25 + 6 = 31$

$31 = 31.$

(2

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(1) $\begin{cases} 3(x^2+y^2)-x^2y^2=3 \\ x^4+y^4-x^2y^2=31 \end{cases} \iff \begin{cases} 3(x^2+y^2)-x^2y^2=3 \\ (x^2+y^2)^2-3x^2y^2=31. \end{cases}$

Зa $= x^2+y^2$, $b = x^2y^2$:

(1) $\iff$ (2) $\begin{cases} 3a-b=3 \\ a^2-3b=31. \ (3) \end{cases}$ $\quad$ (1) $-b=3-3a$

$$\begin{array}{r} 13 \\ \times 13 \\ \hline 39 \\ 13 \\ \hline 169 \end{array} \qquad \begin{array}{r} 31 \\ -\ 9 \\ \hline 22 \end{array} \qquad \begin{array}{r} 88 \\ +\ 81 \\ \hline 169 \end{array}$$

(3) $\iff$

$a^2+3(3-3a)-31=0 \iff$

$a^2-9a-22=0.$

$D = 81+88 = 169 = 13^2$

$a = \frac{9\pm 13}{2} \iff$

$\left[\begin{array}{l} a=\frac{9+13}{2}=11 \\ a=\frac{9-13}{2}=-2 \end{array}\right.$

$b = 3a-3.$

т.е. (2) $\iff \begin{cases} 3a-b=3 \\ a^2-3b=31 \\ \left[\begin{array}{l} a=11 \\ a=-2. \end{array}\right. \end{cases}$

~~$\begin{cases} 3a-b=3 \\ a=11 \end{cases}$~~

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$$\left[\begin{array}{l} 3\cdot 7 - b = 3 \\ 3\cdot(-4) - b = 3 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} b = 21-3=18 \\ b = -12-3=-15. \end{array}\right.$$

$$\left[\begin{array}{l} \begin{cases} b=-15 \\ a=-4 \end{cases} \\ \begin{cases} b=18 \\ a=7 \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} x^2+y^2=-4 \\ x^2y^2=-15 \end{cases} \\ \begin{cases} x^2+y^2=7 \\ x^2y^2=18. \end{cases} \end{array}\right.$$

(посторонние корни

$\Leftrightarrow$

$$\Leftrightarrow \begin{cases} x^2+y^2=7 \quad (3) \\ xy=\pm 3\sqrt{2}. \end{cases}$$

$\cancel{(3\sqrt{2})^2=9}$

$$(3) \Leftrightarrow (x+y)^2-2xy=7 \Leftrightarrow$$

$$(x+y)^2 \mp 6\sqrt{2}=7.$$

$6\sqrt{2}>7 \Rightarrow$

$$(x+y)^2=7\pm 6\sqrt{2} \Leftrightarrow$$

$\Rightarrow 7-6\sqrt{2}$ – не подх.

$$x+y=\pm\sqrt{7+6\sqrt{2}}=\pm\cancel{\sqrt{(\ )^2+}}$$

$6\cancel{+3\cdot 2\sqrt{2}+1}$

$\cancel{3\sqrt{2}}$.

$$7+2\cdot 3\sqrt{2}=7+2\cdot\frac{6\sqrt{2}}{2}$$

$$\left(\frac{\sqrt{2}}{2}\right)^2=.$$

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$S_5 = \frac{\sqrt{3}\cdot 25}{4}$ $\qquad S_4 = \frac{\sqrt{3}\cdot 4^2}{4} = 4\sqrt{3}$.

$S_{ABC\Phi D} = 3S_1 + S_5 + S_4 = 3S_1 + \left(\frac{25}{4}+4\right)\sqrt{3} = 3S_1 + 10{,}25$

$S_{ABCD} = 2S_1 + S_4 + S_5 = 2S_1 + 10{,}25\sqrt{3}$.

$\cancel{S_{ABT} =}$

$S_1 = \frac{1}{2}\sin(120^\circ)\cdot 4\cdot 5 = 10\cdot\sin(60^\circ) = 5\sqrt{3}$.

$S_{ABCD} = 10{,}25\sqrt{3} = \cancel{\frac{20+\frac{1}{4}}{3}\left(20+\frac{1}{4}\right)\sqrt{3}}$

$S_{ABT} = S_{ABCTD} - 2S_1 = S_1 + 10{,}25\sqrt{3} =$

$S_{ABC\Phi D} = 3S_1 + S_4 + S_5$

$S_{ABCD} = 2S_1 + S_4 + S_5$

$S_{ABT} = S_1 + S_4 + S_5 \quad (S_{ABCTD} - 2S_1)$

$\frac{S_{ABT}}{S_{ABCD}} = \frac{S_1 + S_4 + S_5}{2S_1 + S_4 + S_5} = 1 - \frac{S_1}{2S_1 + S_4 + S_5} = \frac{\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\cdot 4\cdot 5}{2S_1 + \frac{25\sqrt{3}+16\sqrt{3}}{4}} =$

$= \frac{5\sqrt{3}}{10\sqrt{3} + \frac{41}{4}\sqrt{3}} = \frac{5\sqrt{3}}{\frac{81}{4}\sqrt{3}} = \frac{20}{81}$

$\frac{25}{16}$

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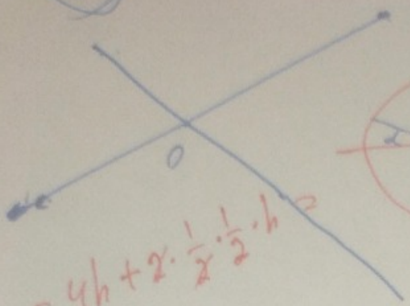
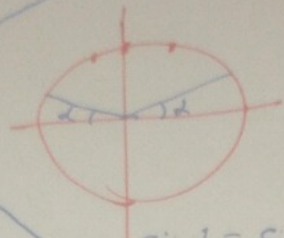
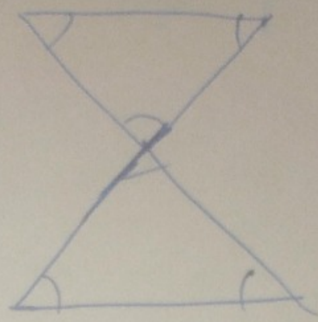
B

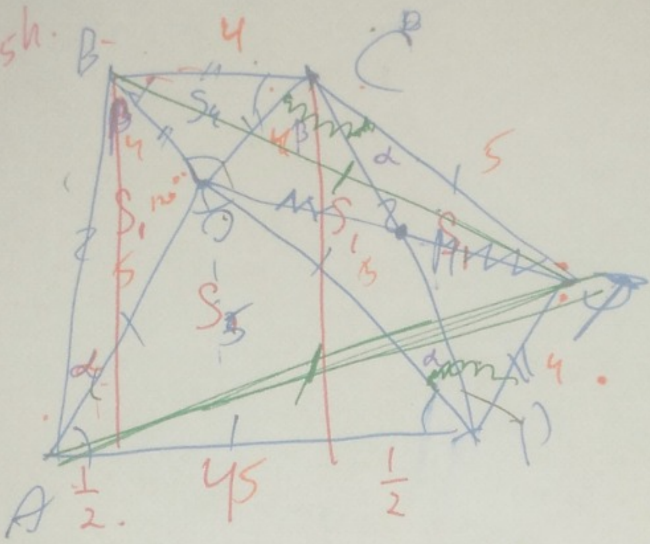






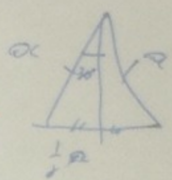
$\sin\alpha = \sin(180° - \alpha)$.

$S_{ABCD} = 4h + 2\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot h = 4{,}5h.$



$\triangle BCT = \triangle ABT$ (СУС)

$\frac{S_{ABT}}{S_{ABCD}} =$



$S = \frac{1}{2}\cdot a \cdot h =$

$= \cancel{\frac{1}{2} a \sqrt{a^2}}$

$= \frac{1}{2} a \cdot \cos 30° \cdot a = \frac{a^2 \cdot \sqrt{3}}{4} = \frac{\sqrt{3}a^2}{4}.$

$\alpha + \beta = 60°$

$\angle BCT = \alpha + \beta + 60° = 120° = \angle BOA \Rightarrow$

$\Rightarrow \triangle BCT = \triangle BOA \Rightarrow BA = BT = AT \Rightarrow$

$\Rightarrow \triangle ATB$ − равная, ■

9кв-17

$\frac{AB}{\sin(120°)} = \frac{5}{\sin\beta} = \frac{4}{\sin\alpha}.$

$14 \times 4 = 56$.

$56 + 25 = 81$.

19×19: 171, 19, 361

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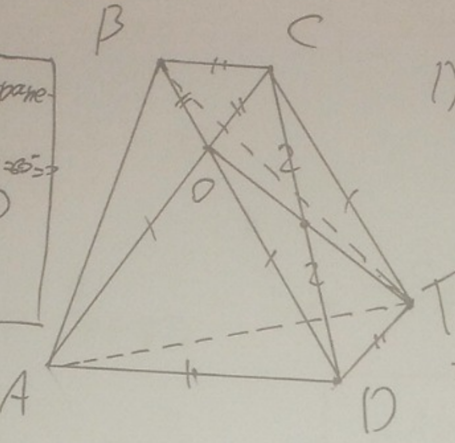
№6

Чистовик



ABCD - р/б трапеция, т.к. $\angle ADB = \angle DBC = 60° \Rightarrow BC \parallel AD$ и 3)

1) ∃ M - середина CD:

$CM = MD$, $OM = MT$ $\Rightarrow$

$\Rightarrow OCTD$ - пар-м $\Rightarrow$

$\Rightarrow \angle ODT = OCT$ ①

2) $\triangle BOC$ и $\triangle AOD$ - равност. $\Rightarrow \angle BCO = 60° = \angle ADO$, $\angle ODT = OCT$ $\Rightarrow$

$\Rightarrow \angle ADT = \angle BCT$, $BC = OC = DT$, $AD = OD = CT$ $\Rightarrow \triangle BCT = \triangle ADT \Rightarrow$

$\Rightarrow AT = BT$

~~3) ∠ AD = α.~~

3) $\triangle ABO = \triangle DOC$ ~~ABCD р/б~~ трапеция ($AO = OD$, $OB = OC$; $\angle BOA = \angle COD$) $\Rightarrow$

$\Rightarrow \angle BAO = \angle CDO = \angle DCT = \angle\alpha$

$\angle ABO = \angle OCD = \angle\beta$.

$\angle\alpha + \angle\beta = 180° - 120° = 60°$.

$\angle ACT = \alpha + \varphi$; $\angle BCT = 60° + \angle\alpha + \angle\beta = 120° =$

$= \angle BOA \Rightarrow$

$\Rightarrow \triangle AOB = \triangle TCB \Rightarrow AB = BT = AT \Rightarrow$

$\Rightarrow \triangle ATB$ - равносторонний, (а)

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(б) Чистовик

$\exists\ S_1 = S_{ABO}$, $S_4 = S_{BOC}$, $S_5 = S_{AOD}$

$S_4 = \frac{16\sqrt{3}}{4}$ ~~ед.~~ ; $S_5 = \frac{25\sqrt{3}}{4}$; $S_1 = \frac{1}{2}\cdot\sin(120^\circ)\cdot 4\cdot 5 = 5\sqrt{3}$.

$S_{ABT} = S_{ABCTD} - S_{BCT} - S_{ATD} = S_{ABCD} + \underbrace{S_{CTD}}_{S_1} - S_1 - S_1 =$

$= S_{ABCD} - S_1$

$\boxed{\begin{array}{l}\triangle ABO = \triangle COD = \triangle DTC \ (OCTD\text{-пар-м}) \\ S_{ABO} = S_{COD} = S_{DTC} = S_1\end{array}}$

$S_{ABCD} = 2S_{ABO} + S_4 + S_5 = 2S_1 + S_4 + S_5$.

$$\frac{S_{ABT}}{S_{ABCD}} = \frac{S_{ABCD} - S_1}{S_{ABCD}} = 1 - \frac{S_1}{2S_1 + S_4 + S_5} =$$

$$= \frac{5\sqrt{3}}{10\sqrt{3} + 4\sqrt{3} + \frac{25}{4}\sqrt{3}} = \frac{5\cdot 4}{56+25} = \frac{20}{81}$$

Ответ: $20/81$.

СТР. 5

№4. Чистовик.

$$\begin{cases} 3(x^2+y^2)-x^2y^2=3 \\ x^4+y^4+2x^2y^2-3x^2y^2=31 \end{cases} \Leftrightarrow \begin{cases} 3(x^2+y^2)-x^2y^2=3 \\ (x^2+y^2)^2-3x^2y^2=31. \end{cases}$$

$(a=x^2+y^2,\ b=x^2y^2)$

$$\Leftrightarrow \begin{cases} 3a-b=3 \\ a^2-3b=31 \end{cases} \Leftrightarrow \begin{cases} -b=3-3a \\ a^2-3b-31=0 \end{cases} \Leftrightarrow$$

$$\begin{cases} b=3a-3 \\ a^2+3(3-3a)-31=0 \end{cases} \Leftrightarrow \begin{cases} b=3a-3 \\ a^2-9a-22=0. \end{cases} \Leftrightarrow$$

$D=81+88=169=13^2$

$$\begin{cases} b=3a-3 \\ \left[\begin{array}{l} a=\frac{9+13}{2}=11 \\ a=\frac{9-13}{2}=-2 \end{array}\right. \end{cases} \Leftrightarrow \left[\begin{array}{l} \begin{cases} b=33-3=30 \\ a=11 \end{cases} \\ \begin{cases} b=-6-3=-9 \\ a=-2 \end{cases} \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} \begin{cases} x^2y^2=30 \\ x^2+y^2=11 \end{cases} \\ \begin{cases} x^2y^2=-9 \\ x^2+y^2=-2 \end{cases} \end{array}\right.$$

$(f=x^2,\ g=y^2)$ $\Leftrightarrow \begin{cases} fg=30 \\ f+g=11 \end{cases}$ $(g\neq 0)$ $\Leftrightarrow$

(посторонние корни $(x,y\notin\mathbb{R})$).

Стр. 1

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Чистовик

$$\Leftrightarrow \begin{cases} f = \frac{30}{g} \quad (g \neq 0) \\ f + g = 11 \end{cases} \Leftrightarrow \begin{cases} f = 30/g \\ \frac{30 + g^2}{g} = 11 \end{cases} \Leftrightarrow \begin{cases} f = \frac{30}{g} \\ g^2 - 11g + 30 = 0 \end{cases}$$

$D = 121 - 120 = 1$

$$\Leftrightarrow \begin{cases} f = \frac{30}{g} \\ \left[\begin{array}{l} g = \frac{11+1}{2} = 6 \\ g = 5 \end{array} \right. \end{cases} \Leftrightarrow \left[\begin{array}{l} \begin{cases} f = 5 \\ g = 6 \end{cases} \\ \begin{cases} g = 5 \\ f = 6. \end{cases} \end{array} \right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} \begin{cases} x = \pm\sqrt{5} \\ y = \pm\sqrt{6} \end{cases} \\ \begin{cases} x = \pm\sqrt{6} \\ y = \pm\sqrt{5} \end{cases} \end{array} \right.$$

Проверка:

$x^2 = 5$
$y^2 = 6$ (уравнения симметричны)

$$\begin{cases} 35 + 18 - 30 = 3 \\ 25 + 36 - 30 = 31 \end{cases} \Leftrightarrow \begin{cases} 33 = 33 \\ 31 = 31. \end{cases}$$

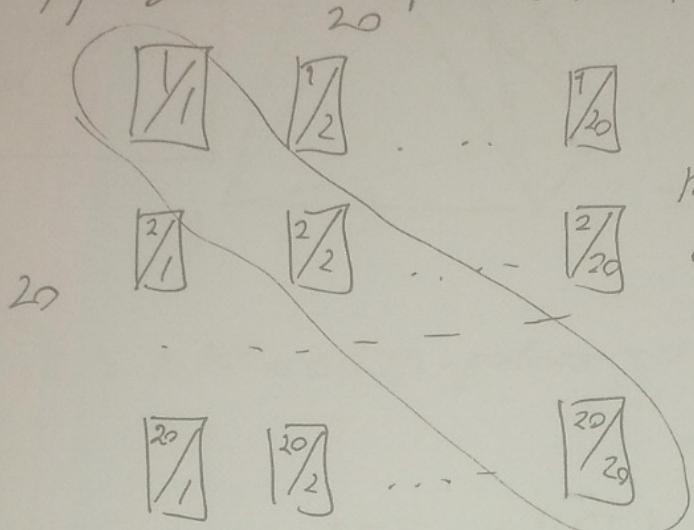
Ответ: $(x; y) = \{(\sqrt{5}; \sqrt{6}); (-\sqrt{5}; \sqrt{6}); (\sqrt{5}; -\sqrt{6}); (-\sqrt{5}; -\sqrt{6}); (\sqrt{6}; \sqrt{5}); (-\sqrt{6}; \sqrt{5}); (\sqrt{6}; -\sqrt{5}); (-\sqrt{6}; -\sqrt{5})\}$.

№5.

Чистовик

1) Т.к. порядок вытаскивания карточек не важен, будем считать, что сначала он достанет дубль;

2) разложим карточки так:



Сначало он достанет карточку из выделенной диагонали (дубль) — 20 способов.

Затем, он не сможет достать карточку из столбца и строки, из которой I-ая карточка — $20^2-20-19 = 20(20-\cancel{1})=19(20-1) = 19^2$ способов.

Всего способов: $20+19^2=381$

Ответ: 381

Стр. 3