

МОСКОВСКИЙ ФИЗИКО-ТЕХНИЧЕСКИЙ ИНСТИТУТ

ОЛИМПИАДА "ФИЗТЕХ" ПО МАТЕМАТИКЕ

11 класс

ВАРИАНТ 4

ШИФР

Заполняется ответственным секретарём

1. [3 балла] Углы α и β удовлетворяют равенствам

$$\sin(2\alpha + 2\beta) = -\frac{1}{\sqrt{17}}; \quad \sin(2\alpha + 4\beta) + \sin 2\alpha = -\frac{2}{17}.$$

Найдите все возможные значения $\tan \alpha$, если известно, что он определён и что этих значений не меньше трёх.

2. [4 балла] Решите систему уравнений

$$\begin{cases} y - 6x = \sqrt{xy - 6x - y + 6}, \\ 9x^2 + y^2 - 18x - 12y = 45. \end{cases}$$

3. [5 баллов] Решите неравенство

$$|x^2 - 26x|^{\log_5 12} + 26x \geq x^2 + 13^{\log_5(26x-x^2)}.$$

4. [5 баллов] Окружности Ω и ω касаются в точке A внутренним образом. Отрезок AB – диаметр большей окружности Ω , а хорда BC окружности Ω касается ω в точке D . Луч AD повторно пересекает Ω в точке E . Прямая, проходящая через точку E перпендикулярно BC , повторно пересекает Ω в точке F . Найдите радиусы окружностей, угол AFE и площадь треугольника AEF , если известно, что $CD = 12$, $BD = 13$.
5. [5 баллов] Функция f определена на множестве положительных рациональных чисел. Известно, что для любых чисел a и b из этого множества выполнено равенство $f(ab) = f(a) + f(b)$, и при этом $f(p) = [p/4]$ для любого простого числа p ($[x]$ обозначает наибольшее целое число, не превосходящее x). Найдите количество пар натуральных чисел $(x; y)$ таких, что $4 \leq x \leq 28$, $4 \leq y \leq 28$ и $f(x/y) < 0$.

6. [5 баллов] Найдите все пары чисел $(a; b)$ такие, что неравенство

$$\frac{8 - 6x}{3x - 2} \geq ax + b \geq 18x^2 - 51x + 28$$

выполнено для всех x на промежутке $(\frac{2}{3}; 2]$.

7. [6 баллов] Данна пирамида $XYZT$, вершина Y которой лежит на одной сфере с серединами всех её рёбер, кроме ребра TY . Известно, что $XY = \sqrt{3}$, $TX = \sqrt{2}$, $TZ = 2$. Найдите длину ребра XZ . Какой наименьший радиус может иметь сфера, описанная около данной пирамиды?

ПИСЬМЕННАЯ РАБОТА

N 5

$$\begin{aligned}
 f(2) &= 0; \quad f(3)=0; \quad f(4)=f(2\cdot 2)=0; \quad f(5)=1 \\
 f(6) &= f(2\cdot 3)=0; \quad f(7)=1; \quad f(8)=f(2\cdot 4)=0; \quad f(9)=f(3\cdot 3)=0 \\
 f(10) &= f(2\cdot 5)=1; \quad f(11)=2; \quad f(12)=f(2\cdot 6)=0; \\
 f(13) &= 3; \quad f(14)=f(2\cdot 7)=1; \quad f(15)=f(3\cdot 5)=1; \\
 f(16) &= f(2\cdot 8)=0; \quad f(17)=4; \quad f(18)=f(2\cdot 9)=0; \\
 f(19) &= 4; \quad f(20)=f(4\cdot 5)=1; \quad f(21)=f(3\cdot 7)=1; \\
 f(22) &= f(2\cdot 11)=2; \quad f(23)=5; \quad f(24)=f(2\cdot 12)=6; \\
 f(25) &= f(5\cdot 5)=2; \quad f(26)=f(13\cdot 2)=3; \quad f(27)=f(3\cdot 9)=6; \\
 f(28) &= f(4\cdot 7)=1
 \end{aligned}$$

Всего: ~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ ~~8~~ ~~9~~ ~~10~~ ~~11~~ ~~12~~ ~~13~~ ~~14~~ ~~15~~ ~~16~~ ~~17~~ ~~18~~ ~~19~~ ~~20~~ ~~21~~ ~~22~~ ~~23~~ ~~24~~ ~~25~~ ~~26~~ ~~27~~ ~~28~~ ~~29~~ ~~30~~ ~~31~~ ~~32~~ ~~33~~ ~~34~~ ~~35~~ ~~36~~ ~~37~~ ~~38~~ ~~39~~ ~~40~~ ~~41~~ ~~42~~ ~~43~~ ~~44~~ ~~45~~ ~~46~~ ~~47~~ ~~48~~ ~~49~~ ~~50~~ ~~51~~ ~~52~~ ~~53~~ ~~54~~ ~~55~~ ~~56~~ ~~57~~ ~~58~~ ~~59~~ ~~60~~ ~~61~~ ~~62~~ ~~63~~ ~~64~~ ~~65~~ ~~66~~ ~~67~~ ~~68~~ ~~69~~ ~~70~~ ~~71~~ ~~72~~ ~~73~~ ~~74~~ ~~75~~ ~~76~~ ~~77~~ ~~78~~ ~~79~~ ~~80~~ ~~81~~ ~~82~~ ~~83~~ ~~84~~ ~~85~~ ~~86~~ ~~87~~ ~~88~~ ~~89~~ ~~90~~ ~~91~~ ~~92~~ ~~93~~ ~~94~~ ~~95~~ ~~96~~ ~~97~~ ~~98~~ ~~99~~ ~~100~~ ~~101~~ ~~102~~ ~~103~~ ~~104~~ ~~105~~ ~~106~~ ~~107~~ ~~108~~ ~~109~~ ~~110~~ ~~111~~ ~~112~~ ~~113~~ ~~114~~ ~~115~~ ~~116~~ ~~117~~ ~~118~~ ~~119~~ ~~120~~ ~~121~~ ~~122~~ ~~123~~ ~~124~~ ~~125~~ ~~126~~ ~~127~~ ~~128~~ ~~129~~ ~~130~~ ~~131~~ ~~132~~ ~~133~~ ~~134~~ ~~135~~ ~~136~~ ~~137~~ 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№4

 R - радиус Ω r - радиус ω

$\angle FEA = \alpha$

~~AB ⊥ BC~~

$BD^2 = BM \cdot AB$

$13^2 = (2R - 2r) \cdot 2R$

 $\angle BCA$ - прямой (биссектрисы, опущенные на диаметр) $\triangle BDO_2 \sim \triangle BCA$ ($\angle BCA$ - прямой; $\angle BDO_2 = \angle BCA = 90^\circ$)

$$\frac{BO_2}{BA} = \frac{BD}{BC}$$

$$\frac{2R-r}{2R} = \frac{13}{25}; 26R = 50R - 25r;$$

$$r = \frac{24}{25} R$$

$$13^2 = \frac{24}{25} R \cdot 2R; R = \sqrt{\frac{13^2 \cdot 5^2}{48}} = 32,5$$

$$r = \frac{24}{25} \cdot 32,5 = 31,2$$

 $EF \perp BC \Rightarrow EF \parallel O_2D \Rightarrow \angle O_2DA = \alpha$
 $O_2D \perp BC$

$$O_2D = O_2A = r \Rightarrow \angle EAB = \alpha$$

$$\angle EAB = \angle FEA \Rightarrow \angle BEF = \angle AFE$$

$$\angle BEA = 90^\circ \text{ (отмечено на диаметре)} \Rightarrow \angle BEF = 90^\circ - \alpha$$

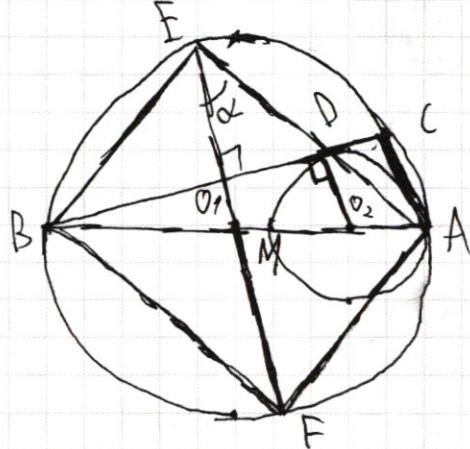
$$\angle BAF = \angle BEF \text{ (отмечено на отрезке)} = 90^\circ - \alpha$$

$$\angle EAF = \angle EAB + \angle BAF = \alpha + (90^\circ - \alpha) = 90^\circ$$

 $\Rightarrow EF$ - диаметр Ω ~~($BE = AF$, $BE \parallel AF$)~~

$$\angle EMA = 180^\circ - 2\alpha; \angle HMD_1 = 2\alpha; \angle QBM = 90^\circ - 2\alpha$$

$$\sin(90^\circ - 2\alpha) = \frac{BC}{2R} = \frac{13}{25}; \sin 2\alpha = \frac{5}{13}; \alpha = \arcsin\left(\frac{5}{13}\right)$$



ПИСЬМЕННАЯ РАБОТА

$$MF = MA = R; \angle EFA = \angle BAF = 90^\circ - \alpha$$

$$\angle EFA = 90^\circ - \arcsin\left(\frac{5}{13}\right)$$

$$S_{\Delta EAF} = EA \cdot AF = 2R \cdot \sin(\angle EFA) \cdot 2R \cdot \cos(\angle EFA)$$

$$S_{\Delta EAF} = 4225 \cdot \sin(90^\circ - \arcsin\left(\frac{5}{13}\right)) \cdot$$

$$\cos(90^\circ - \arcsin\left(\frac{5}{13}\right))$$

$$\text{Ответ: } R = 32,5; r = 32,5;$$

$$S_{\Delta EAF} = EF \cdot \sin \alpha + EF \cdot \cos \alpha =$$

$$= 2R \cdot \sin\left(\arcsin\left(\frac{5}{13}\right)\right) \cdot \cos\left(\arcsin\left(\frac{5}{13}\right)\right) =$$

$$= 4225 \cdot \sin\left(\arcsin\left(\frac{5}{13}\right)\right) \cdot \cos\left(\arcsin\left(\frac{5}{13}\right)\right)$$

$$\text{Ответ: } R = 32,5; r = 32,5; \angle A.$$

$$MF = MA = R; \angle EFA = \angle BAF = 90^\circ - \alpha; \angle AFE = 90^\circ - \arcsin\left(\frac{5}{13}\right)$$

$$S_{\Delta EAF} = EA \cdot AF = 2R \cdot \sin(\angle EFA) \cdot 2R \cdot \cos(\angle EFA)$$

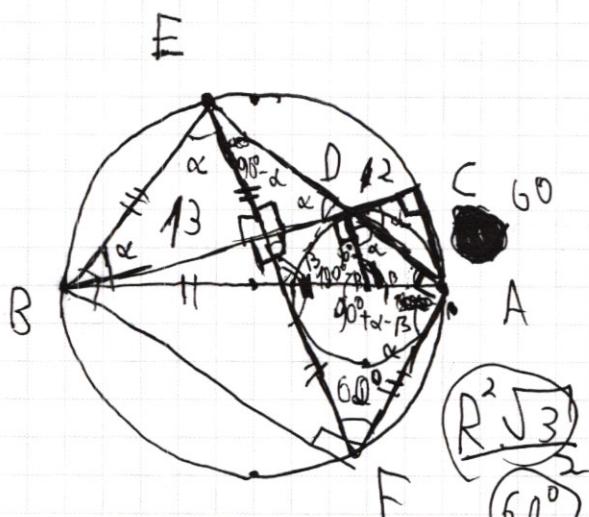
$$S_{\Delta EAF} = 4225 \cdot \sin(90^\circ - \arcsin\left(\frac{5}{13}\right)) \cdot \cos(90^\circ - \arcsin\left(\frac{5}{13}\right))$$

$$\text{Ответ: } R = 32,5; r = 32,5; \angle EFA = 90^\circ - \arcsin\left(\frac{5}{13}\right); S_{\Delta EAF} = 4225 \cdot \sin\left(90^\circ - \arcsin\left(\frac{5}{13}\right)\right) \cdot \cos\left(90^\circ - \arcsin\left(\frac{5}{13}\right)\right).$$

черновик чистовик
(Поставьте галочку в нужном поле)

Страница № _____
(Нумеровать только чистовики)

$$2R \cdot \frac{2}{25}R = 13^2 \quad R = \frac{13 \cdot 5}{2}$$



$$13^2 = 2R(2R - 2r) \quad 32,5$$

$$90^\circ - \alpha = 180^\circ - \alpha - \beta \quad \frac{13 \cdot 12}{5} \\ 2\alpha = 180^\circ - \beta \quad 31,2$$

$$90^\circ - \alpha = 90^\circ + \alpha - \beta \\ \beta = 2\alpha$$

$$25r = 24R \quad \frac{2R - r}{2R} = \frac{13}{25}$$

$$r = \frac{24}{25}R \quad 24R = 50R - 25r$$

$$13^2 = 2R(2R - \frac{24}{25}R) \quad 25r = 12R$$

$$13^2 = 2R \cdot \frac{26}{25}R$$

$$R = \frac{25}{12}r$$

$$180^\circ - 90^\circ - (180^\circ - \beta) = \beta - 90^\circ = 2\alpha - 90^\circ$$

~~$$180^\circ - 2\alpha - (180^\circ - \beta) = 2\beta - 2\alpha = 180^\circ - 4\alpha$$~~

$$13^2 = \frac{R^2 \cdot 13 \cdot 4}{25} \sin 2\alpha \cdot (\cos 2\beta + \sin 2\beta \cdot \cos 2\alpha) = -\frac{1}{\sqrt{17}}$$

$$R^2 = 13 \cdot 25 \quad \sin 2\alpha \cdot (\cos 4\beta + \sin 4\beta \cdot (\cos 2\alpha + \sin 2\alpha)) = -\frac{2}{17}$$

$$R = \frac{5}{2}\sqrt{17} \quad 7 \cdot 4 \cdot 9 \cdot 2 \quad 7 \cdot 2^2 \cdot 2^3 \quad 3^2 \cdot 17^2$$

$$r = \frac{6}{5}\sqrt{17} \quad 224 \quad 289 \quad 32 - 68 + 28$$

$$23d + b \geq -5 \quad -\frac{2}{3} \quad 50 - 85 + 28 \quad -8 \\ -23d + b \geq -12 \quad -1 \quad 18 \cdot 4 - 51 \cdot 2 + 28$$

$$2\sin \alpha \cos \alpha \cdot (\cos 2\beta - \sin 2\beta) + 2\sin 2\alpha \cos 2\beta \cdot (\cos^2 \alpha + \sin^2 \alpha)$$

$$(-t)^{\log_5 12} \geq -t + 13^{\log_5 (-t)}$$

$$\begin{array}{r} 65 \\ \times 65 \\ \hline 325 \\ 0 \\ \hline 4225 \end{array}$$

ПИСЬМЕННАЯ РАБОТА

$$y - 6x = \sqrt{(x-1)(y-6)} \quad |(x^2-26x|^{log_5 12} + 13) \geq x^2-26x+13^{log_5 t}$$

$$|t|^{log_5 12} \geq t + 13^{log_5 (t)}$$

~~$27x^2 + 18x + 12y - 12xy = 45$~~

$3(x+3)^2 +$

$y^2 - 12xy + 36) = (x-1)(y-6) \quad (3x-3)^2 + (y-6)^2 = 90$

~~$t^{log_5 12-1} + 1 \geq log_5(t \cdot \frac{13}{5})$~~
 $t^{log_5 12-1} + 1 \geq 13^{log_5 t}$

$f(1)=0 \quad f(2)=0 \quad f(3)=0 \quad f(4)=0 \quad f(5)=1 \quad f(6)=0$

$t^{log_5 12-1} \geq \frac{13}{5} t \quad f(7)=1 \quad f(8)=0 \quad f(9)=0 \quad f(10)=1$

$f(11)=2 \quad f(12)=0 \quad f(13)=3 \quad f(14)=1 \quad f(15)=1$

$f(16)=0 \quad f(17)=4 \quad f(18)=0 \quad f(19)=4$

$f(20)=1 \quad f(21)=1 \quad f(22)=2 \quad f(23)=5$

$f(24)=0 \quad f(25)=2 \quad f(26)=3 \quad f(27)=0$

$f(28)=1$

$0: 9 \quad 1: 8 \quad 2: 3 \quad 3: 2 \quad 4: 2 \quad 5: 1$

$9 \cdot 16 + 8 \cdot 8 + 3 \cdot 5 + 2 \cdot 3 + 2$

$$\begin{array}{r} 76x \\ \times 16 \\ \hline 144 \\ + 75 \\ \hline 1231 \end{array}$$

$144 + 64 + 15 + 8$

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ФЕДЕРАЛЬНОЕ ГОСУДАРСТВЕННОЕ АВТОНОМНОЕ
ОБРАЗОВАТЕЛЬНОЕ УЧРЕЖДЕНИЕ ВЫСШЕГО
ОБРАЗОВАНИЯ

«МОСКОВСКИЙ ФИЗИКО-ТЕХНИЧЕСКИЙ ИНСТИТУТ
(НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ
УНИВЕРСИТЕТ)»

ШИФР

(заполняется секретарём)

ПИСЬМЕННАЯ РАБОТА

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черновик чистовик
(Поставьте галочку в нужном поле)

Страница №__
(Нумеровать только чистовики)

ПИСЬМЕННАЯ РАБОТА

$$\frac{8-6x}{3x-2} \geq dx+b$$

$$8-6x-3dx^2+2dx-3bx+2b$$

$$-3dx^2 + (2d-6)x + 2b + 8$$

$$(2d-3c)^2 + 48dc$$

$$4d^2 + 36dc + 9c^2$$

$$c = \sqrt{16}$$

$$(d+6)^2 - 2d6 = 90$$

$$(d+6)^2 - 2c^2 = 90$$

$$(9d+6-2c)(d+6+2c) = 90$$

$$9x-9+4y-6-2y-12x \quad 9x+4y-7+2y+12x$$

$$(-11x-4y-7)(13x+3y-7) = 90$$

$$0(3d-9)(3d+9) + (6-3)(6+3) = 0$$

$$t^{\log_5 12} + t^{-\log_5 13} \geq 0$$

$$\log_5 12 t^{\log_5 12 - 1} + 1 - \log_5 13 t^{\log_5 13 - 1}$$

$$(18x^2 - 57x + 28)(3x - 2) = 8 - 6x$$

$$\cancel{54x^3} - \cancel{153x^2} + \cancel{84x} - \cancel{36x^2} + \cancel{102x} \cancel{86} - \cancel{8} \cancel{6x}$$