Часть 1

Олимпиада: **Физика, 11 класс (1 часть)**

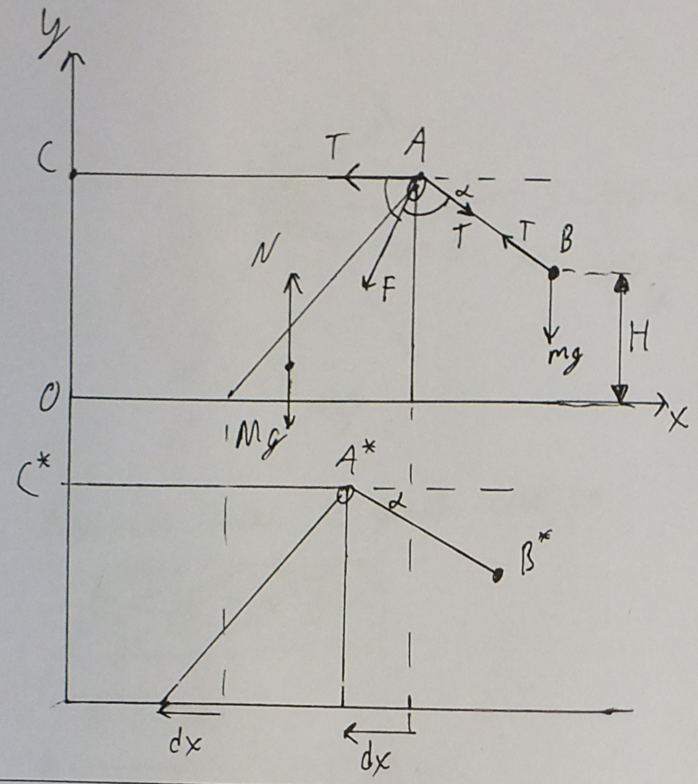
Шифр: **21200899**

ID профиля: **315863**

Вариант 2

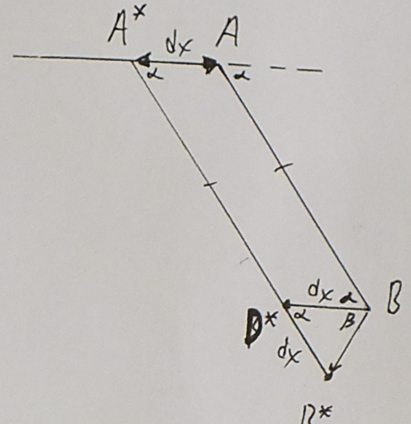
№1

Чистовик



$\cos\alpha = \frac{4}{5}$

А) Пусть клин сместился на расстояние dx влево.



$A^*A = D^*B = dx$

$A^*D^* = AB \Rightarrow D^*B^* = dx$

Рассмотрим ΔD^*B^*B: он равнобедренный $\Rightarrow$

$B^*B = 2dx \cdot \sin\frac{\alpha}{2}$

Тогда по теореме синусов: $\frac{B^*B}{\sin\alpha} = \frac{B^*D^*}{\sin\beta}$

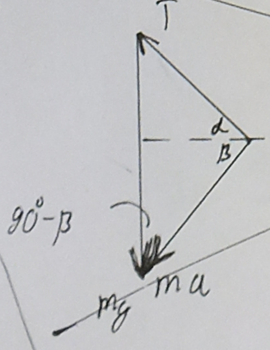
$$\frac{2dx\sin\frac{\alpha}{2}}{\sin\alpha} = \frac{dx}{\sin\beta} \Rightarrow \sin\beta = \frac{\sin\alpha}{2\sin\frac{\alpha}{2}} = \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}} = \cos\frac{\alpha}{2}.$$

Перемещение будет происходить под углом β к горизонту, все время вдоль прямой BB^* $\Rightarrow$ и ускорение будет направлено под углом β к горизонту $\boxed{\sin\beta = \cos\frac{\alpha}{2}}$

$\cos(90^\circ - \beta) = \cos\frac{\alpha}{2}$

$\beta = 90^\circ - \frac{\alpha}{2}$

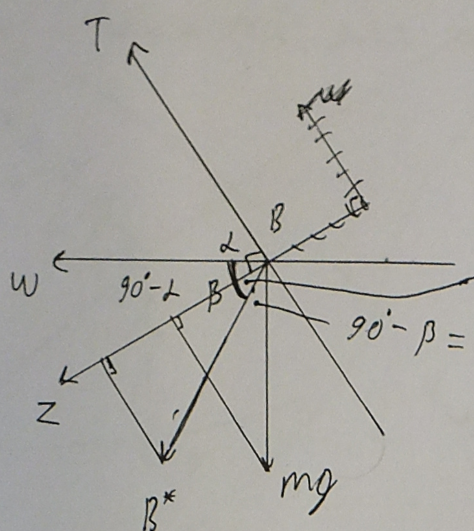
г) Δ скоростей для шара:



По т. синусов:

$$\frac{T}{\sin(90^\circ - \beta)} = \frac{mg}{\sin(\alpha+\beta)} \Rightarrow T = \frac{mg\cos\beta}{\sin(\alpha+\beta)}$$

2)



$\beta-(90°-\alpha) = 90° - \frac{\alpha}{2} - 90° + \alpha = \frac{\alpha}{2}$

$90°-\beta = 90° - 90° + \frac{\alpha}{2} = \frac{\alpha}{2}$

Вдоль оси z для шара:

$$\cancel{\frac{B^*B}{dt} = mg\cos\alpha}$$

$$m\cdot\frac{B^*B\cdot\cos\frac{\alpha}{2}}{(dt)^2} = mg\cos\alpha$$

$$m\cdot\frac{2\,dx\cdot\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{(dt)^2} = mg\cos\alpha$$

$$m\cdot\frac{\sin\alpha\cdot dx}{(dt)^2} = mg\cos\alpha$$

$$m\cdot\frac{dx}{(dt)^2} = mg\,\frac{\cos\alpha}{\sin\alpha} = m a_{клина} \Rightarrow \boxed{a_{клина} = g\cot\alpha}$$

3) $F = 2T\cos\left(\frac{\pi-\alpha}{2}\right) = 2T\cos\left(\frac{\pi}{2}-\frac{\alpha}{2}\right) = 2T\sin\frac{\alpha}{2}$

Вдоль оси x для клина:

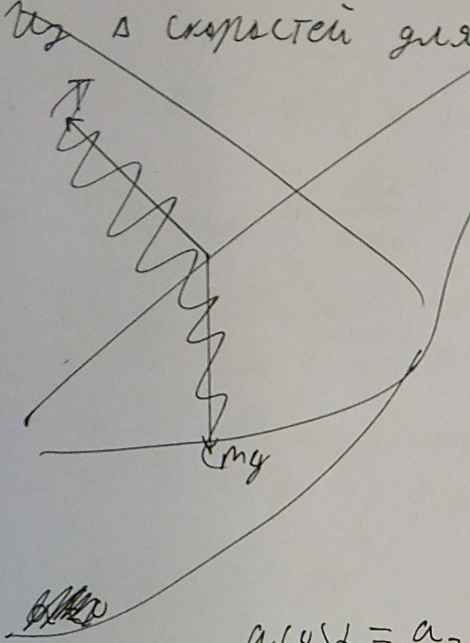
$$-F\cos\left(\frac{\pi-\alpha}{2}\right) = -Ma_{клина}$$

$$F\sin\frac{\alpha}{2} = Ma_{клина}$$

$$2T\sin^2\frac{\alpha}{2} = Mg\cot\alpha$$

$$T = \frac{Mg\cot\alpha}{2\sin^2\frac{\alpha}{2}}$$

Из Δ скоростей для шара:

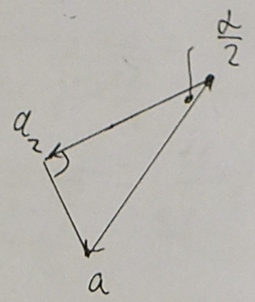


Чистовик

Вдоль оси Z для шара:

$$mg\cos\alpha = ma_z$$

$$a_z = g\cos\alpha$$



$$a\cos\alpha = a_z = g\cos\alpha \Rightarrow$$

$$a = g$$

Вдоль оси ω:

$$T\cos\alpha = m\cdot g\cos\beta$$

$$T = \frac{mg\cos\beta}{\cos\alpha}$$

Итого:

$$\frac{Mg\cos\alpha}{2\sin^2\frac{\alpha}{2}\sin\alpha} = \frac{mg\cos\beta}{\cos\alpha}$$

$$\boxed{\frac{m}{M} = \frac{\cos^2\alpha}{2\sin^2\frac{\alpha}{2}\sin\alpha\cos\beta} = \frac{\cos^2\alpha}{2\sin^2\frac{\alpha}{2}\sin\alpha\sin\frac{\alpha}{2}}}$$

4) Вдоль оси y для шара:

$$g\cos\frac{\alpha}{2} = a_y$$

$$\frac{a_y t^2}{2} = H \Rightarrow t = \sqrt{\frac{2H}{a_y}} = \sqrt{\frac{2H}{g\cos\frac{\alpha}{2}}}$$

Ответ: 1) $\sin\beta = \cos\frac{\alpha}{2}$; 2) $a = g\cot\alpha$; 3) $\frac{m}{M} = \frac{\cos^2\alpha}{2\sin^3\frac{\alpha}{2}\sin\alpha}$

4) $t = \sqrt{\frac{2H}{g\cos\frac{\alpha}{2}}}$

$\boxed{\text{Чистовик}}$

1) $\sin\beta = \sqrt{\cos^2\frac{\alpha}{2}} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+\frac{4}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$

2) $a_{\text{клина}} = g\,\frac{\cos\alpha}{\sin\alpha} = g\cdot\frac{4\cdot 5}{5\cdot 3} = \frac{4}{3}g$

3) $\frac{m}{M} = \frac{\cos^2\alpha}{\cancel{2}\,\frac{1-\cos\alpha}{\cancel{2}}\cdot\sin\alpha\cdot\sqrt{\sin^2\frac{\alpha}{2}}} = \frac{\cos^2\alpha}{(1-\cos\alpha)\sin\alpha\sqrt{\frac{1-\cos\alpha}{2}}} =$

$= \frac{16}{25\cdot\left(1-\frac{4}{5}\right)\cdot\frac{3}{5}\sqrt{\frac{1-\frac{4}{5}}{2}}} = \frac{16}{\cancel{25}\cdot\frac{1}{\cancel{5}}\cdot\frac{3}{\cancel{5}}\cdot\frac{1}{\sqrt{10}}} = \frac{16}{3\sqrt{10}} = \frac{16\sqrt{10}}{30} = \frac{8\sqrt{10}}{15}$

4) $t = \sqrt{\frac{2M\cdot\sqrt{10}}{g\cdot 3}}$

Ответ: 1) $\frac{3\sqrt{10}}{10}$; 2) $\frac{4}{3}g$; 3) $\frac{16\sqrt{10}}{30} = \frac{8\sqrt{10}}{15}$; 4) $\sqrt{\frac{2M\sqrt{10}}{3g}}$

№2 $\boxed{\text{Чистовик}}$

Дано: $c(T) = \frac{5}{2} R \frac{T}{T_0}$; ν

Решение:

1) $c = \frac{\delta Q}{\nu dT} \Rightarrow \delta Q = c \nu dT \Rightarrow \delta Q = c(T) \nu dT \Rightarrow \delta Q = \frac{5}{2} R \frac{T}{T_0} \nu dT$

$\delta Q = \frac{5}{2} \nu R \frac{T dT}{T_0}$

$Q_{подв} = \frac{5}{2} \frac{\nu R}{T_0} \int_{T_0}^{T_0/2} T dT = \frac{5}{2} \frac{\nu R}{T_0} \cdot \frac{T^2}{2} \Big|_{T_0}^{T_0/2} = \frac{5}{2} \frac{\nu R}{T_0} \cdot \left(\frac{T_0^2}{8} - \frac{T_0^2}{2} \right) =$

$= -\frac{5}{2} \frac{\nu R}{T_0} \cdot \frac{3}{8} T_0^2 = -\frac{15}{16} \nu R T_0$

$Q_1 = -Q_{подв} = \boxed{\frac{15}{16} \nu R T_0 = Q_1}$

2) Первое начало термодинамики:

$\delta Q = \delta A + dU \Rightarrow \delta A = \delta Q - dU \Rightarrow$

$\delta A = \frac{5}{2} \nu R \frac{T dT}{T_0} - \frac{3}{2} \nu R dT$

$A = \frac{5}{2} \frac{\nu R}{T_0} \int_{T_0}^{T_1} T dT - \frac{3}{2} \nu R \int_{T_0}^{T_1} dT = \frac{5}{2} \frac{\nu R}{T_0} \frac{T^2}{2} \Big|_{T_0}^{T_1} - \frac{3}{2} \nu R dT \Big|_{T_0}^{T_1} =$

$= \frac{5}{2} \frac{\nu R}{T_0} \left(\frac{T_1^2}{2} - \frac{T_0^2}{2} \right) - \frac{3}{2} \nu R (T_1 - T_0) =$

$= \frac{5}{2} \frac{\nu R}{T_0} \frac{T_1^2}{2} - \frac{5}{2} \frac{\nu R}{T_0} \frac{T_0^2}{2} - \frac{3}{2} \nu R T_1 + \frac{3}{2} \nu R T_0 \Rightarrow$

$A(T_1) = \frac{5}{4T_0} \nu R T_1^2 - \frac{3}{2} \nu R T_1 - \frac{5}{2} \nu R \frac{T_0}{2} + \frac{3}{2} \nu R T_0$ – парабола ветви вверх $\Rightarrow$

$T_{min} = \frac{\frac{3}{2} \nu R}{2 \cdot \frac{5}{4T_0} \nu R} = \frac{3}{5} T_0$ $\boxed{\frac{3}{5} T_0 = T_{min}}$

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$$3)\ A(T_{min}) = \frac{5}{4T_0}\nu R\cdot\frac{9}{25}T_0^2 - \frac{3}{2}\nu R\cdot\frac{3}{5}T_0 - \frac{5}{2}\nu R\frac{T_0}{2} + \frac{3}{2}\nu RT_0 =$$

$$= \frac{9}{4\cdot 5}\nu RT_0 - \frac{9}{10}\nu RT_0 - \frac{5}{4}\nu RT_0 + \frac{3}{2}\nu RT_0 =$$

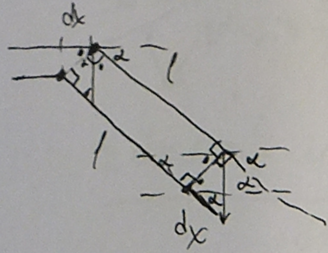
$$= \nu RT_0\left(\frac{9}{20} - \frac{9}{10} - \frac{5}{4} + \frac{3}{2}\right) =$$

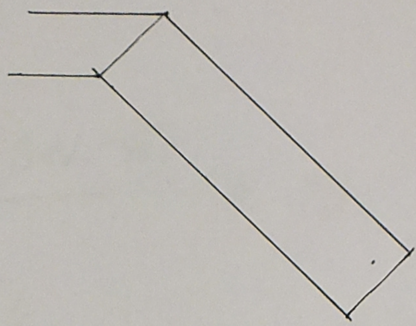
$$= \nu RT_0\left(\frac{9-18-25+30}{20}\right) =$$

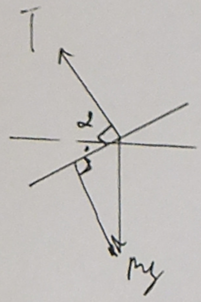
$$= \nu RT_0\,\frac{14-18}{20} = \nu RT_0\left(-\frac{4}{20}\right) = -\nu RT_0\frac{1}{5}$$

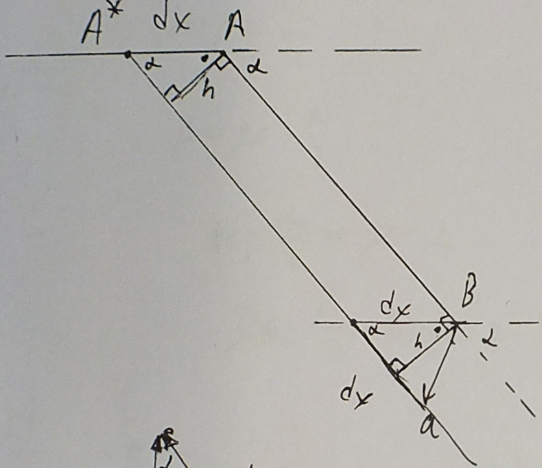
$$A = -\frac{1}{5}\nu RT_0$$

Ответ: 1) $Q_1 = \frac{15}{16}\nu RT_0$; 2) $\frac{3}{5}T_0$; 3) $-\frac{\nu RT_0}{5}$

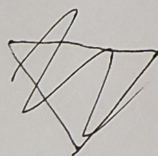


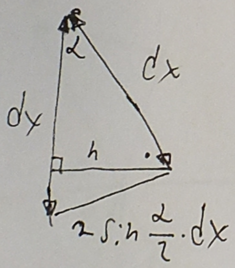






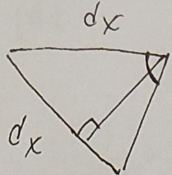
$\sin\alpha = \frac{h}{dx}$





sin

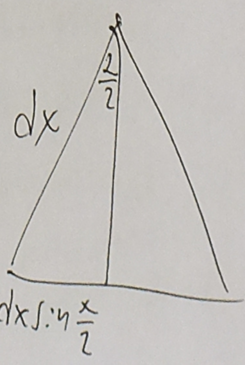
$$\frac{2\sin\frac{\alpha}{2}\cancel{dx}}{\sin\alpha} = \frac{\cancel{dx}}{\sin\beta}$$



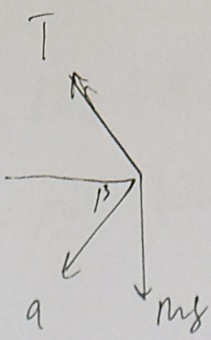
$$\sin\beta = \frac{\sin\alpha}{2\sin\frac{\alpha}{2}}$$

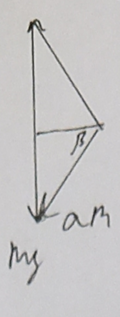
$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

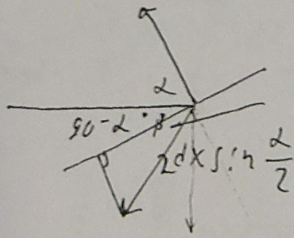
$$\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$



$\frac{1}{2} T\cos\alpha \neq mg =$







$$\beta-(90-\alpha)=\beta+\alpha-90$$

$$\frac{2dx\sin\frac{\alpha}{2}\cdot\cos(\beta+\alpha-90^\circ)}{dt}M=mg$$

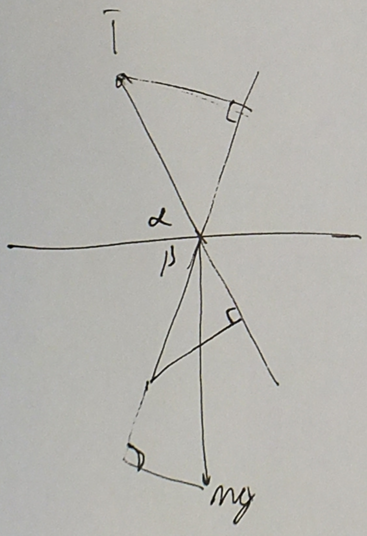
$$\cos 90-\beta=\cos\frac{\alpha}{2}$$

$$90-\beta=\frac{\alpha}{2}$$

$$\beta=90-\frac{\alpha}{2}$$

$$90-\frac{\alpha}{2}=90-\alpha$$

$$\cancel{90}-\frac{\alpha}{2}-\cancel{90}+\alpha=\frac{\alpha}{2}$$



$$\frac{2dx\sin\frac{\alpha}{2}}{(dt)^2}$$

He

$\cancel{\cos\beta=\cos}$

$$\cos\beta=\sin(90-\beta)$$

$$\cos\beta=\cos\left(90-\frac{\alpha}{2}\right)=\sin\frac{\alpha}{2}$$

$$pV=\nu RT$$

$$pdV+Vdp=\nu R\,dT$$

$$\frac{5}{2}\nu R\frac{TdT}{T_0}=\delta A+\frac{3}{2}\nu R dT$$

$$\frac{5}{2}\nu R\frac{TdT}{T_0}=pdV+\frac{3}{2}\nu R dT$$

$$Q_{подв} = \frac{\frac{5}{2}\nu R}{T_0}\int_{T_0}^{T_0/2} T\,dT = \frac{5}{2}\frac{\nu R}{T_0}\left(\frac{T_0^2}{4} - T_0^2\right) = \frac{5}{2}\frac{\nu R}{T_0}\cdot\frac{3T_0^2}{4}$$

$$\frac{\frac{3}{2}\nu R\cancel{}}{2\frac{5}{4T_0}\nu R} = \frac{3}{4\cdot\frac{5}{4T_0}} = \frac{3T_0}{5}$$

$$\cos\alpha = \frac{4}{5}$$

$$\sin\alpha = \frac{3}{5}$$

Часть 2

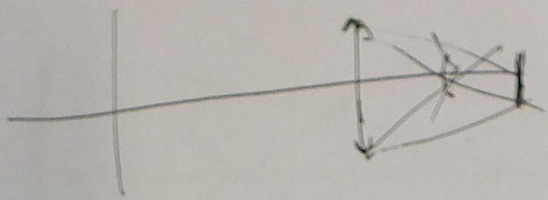
Олимпиада: **Физика, 11 класс (2 часть)**

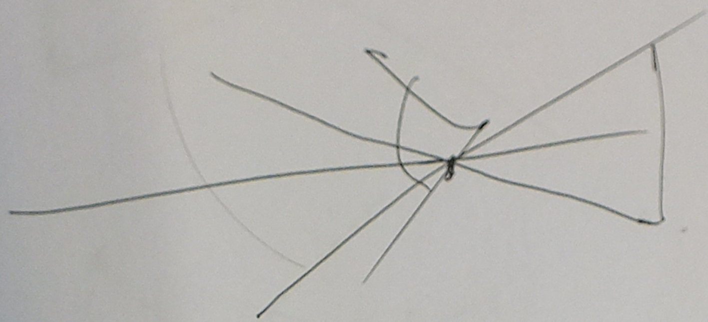
Шифр: **21200899**

ID профиля: **315863**

Вариант 2

N4

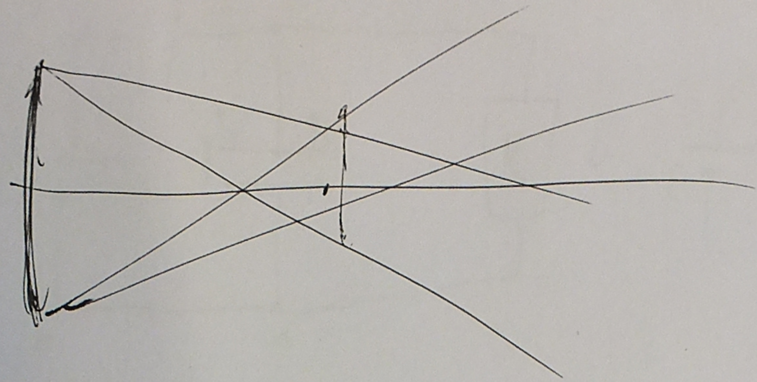


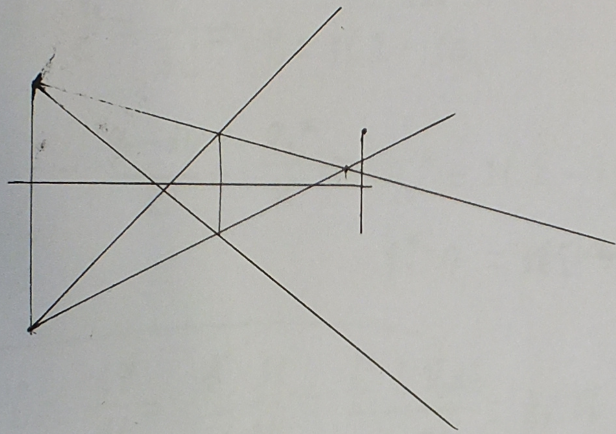


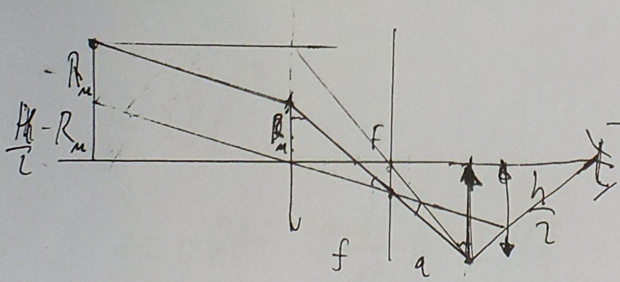
N4

f +

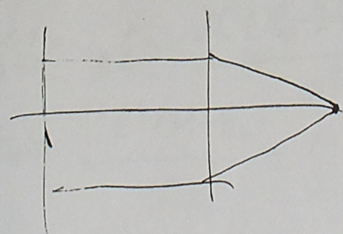
3)

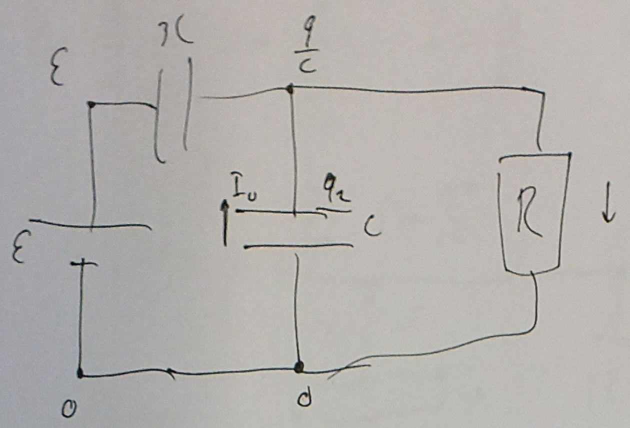


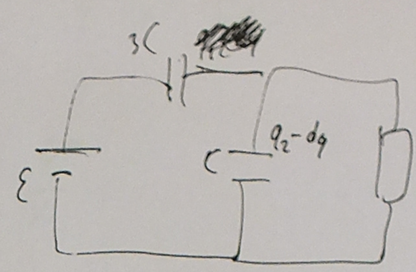




$\frac{\frac{D_u}{2}}{f} =$







$$P = U I_0$$

$$\mathcal{E} - \frac{q_2}{C} = \frac{q_1}{3C} \Rightarrow q_1 = 3C\mathcal{E} - 3q_2$$

$$\mathcal{E} - \frac{q_2 - dq}{C} = \frac{q_1^*}{3C} \Rightarrow q_1^* = 3C\mathcal{E} - 3q_2 + 3dq$$

$$q_1^* - q_1 = 3dq$$

$$\mathcal{E} = \frac{d\varphi}{dt} = \frac{B \cdot dS}{dt} = \frac{B \cdot L v_0 dt}{dt} = BLv_0$$

$$F_A = IBL$$

$$F = qvB = q\frac{dl}{dt}B = IdlB$$

$$\mathcal{E} = \frac{BL(v_1 - v_2)dt}{dt} = BL(v_1 - v_2) = I5R$$

$$I = \frac{BL(v_1 - v_2)}{5R}$$

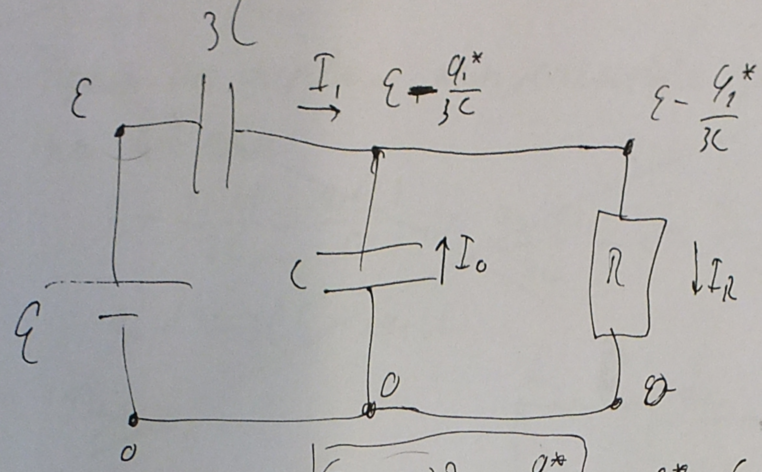
$$F_A = \frac{B^2L^2(v_1 - v_2)}{5R} = \frac{m}{2}a_2 = ma_1$$

$\longrightarrow x$

$$F_A = \frac{m}{2} \cdot a_{2x} \qquad -F_A = ma_{1x}$$

$$-\frac{m}{2}\frac{dv_{2x}}{dt} = \frac{m\, dv_{1x}}{dt}$$

$C=\frac{q}{U} \Rightarrow U=\frac{q}{C}$ $\quad I=\frac{U}{R}$ $\quad \frac{CU^2}{2}=\frac{C}{2}\cdot\frac{q^2}{C^2}=\frac{q^2}{2C}$



$U_R = \boxed{(I_0+I_1)R = \frac{q_2^*}{C}} \Rightarrow q_2^* = C(I_0+I_1)R$

$q_2^* = I_0CR + I_1CR$

$\boxed{\frac{q_1^*}{3C}+\frac{q_2^*}{C}=\mathcal{E}} \Rightarrow q_1^*+3q_2^*=3C\mathcal{E}$

$P=UI_0$

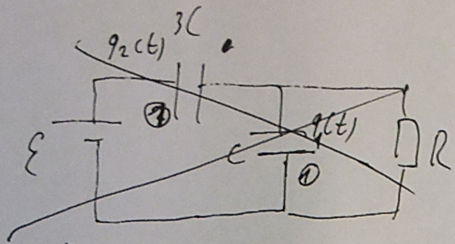
$\boxed{\mathcal{E}=\frac{q_1^*}{3C}+(I_0+I_1)R}$

$3C\mathcal{E}=q_1^*+3C(I_0+I_1)R$

$q_1^*=3C\mathcal{E}-3CR(I_0+I_1)$

$\mathcal{E}\cancel{-R(I_0+I_1)}+\cancel{(I_0+I_1)R=\mathcal{E}}$

3)

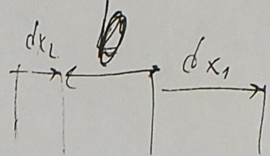


~~Пусть на первом конденсаторе заряд $q_1(t)$, тогда на втором:~~

$$\cancel{\varepsilon - \frac{q_2(t)}{3C} = \frac{q_1(t)}{C} \Rightarrow \frac{q_2(t)}{3C} = \varepsilon - \frac{q_1(t)}{C}}$$

$$\cancel{q_2(t) = 3C\varepsilon - 3q_1(t)}$$

~~Пусть~~



$$dx = (dx_1 + b) - (dx_2 + b) = dx_1 - dx_2$$

$$-\frac{dx_2}{\cancel{(dt)^2}} = 2\frac{dx_1}{\cancel{(dt)^2}}$$

$$dx = dx_1 - dx_2 = 3dx_1$$

$$-dx_2 = 2dx_1$$

$$\cancel{dx_{отн} =}$$

$$a_2 = \frac{2B^2L^2(v_1 - v_2)}{5Rm}; \quad a_1 = \frac{B^2L^2(v_1 - v_2)}{5Rm}$$

$$a_{отн} = a_1 - a_2$$

$$dx_1 = v_1 t + \frac{at^2}{2}$$

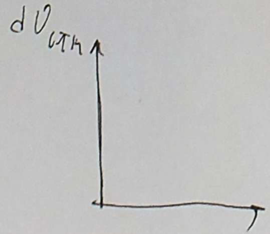
$$dx_2 = v_2 t + \frac{at^2}{2}$$

$$dx_1 = v_1 dt - \frac{a_1 dt^2}{2}$$

$$dx_2 = v_2 dt + \frac{at^2}{2}$$

$$dx_1 - dx_2 = \cancel{a(dt)^2 + (v_1 - v_2)dt} =$$

$$= (v_1 - v_2)dt - \frac{(a_1 - a_2)dt^2}{2}$$



$$dv_1 - dv_2 = 3dv_1 \Rightarrow a_{отн} = 3a_1$$

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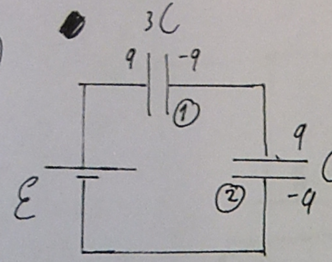
Чистовик

№3

1) I)

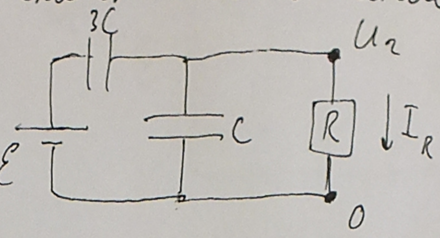


$$\mathcal{E} = \frac{q}{3C} + \frac{q}{C} = \frac{4q}{3C} \Rightarrow$$

$$q = \frac{3C\mathcal{E}}{4} \Rightarrow$$

$$U_2 = \frac{q}{C} = \frac{3C\mathcal{E}}{4C} = \frac{3\mathcal{E}}{4}$$

II) Напряжение на конденсаторах после замыкания ключа скачком не поменяется.



$$I_R = \frac{U_2}{R} = \boxed{\frac{3\mathcal{E}}{4R} = I}$$

2) В установившемся состоянии $I_R(t_{уст}) = 0 \Rightarrow U_2(t_{уст}) = 0 \Rightarrow$

$\Rightarrow U_1 = \mathcal{E} \Rightarrow \frac{q_1}{3C} = \mathcal{E} \Rightarrow q_1 = 3C\mathcal{E}$, т.е. через источник протек заряд $q_{\mathcal{E}} = q_1 - q = 3C\mathcal{E} - \frac{3C\mathcal{E}}{4} = \frac{9C\mathcal{E}}{4}$

По закону сохранения энергии:

$$A_{\mathcal{E}} = Q + \Delta W_1 + \Delta W_2$$

$$q_{\mathcal{E}}\mathcal{E} = Q + \left(0 - \frac{q^2}{2C}\right) + \left(\frac{q_1^2}{2\cdot 3C} - \frac{q^2}{2\cdot 3C}\right)$$

$$\frac{9C\mathcal{E}^2}{4} = Q + \frac{q_1^2}{6C} - \frac{q^2}{2C} - \frac{q^2}{6C}$$

$$\frac{9C\mathcal{E}^2}{4} = Q + \frac{9C^2\mathcal{E}^2}{6C} - \frac{9}{16}C^2\mathcal{E}^2\cdot\frac{1}{2C} - \frac{9}{16}C^2\mathcal{E}^2\cdot\frac{1}{6C}$$

$$\frac{9C\mathcal{E}^2}{4} = Q + \frac{3C\mathcal{E}^2}{2} - \frac{9}{16}C\mathcal{E}^2\left(\frac{1}{2} - \frac{1}{6}\right)$$

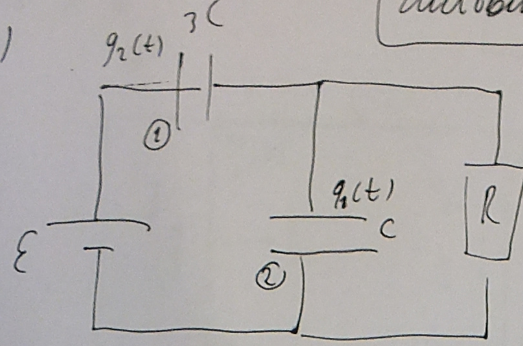
$$\frac{9C\mathcal{E}^2}{4} = Q + \frac{6C\mathcal{E}^2}{4} - \frac{9}{16}C\mathcal{E}^2\cdot\frac{2}{6}$$

$$\frac{3C\mathcal{E}^2}{4} = Q - \frac{3}{16}C\mathcal{E}^2$$

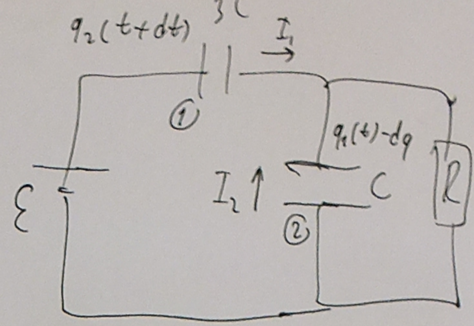
$$Q = \frac{3C\mathcal{E}^2}{4} + \frac{3}{16}C\mathcal{E}^2 = \frac{12+3}{16}C\mathcal{E}^2 = \boxed{\frac{15}{16}C\mathcal{E}^2 = Q}$$

(Чистовик)

3)



$\Rightarrow$



$$\begin{cases} \mathcal{E} = \dfrac{q_2(t)}{C} + \dfrac{q_1(t)}{3C} \\ \mathcal{E} = \dfrac{q_2(t)-dq}{C} + \dfrac{q_1(t+dt)}{3C} \end{cases} \Rightarrow -\frac{dq}{C} + \frac{q_1(t+dt)-q_1(t)}{3C} = 0$$

$3\,dq = q_1(t+dt) - q_1(t) \Rightarrow 3I_2 = I_1 \Rightarrow$

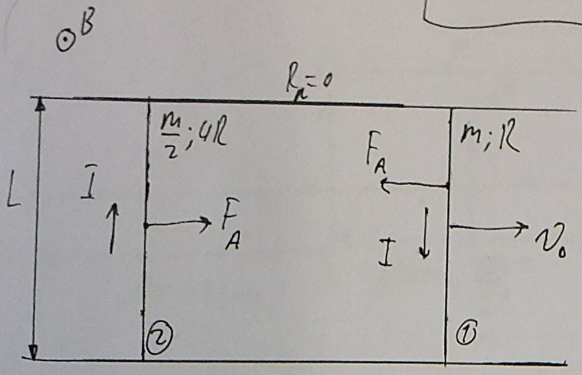
Если $I_2 = I_0$, то $I_1 = 3I_0$.

Тогда $I_R(t_0) = I_2 + I_1 = 4I_0 \Rightarrow U_R = \dfrac{4I_0}{R}$

Ответ: 1) $\dfrac{3\mathcal{E}}{4R}$; 2) $\dfrac{15}{16}C\mathcal{E}^2$; 3) $\dfrac{4I_0}{R}$

~~№~~ № 4

Чистовик



1) Скорость второй перемычки мгновенно не поменялась.

$$\mathcal{E}_i = I \cdot 5R$$

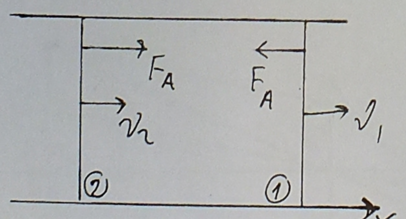
$$\mathcal{E}_i = \frac{d\Phi}{dt} = \frac{B\,dS}{dt} = \frac{BLv_0\,dt}{dt} = BLv_0$$

$$BLv_0 = 5IR \Rightarrow I = \frac{BLv_0}{5R}$$

$$F_A = IBL = \frac{BLv_0}{5R} \cdot BL = \frac{B^2L^2v_0}{5R} = \frac{m}{2} a_2(0)$$

$$\boxed{a_2(0) = \frac{2B^2L^2v_0}{5Rm}}$$

2)



На перемычки действуют в каждый момент времени одинаковые сила ⇒

Для тел вдоль оси x:

$$\begin{cases} F_A = \frac{m}{2} a_{2x} \\ -F_A = m a_{1x} \end{cases} \Rightarrow$$

$$-\frac{m}{2} a_{2x} = m a_{1x} \Rightarrow -\frac{dv_{2x}}{dt} = 2\frac{dv_{1x}}{dt} \Rightarrow -dv_{2x} = 2dv_{1x} \Rightarrow$$

$$\cancel{-(0 - v_k) = 2(v_k - v_1) \Rightarrow v_k}$$

$$-(v_k - 0) = 2(v_k - v_1) \Rightarrow -v_k = 2v_k - 2v_1 \Rightarrow 3v_k = 2v_1 \Rightarrow$$

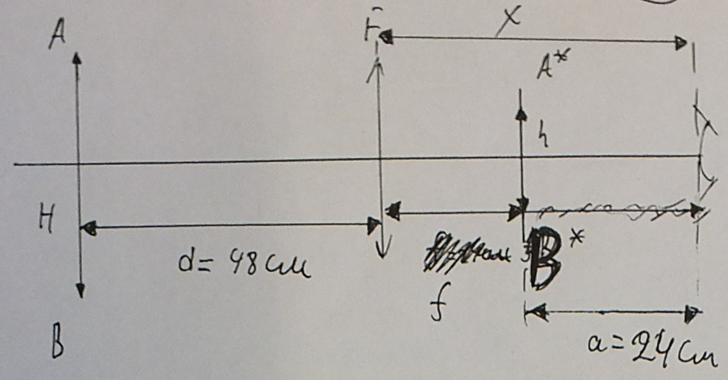
$$\boxed{v_k = \frac{2v_1}{3}}$$

№4

(чистовик)

F = 12 см

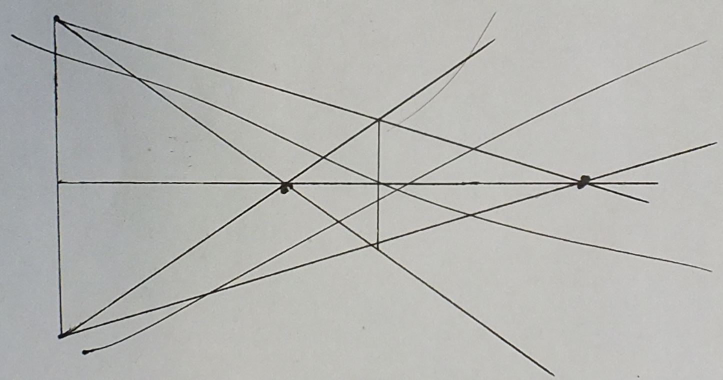
H = 9 см

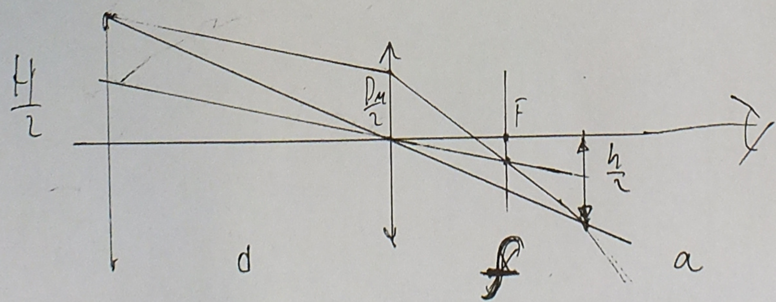


1) $\frac{1}{F} = \frac{1}{d} + \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{1}{F} - \frac{1}{d} = \frac{d-F}{Fd} \Rightarrow f = \frac{Fd}{d-F} = \frac{12 \cdot 48}{48-12} = \frac{12 \cdot 48}{36} = 16$ см

$x = f + a = 16 + 24 = \boxed{40 \text{ см} = x}$

2) $\frac{h}{H} = \frac{f}{d} \Rightarrow h = \frac{f}{d} \cdot H = \frac{16 \cdot 9}{48} = 3$ см





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