Часть 1

Олимпиада: **Физика, 11 класс (1 часть)**

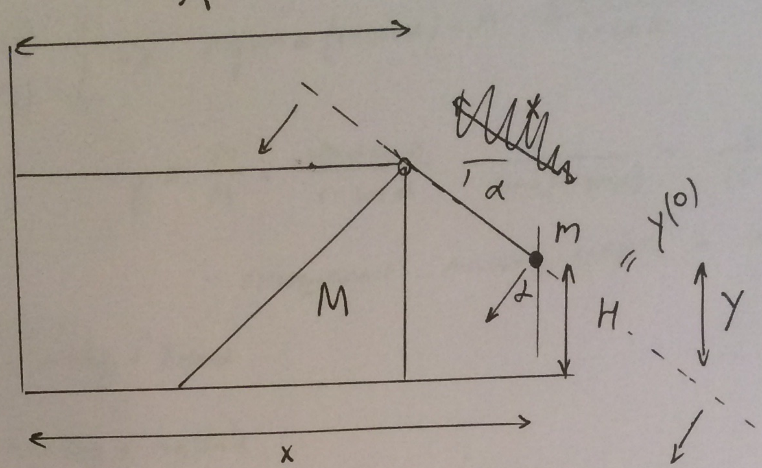
Шифр: **21202579**

ID профиля: **860053**

Вариант 2

Чистовик

2.

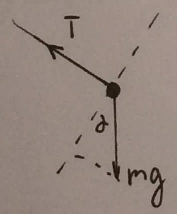


1) Угол α не меняется $\Rightarrow$ шар движется перпендикулярно нити

$\Rightarrow$ ускорение направлено под углом α к вертикали

$\cos\alpha = 4/5$ (по условию)

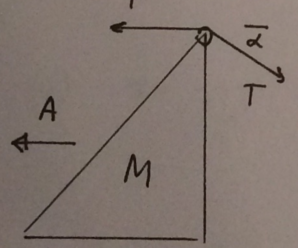
2)



$\Rightarrow T = mg\sin\alpha$

$T - T\cos\alpha = MA$ } $F = ma$ для клина

$mg\sin\alpha\,(1-\cos\alpha) = MA$ (1)



условие нерастяжимости

$$X + \frac{x - X}{\cos\alpha} = \text{constant}$$

$$\Rightarrow \ddot{x} = \underbrace{\ddot{X}}_{=-A}(1-\cos\alpha)$$

$m\ddot{x} = -T\cos\alpha$ } проекция $F = ma$ для шара

$$mA = \frac{T\cos\alpha}{1-\cos\alpha} = \frac{(mg\sin\alpha)\cos\alpha}{1-\cos\alpha}$$

$$\boxed{A = \frac{g\sin\alpha\cos\alpha}{1-\cos\alpha}}$$ ускорение клина (2)

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Черновик

Чистовик Вариант 11-02

2.

1) $\nu C = \dfrac{\delta Q}{dT}$ (по определению молярной тепл.)

$$\Rightarrow \quad Q_1 = -\frac{5}{2}\,\frac{\nu R}{T_0}\int\limits_{T_0}^{\frac{1}{2}T_0} T\,dT = -\frac{5}{2}\,\frac{\nu R}{T_0}\left(\frac{T^2}{2}\right)\Bigg|_{T_0}^{\frac{1}{2}T_0}$$

$$= -\frac{5}{4}\,\nu R T_0\left(\frac{1}{4} - 1\right)$$

$$= \frac{15}{16}\,\nu R T_0$$

2) Закон сохранения энергии $\delta Q = \delta A + dU$

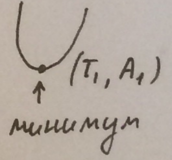
$$\Longrightarrow \quad \frac{5}{2} R \frac{T}{T_0}\nu\, dT = \delta A + \frac{3}{2}\nu R\, dT$$

$$\Longrightarrow \quad A = \frac{5}{2}\,\frac{\nu R}{T_0}\left(\frac{T^2}{2} - \frac{T_0^2}{2}\right) - \frac{3}{2}\nu R (T - T_0)$$

работа, как функция температуры, до которой охл. газ

$$A(T) = \left(\frac{5}{4}\,\frac{\nu R}{T_0}\right)T^2 - \left(\frac{3}{2}\nu R\right)T + \frac{1}{4}\nu R T_0$$ ← парабола с ветвями вверх

минимум достигается при температуре



$$T_1 = \frac{\left(\frac{3}{2}\nu R\right)}{\left(\frac{5}{2}\,\frac{\nu R}{T_0}\right)} = \frac{3}{5}T_0$$

3) Минимальная работа

$$A_1 = A(T_1) = \left(\frac{5}{4}\,\frac{\nu R}{T_0}\right)\frac{9}{25}T_0^2 - \left(\frac{3}{2}\nu R\right)\frac{3}{5}T_0 + \frac{1}{4}\nu R T_0$$

$$= \left(\frac{9}{20} - \frac{9}{10} + \frac{1}{4}\right)\nu R T_0 = -\frac{1}{5}\nu R T_0$$

Ответ: 1) газ отдаст $Q_1 = \frac{15}{16}\nu R T_0$

2) до температуры $T_1 = \frac{3}{5}T_0$

3) мин. работа $A_1 = -\frac{1}{5}\nu R T_0$

Чистовик

3) $\left.\begin{matrix}(1)\\(2)\end{matrix}\right\} \Rightarrow mg\sin\alpha(1-\cos\alpha) = M\,\dfrac{g\sin\alpha\cos\alpha}{1-\cos\alpha}$

$$\gamma \equiv \frac{m}{M} = \frac{\cancel{\sin\alpha}\cos\alpha}{1-\cos\alpha}\,\frac{1}{\cancel{\sin\alpha}(1-\cos\alpha)} = \frac{\cos\alpha}{(1-\cos\alpha)^2}$$

– отношение масси шара к массе клина

4) $m\ddot{y} = -mg + T\sin\alpha$

$m\ddot{y} = -mg + mg\sin^2\alpha$

$\left.\begin{matrix}\cancel{m}\ddot{y} = -\cancel{m}g\cos^2\alpha\\ -H = \dfrac{\ddot{y}\tau^2}{2}\end{matrix}\right\} \tau = \sqrt{2\dfrac{H}{g\cos^2\alpha}} = \dfrac{1}{\cos\alpha}\sqrt{\dfrac{2H}{g}}$

Ответ: 1) $\cos\alpha = 4/5$

2) ускорение клина $A = g\,\dfrac{\sin\alpha\cos\alpha}{1-\cos\alpha}$

$$= g\,\frac{\sqrt{1-(4/5)^2}\,(4/5)}{1-(4/5)} = g\,\frac{\frac{3}{25}\ \frac{4}{5}}{\frac{1}{5}} = g\,\frac{3\cdot 4}{25}$$

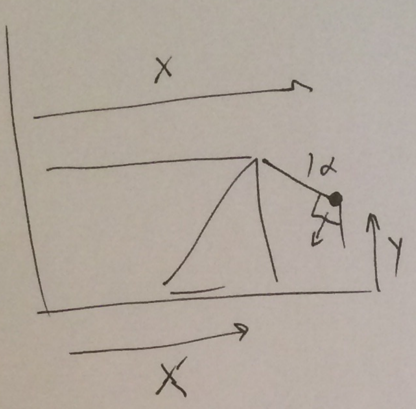
$$= \frac{12}{25}g$$

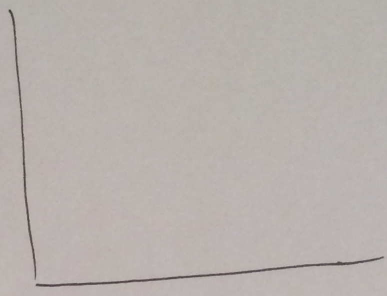
3) $\gamma \equiv \dfrac{m}{M} = \dfrac{\cos\alpha}{(1-\cos\alpha)^2} = \dfrac{4/5}{(1-4/5)^2} = \dfrac{4/5}{1/25} = 20$

4) $\tau = \dfrac{1}{4/5}\sqrt{\dfrac{2H}{g}}$

③

Черновик





$$L = T - V$$

$$L = \frac{1}{2}M\dot{X}^2 + \frac{1}{2}m\dot{x}^2 - mg\left((x - X)\tan\alpha + H\right)$$

$$\frac{x - X}{\cos\alpha} + X = const$$

$$\ddot{x} - \ddot{X} = -\ddot{X}\cos\alpha$$

$$\ddot{x} = \ddot{X}(1 - \cos\alpha)$$

$$L = \frac{1}{2}M\left(\frac{\dot{x}}{1-\cos\alpha}\right)^2 + \frac{1}{2}m\dot{x}^2 - mg\left(\left(x - \frac{x}{1-\cos\alpha}\right)\tan\alpha + H\right)$$

$$\frac{\partial L}{\partial \dot{x}} = \frac{1}{2}M\left(\frac{1}{1-\cos\alpha}\right)^2 2\dot{x} + \frac{1}{2}m\, 2\dot{x}$$

$$\dot{x} = V$$

$$\frac{\partial L}{\partial x} = -mg\left(1 - \frac{1}{1-\cos\alpha}\right)\tan\alpha$$

$$M\left(\frac{1}{1-\cos\alpha}\right)^2\ddot{x} + m\ddot{x} = mg\tan\alpha\left(\frac{1}{1-\cos\alpha} - 1\right)$$

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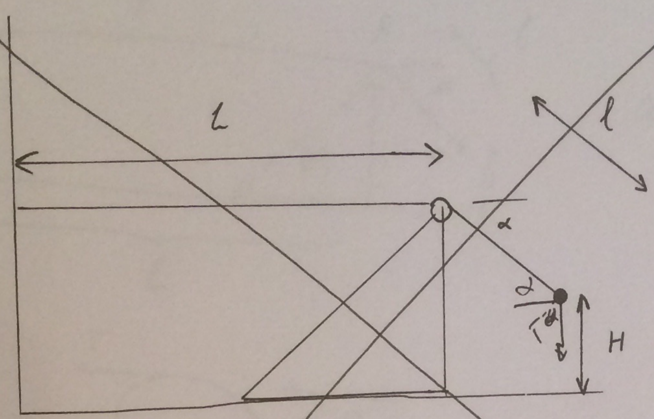
Черновик

~~Чистовик~~ Черновик

1.



1) Уравнения движения системы.

нерастяжимость нити $\quad L + l = constant$

$$\Rightarrow \frac{d}{dt}(L+l) = \frac{d}{dt}(constant)$$

$$\Rightarrow (-\dot{L}) = \dot{l}$$

$$-M\ddot{X} = +T(1-\cos\alpha)$$

$$-m\ddot{x} = T\cos\alpha$$

$$m\ddot{X} = \frac{m\ddot{x}}{1-\cos\alpha}$$

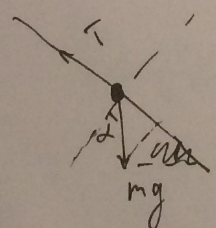
$$\ddot{x} = \frac{mg\tan\alpha\left(\frac{\cos\alpha}{1-\cos\alpha}\right)}{m + M\left(\frac{1}{1-\cos\alpha}\right)^2}$$

mg

9/25

$\ddot{l} =$

~~mg~~ $\quad T\cos\alpha = m\ddot{x}$



~~T = mg sin α~~

~~T cos α~~ =

$$mA = \frac{T\cos\alpha}{1-\cos\alpha}$$

$$MA = \frac{mg\sin\alpha\cos\alpha}{1-\cos\alpha}$$

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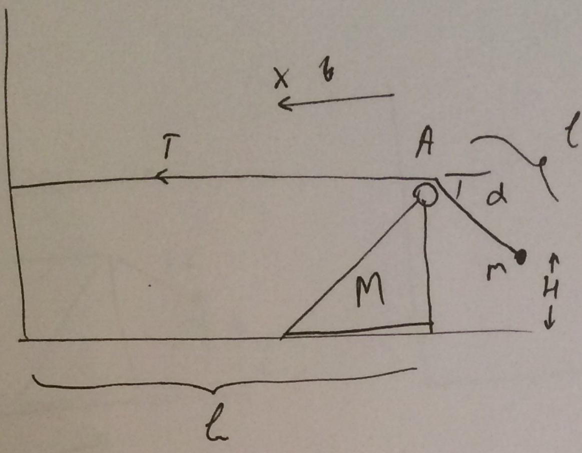
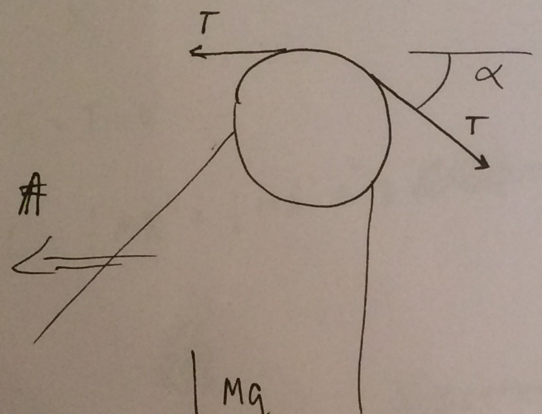
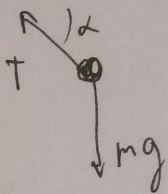
Черновик

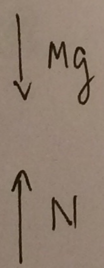


F = ma для всей сис.

$\cancel{a_{cm}}$ $T = (M+m)\, a_{cm}$







$$x_{cm} = \frac{xm + XM}{m+M} \Rightarrow$$

$$\cancel{a_{cm}} \quad T = (M+m)\,\frac{m a_x + M A_x}{m+M}$$

$$= m a_x + M A_x, \quad A_x = A$$

$$l = \sqrt{l_x^2 + l_y^2}$$

$$\dot l = \frac{1}{2}\,\frac{1}{\sqrt{l_x^2 + l_y^2}}\left(2 l_x \dot l_x + 2 l_y \dot l_y\right)$$

$$T - T\cos\alpha = MA$$

$$m a_x = T\cos\alpha$$

$$\ddot l = \frac{1}{2}\,\frac{\left(-\frac{1}{2}\right)}{\left(l_x^2 + l_y^2\right)^{3/2}}\left(2\dot l_x^2 + 2 l_x \ddot l_x + 2 \dot l_y^2 + 2 l_y \ddot l_y\right) \cancel{m a_x = T\cos\alpha =}$$

$$L + l = \text{constant}$$

$$\frac{d}{dt} L = \frac{d}{dt} l$$

6

Черновик

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номер 6

$$C(T) = \frac{5}{2} R \frac{T}{T_0} \qquad C = \frac{\delta Q}{dT}$$

$$\delta Q = \nu C(T)\, dT$$

$$Q_1 = -\frac{5}{2}\frac{R}{T_0}\nu \int_{T_0}^{\frac{1}{2}T_0} T\, dT = -\frac{5}{2}\frac{R}{T_0}\left(\frac{T^2}{2}\right)\Bigg|_{T_0}^{\frac{1}{2}T_0} = -\frac{5}{4}\nu R T_0\left(\frac{1}{4}-1\right)$$

$$= \frac{5}{4}\nu R T_0\left(1-\frac{1}{4}\right)$$

$$= \frac{5}{4}\nu R T_0 \frac{3}{4}$$

$$\delta A = p\, dV \qquad \delta Q = \frac{5}{2} R \frac{T}{T_0} dT$$

$$\delta Q = \delta A + dU$$

$$\frac{5}{2}\nu R \frac{T}{T_0} dT = \overset{\delta A}{\cancel{pdV}} + \frac{3}{2}\nu R\, dT$$

$$\delta A = \frac{5}{2}\frac{\nu R}{T_0} T\, dT - \frac{3}{2}\nu R\, dT$$

$$A = \frac{5}{2}\frac{\nu R}{T_0}\int_{T_0}^{T} T'\, dT' - \frac{3}{2}\nu R \int_{T_0}^{T} dT'$$

$$= \frac{5}{2}\frac{\nu R}{T_0}\left(\frac{T^2}{2} - \frac{T_0^2}{2}\right) - \frac{3}{2}\nu R (T - T_0)$$

$$= \frac{5}{4}\frac{\nu R T^2}{T_0} - \frac{5}{4}\nu R T_0 - \frac{3}{2}\nu R T + \frac{6}{4}\nu R T_0$$

$$= \left(\frac{5}{4}\frac{\nu R}{T_0}\right) T^2 - \left(\frac{3}{2}\nu R\right) T + \frac{1}{4}\nu R T_0$$

$$T_1 = \frac{3}{5} T_0$$

$$A_1 = \frac{\cancel{5}^{1}}{4}\nu R T_0 \frac{9}{\cancel{25}_{5}} - \frac{3}{2}\nu R \frac{3}{5} T_0 + \frac{1}{4}\nu R T_0$$

$$= \frac{9}{20}\nu R T_0 - \frac{9}{10}\nu R T_0 + \frac{1}{4}\nu R T_0$$

$$= \left(\frac{9}{20} - \frac{18}{20} + \frac{5}{20}\right)\nu R T_0$$

$$= -\frac{4}{20}\nu R T_0 = -\frac{1}{5}\nu R T_0$$

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Часть 2

Олимпиада: **Физика, 11 класс (2 часть)**

Шифр: **21202579**

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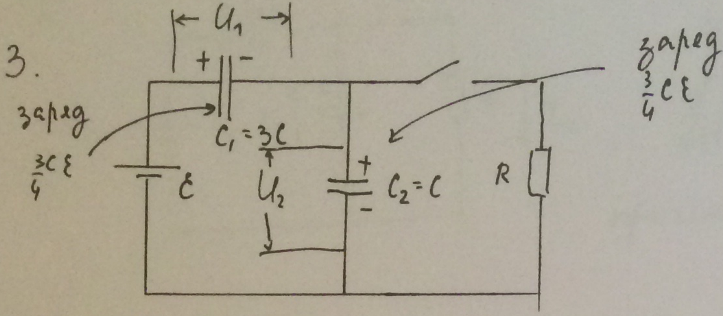
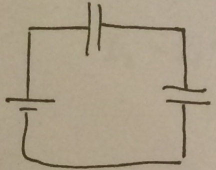
Вариант 2

Чистовик Вариант 11·02

3.



1) Установившийся режим с разомкнутым ключом



$U_1+U_2=\mathcal{E}$

заряды на конд-х равны $U_1C_1=U_2C_2$

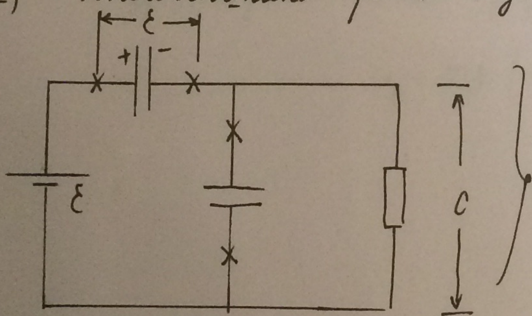
$(\mathcal{E}-U_2)3C=U_2C$

$3\mathcal{E}-3U_2=U_2$

$U_2=\frac{3}{4}\mathcal{E}$ $\quad U_1=\frac{1}{4}\mathcal{E}$

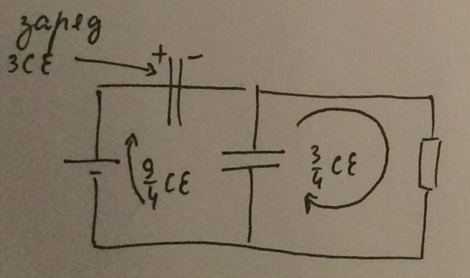
=> ток через резистор сразу после замыкания $I_{11}=\frac{\frac{3}{4}\mathcal{E}}{R}=\frac{3}{4}\frac{\mathcal{E}}{R}$

2) Установившийся режим с замкнутым ключом



ток в цепи не течёт

уравнение для энергии



$$\underbrace{\frac{1}{2}(3C)\left(\frac{1}{4}\mathcal{E}\right)^2+\frac{1}{2}C\left(\frac{3}{4}\mathcal{E}\right)^2}_{\text{было}}+\underbrace{\frac{9}{4}C\mathcal{E}^2}_{\text{раб. источника}}=$$

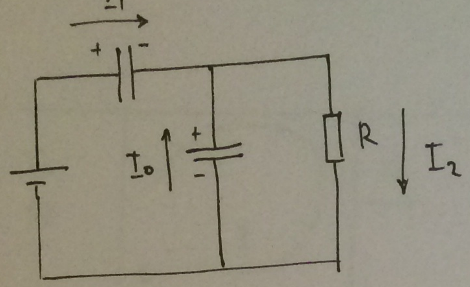
$$=\underbrace{\frac{1}{2}(3C)\mathcal{E}^2}_{\text{стала}}+Q$$

$$\Rightarrow \frac{1}{2}\cdot 3\cdot\frac{1}{16}+\frac{1}{2}\cdot\frac{9}{16}+\frac{9}{4}=\frac{1}{2}\cdot 3+\left(\frac{Q}{C\mathcal{E}^2}\right)$$

$$\Rightarrow Q=\frac{9}{8}C\mathcal{E}^2$$

Чистовик

3)



$$U_1 + U_2 = \mathcal{E}$$

$$\frac{d}{dt}(U_1 + U_2) = 0,$$

причём $I_1 = \frac{d}{dt}(C_1 U_1) = C_1 \frac{dU_1}{dt}$

$I_0 = -\frac{d}{dt}(C_2 U_2) = -C_2 \frac{dU_2}{dt}$

$$\Rightarrow \left.\begin{array}{l} \frac{I_1}{C_1} - \frac{I_0}{C_2} = 0 \\ I_2 = I_1 + I_0 \end{array}\right\} \Rightarrow I_2 = I_0 + \frac{I_0 C_1}{C_2}$$

$$= I_0\left(1 + \frac{C_1}{C_2}\right)$$

$$\Rightarrow U_R = I_0 R\left(1 + \frac{C_1}{C_2}\right)$$

Ответ: 1) ток через резистор сразу после замыкания кл.

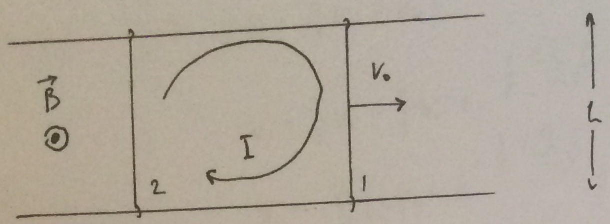
$I_{11} = \frac{3}{4}\frac{\mathcal{E}}{R}$

2) на резисторе выделится $Q = \frac{9}{8} C\mathcal{E}^2$

3) напряжение на резисторе в данный момент

$$U_R = I_0 R\left(1 + \frac{C_1}{C_2}\right) = I_0 R\left(1 + \frac{3}{1}\right) = 4 I_0 R$$

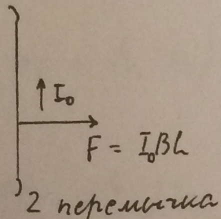
4.



1) $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d\Phi}{dt}$ (по з-ну Фарадея)

$$\frac{d\Phi}{dt} = B(V_1 - V_2)L$$

$$I(R+4R) = B(V_1-V_2)L \xrightarrow[\text{момент}]{\text{в начальный}} 5RI_0 = BV_0L, \quad I_0 = \frac{BV_0L}{5R}$$



$F = ma$ для второй перемычки

$$\frac{m}{2}a_0 = B\left(\frac{BV_0L}{5R}\right)L$$

$$a_0 = \frac{2}{5}\,\frac{B^2L^2V_0}{mR}$$ } искомое ускорение

2) Через продолжительный промежуток времени

$$I = \frac{B(V_1-V_2)L}{5R}$$

$$\frac{m}{2}\dot{V}_2 = BIL \;\Big|\times 2$$

$$m\dot{V}_1 = -BIL$$

$$\Rightarrow \quad m\frac{d}{dt}(V_1-V_2) = -3BIL$$

$$m\frac{d}{dt}(V_1-V_2) = -3BL\,\frac{BL(V_1-V_2)}{5R}$$

$$\int\frac{d(V_1-V_2)}{V_1-V_2} = -3\frac{B^2L^2}{5mR}\int dt$$

$$\underline{V_1 - V_2 = V_0\, e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}} \qquad (1)$$

$$m\frac{d}{dt}(V_1+V_2) = \frac{B^2L^2}{5R}(V_1-V_2)$$

$$\int d(V_1+V_2) = \frac{B^2L^2}{5mR}V_0\int e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}dt$$

$$\underline{V_1 + V_2 = \frac{4V_0}{3} - \frac{V_0}{3}e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}} \qquad (2)$$

Чистовик

$$(1)+(2) \quad \longrightarrow \quad 2V_1 = \frac{4}{3}V_0 + V_0 e^{-\frac{3}{5}\frac{B^2L^2t}{mR}} - \frac{V_0}{3} e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}$$

$$V_1 = \frac{2}{3}V_0 + \frac{1}{3}V_0 e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}$$

$$V_1 = \frac{V_0}{3}\left(2\cancel{V_0} + e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}\right)$$

$$(2)-(1) \quad \longrightarrow \quad 2V_2 = \frac{4}{3}V_0 - \frac{V_0}{3} e^{-\frac{3}{5}\frac{B^2L^2t}{mR}} - V_0 e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}$$

$$V_2 = \frac{2}{3}V_0\left(1 - e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}\right)$$

через продолжительный промежуток времени

$$V_1' = V_2' = \frac{2}{3}V_0$$

3)

$$ds = (V_1 - V_2)\,dt \quad \text{— расстояние между перемычками}$$

$$\Delta s = V_0 \int e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}\,dt$$

$$\Delta s = V_0\left(-\frac{5}{3}\frac{mR}{B^2L^2}\right)\int e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}\,d\left(-\frac{3}{5}\frac{B^2L^2t}{mR}\right)$$

$$\Delta s = -\frac{5}{3}\frac{mRV_0}{B^2L^2} e^{-\frac{3}{5}\frac{B^2L^2t}{mR}} + \frac{5}{3}\frac{mRV_0}{B^2L^2}$$

Ответ:

1) $a_0 = \frac{2}{5}\frac{B^2L^2V_0}{mR}$

2) $V_1' = V_2' = \frac{2}{3}V_0$

3) $\Delta s = \frac{5}{3}\frac{mRV_0}{B^2L^2}$

Черновик

$2\frac{1}{4} = \frac{8}{4} + \frac{1}{4} = \frac{9}{4}$ $\cancel{16\cdot 2}$

$3 + 9 + 9\cdot 8 = 16\cdot 3 + 9\cdot 4$

$3 + 9\cdot 8 = 16\cdot 3 + 9\cdot 3$

$3 + 9\cdot 5 = 16\cdot 3$

$\frac{1}{2}\cdot 3\cdot \frac{1}{16} + \frac{1}{2}\cdot\frac{9}{16} + \frac{9}{4} = \frac{3}{2} + Q$ $\Big|\ \cancel{16}\ 32$

$\cancel{3}\ \ 3 + 9 + 9\cdot 8 = 48 + 32Q$

$12 + 81 - 9 = 48 + 32Q$

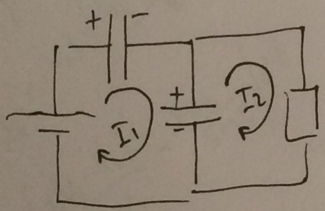
$3 + 81 - 48 = 32Q$

$36 = 32Q$

$Q = \frac{36}{32} C\mathcal{E}^2 = \frac{18}{16} C\mathcal{E}^2 = \frac{9}{8} C\mathcal{E}^2$

$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt}$



$$\begin{cases} \mathcal{E} - \frac{U_1}{C_1} - \frac{U_2}{C_2} = 0 \\ \frac{U_2}{C_2} - \cancel{I_2 R} = 0 \\ \quad U_R \end{cases}$$

$\cancel{\text{}}\ \ I_0 = -\frac{d}{dt}(C_2 U_2)$

$\frac{d}{dt}U_2 = -\frac{I_0}{C_2}$

$\cancel{\text{}}\ \ I_2 - I_1 = I_0$

$\cancel{\frac{d}{dt}}\ \ U_1 + U_2 = \mathcal{E}$

$\frac{d}{dt}(U_1 + U_2) = 0$

$I_1 = \frac{d}{dt}(C_1 U_1)$

$\frac{d}{dt}U_1 = \frac{I_1}{C_1}$

$-\frac{I_0}{C_2} + \frac{I_1}{C_1} = 0$

$I_2 = I_0 + I_1$

$U_R = I_2 R = I_0 + I_0\frac{C_1}{C_2}$

Черновик

$$-(V_1-V_2)+V_1+V_2=\frac{4}{3}V_0-\frac{V_0}{3}e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}-V_0e^{-\frac{3}{5}}$$

$$2V_2=\frac{4}{3}V_0-\frac{V_0}{3}-V_0=0$$

$$\frac{d}{dt}(V_1+V_2)=\cancel{\frac{3BL(V_1-V_2)}{5R}}$$

$$V_1+V_2=e$$

$$m(\dot V_1+\dot V_2)=BL\frac{(V_1-V_2)BL}{5R}$$

$$\frac{d}{dt}(V_1+V_2)=\frac{B^2L^2}{5mR}V_0e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}$$

$$V_1+V_2=\frac{B^2L^2}{5mR}V_0\int e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}dt$$

$$V_1+V_2=V_0+\frac{\cancel{B^2L^2}}{\cancel{5mR}}V_0\left(-\frac{\cancel{5mR}}{3\cancel{B^2L^2}}\right)e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}$$

$$V_1+V_2=V_0-\frac{V_0}{3}e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}$$

$$m(\dot V_1-\dot V_2)=-3BIL$$

$$m\frac{d}{dt}(V_1-V_2)=-3\frac{B(V_1-V_2)L}{5R}$$

$$\ln\frac{V_1-V_2}{V_{1_0}-V_{2_0}}=-\frac{BLt}{5mR}$$

$$\int\frac{d(V_1-V_2)}{V_1-V_2}=-\frac{3BL}{5mR}\int dt$$

$$V_1-V_2=V_0e^{-\frac{3BLt}{5mR}}$$

$$2V_1=-\frac{V_0}{3}e^{-\frac{3BL^2t}{5mR}}+V_0e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}+V_0$$

$$2V_1=V_0\Rightarrow V_1=\frac{V_0}{2}$$

$$V_2=V_0-V_1-\frac{V_0}{3}e^{-3/5\frac{B^2L^2t}{mR}}$$

$$V_2=V_0-\frac{V_0}{2}-0=\frac{V_0}{2}$$

$$\frac{d}{dt}(V_1+V_2)=\frac{B^2L^2}{5mR}V_0^{-\frac{3}{5}\frac{B^2L^2t}{mR}}$$

$$-\frac{1}{3}+2=$$

$$\frac{d}{dt}\left(-\frac{V_0}{3}e^{-\frac{3}{5}\frac{B^2L^2t}{mR}}\right)=-\frac{V_0}{\cancel{3}}\left(-\frac{\cancel{3}}{5}\frac{B^2L^2t}{mR}\right)e^{\cdots}$$

$$\frac{d}{dt}(V_1-V_2)=V_0\left(-\frac{3}{5}\frac{B^2L^2}{mR}\right)e^{\cdots}$$